versione 0

Equazioni differenziali – 0

```
DSolve[{y'[x] == (8 x y[x])/(4 x^2 + 1) + 12 x^2, y[0] == 2},
 y[x], x]
```

$$\left\{\left\{y[x] \to \frac{1}{2}\left(4 + 6x + 16x^2 + 24x^3 - 3\,\text{ArcTan}[2x] - 12x^2\,\text{ArcTan}[2x]\right)\right\}\right\}$$

Funzioni di due variabili, punti critici – 0

```
g[x_, y_] := x^4 - 2 x^2 y + 2 y^2 - 8 y;
f[x_, y_] := g[x, y]; Expand[f[x, y]]; Print[Expand[f[x, y]]];
Print[{Together[D[f[x, y], x]], Together[D[f[x, y], y]]}];
Print[Solve[{D[f[x, y], x] == 0, D[f[x, y], y] == 0}, {x, y}]];
H[x_, y_] = {{D[f[x, y], x, x], D[f[x, y], x, y]}, {D[f[x, y], x, y], D[f[x, y], y, y]}};
H[x, y];
Print[MatrixForm[H[x, y]]];
Print[{
  MatrixForm[H[0, 2]],
  MatrixForm[H[-2, 4]],
  MatrixForm[H[2, 4]]
 }]
```

$x^4 - 8y - 2x^2y + 2y^2$

$\{4(x^3 - xy), -2(4 + x^2 - 2y)\}$

$\{\{y \to 2, x \to 0\}, \{y \to 4, x \to -2\}, \{y \to 4, x \to 2\}\}$

$$\begin{pmatrix} 12x^2 - 4y & -4x \\ -4x & 4 \end{pmatrix}$$

$$\left\{\begin{pmatrix} -8 & 0 \\ 0 & 4 \end{pmatrix}, \begin{pmatrix} 32 & 8 \\ 8 & 4 \end{pmatrix}, \begin{pmatrix} 32 & -8 \\ -8 & 4 \end{pmatrix}\right\}$$

Integrale doppio – 0

```
f[x_, y_] := x/(x^2 + y^2)^2;
Assuming[x ≥ 0 && x ≤ 10 && y < 10 && y > -10,
 Simplify[{∫_5^Sqrt[100-y^2] f[x, y] dx,
   ∫_(-5 Sqrt[3])^(5 Sqrt[3]) ∫_5^Sqrt[100-y^2] f[x, y] dx dy}]]
```

$$\left\{-\frac{1}{200} + \frac{1}{50 + 2y^2}, \frac{1}{60}\left(-3\sqrt{3} + 4\pi\right)\right\}$$

```
f[x_, y_] := x/(x^2 + y^2)^2;
Assuming[-π/3 ≤ t ≤ π/3,
 Simplify[{∫_{5/Cos[t]}^{10} r * f[r Cos[t], r Sin[t]] ⅆr,
   ∫_{-π/3}^{π/3} ∫_{5/Cos[t]}^{10} r * f[r Cos[t], r Sin[t]] ⅆr ⅆt}]]
```

$$\left\{\frac{1}{10}\, \mathrm{Cos}[t]\, (-1 + 2\, \mathrm{Cos}[t]),\ \frac{1}{60}\left(-3\sqrt{3} + 4\pi\right)\right\}$$

Numero complesso – 0

```
Reduce[(z)^3 + 8 I == 0 + 0 I, z]
```

$z == 2\,i \;||\; z == -i - \sqrt{3} \;||\; z == -i + \sqrt{3}$

Matrice, autovalori… – 0

```
a = ( 1    0  -1
     -√3   1   0 );
b = a.Transpose[a]; MatrixForm[b]
```

$$\begin{pmatrix} 2 & -\sqrt{3} \\ -\sqrt{3} & 4 \end{pmatrix}$$

```
Eigenvalues[b]
```

{5, 1}

```
Eigenvectors[b]
```

$$\left\{\left\{-\frac{1}{\sqrt{3}}, 1\right\}, \left\{\sqrt{3}, 1\right\}\right\}$$

```
v2 = %[[1]]/Norm[%[[1]]]; v1 = %[[2]]/Norm[%[[2]]];
m = Transpose[{v1, v2}]; MatrixForm[m]
```

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

```
MatrixForm[Transpose[m].b.m]
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}$$

versione 1

Equazioni differenziali – 1

```
DSolve[{y'[x] == (2 x y[x])/(x^2 + 9) + 3 x^2, y[0] == 8},
 y[x], x]
```

$$\left\{\left\{y[x] \to \frac{1}{9}\left(72 + 243\,x + 8\,x^2 + 27\,x^3 - 729\,\mathrm{ArcTan}\left[\frac{x}{3}\right] - 81\,x^2\,\mathrm{ArcTan}\left[\frac{x}{3}\right]\right)\right\}\right\}$$

Funzioni di due variabili, punti critici – 1

```
g[x_, y_] := x^4 - 2 x^2 y + 2 y^2 - 8 y;
f[x_, y_] := 1/2 g[2 y, x]; Expand[f[x, y]]; Print[Expand[f[x, y]]];
Print[{Together[D[f[x, y], x]], Together[D[f[x, y], y]]}];
Print[Solve[{D[f[x, y], x] == 0, D[f[x, y], y] == 0}, {x, y}]];
H[x_, y_] = {{D[f[x, y], x, x], D[f[x, y], x, y]}, {D[f[x, y], x, y], D[f[x, y], y, y]}};
H[x, y];
Print[MatrixForm[H[x, y]]];
Print[{
  MatrixForm[H[2, 0]],
  MatrixForm[H[4, -1]],
  MatrixForm[H[4, 1]]
 }]
```

$$-4\,x + x^2 - 4\,x\,y^2 + 8\,y^4$$

$$\left\{2\left(-2 + x - 2\,y^2\right), -8\left(x\,y - 4\,y^3\right)\right\}$$

$$\{\{x \to 2, y \to 0\}, \{x \to 4, y \to -1\}, \{x \to 4, y \to 1\}\}$$

$$\begin{pmatrix} 2 & -8\,y \\ -8\,y & \frac{1}{2}\left(-16\,x + 192\,y^2\right) \end{pmatrix}$$

$$\left\{\begin{pmatrix} 2 & 0 \\ 0 & -16 \end{pmatrix}, \begin{pmatrix} 2 & 8 \\ 8 & 64 \end{pmatrix}, \begin{pmatrix} 2 & -8 \\ -8 & 64 \end{pmatrix}\right\}$$

Integrale doppio – 1

In[5]:=
```
f[x_, y_] := y/(x^2 + y^2)^2;
Assuming[x ≥ 0 && y ≤ 4 && x < 4 && x > -4,
 Simplify[{Integrate[f[x, y], {y, 2, Sqrt[16 - x^2]}],
   Integrate[f[x, y], {x, -2 Sqrt[3], 2 Sqrt[3]}, {y, 2, Sqrt[16 - x^2]}]}]]
```

Out[6]= $\left\{-\frac{1}{32} + \frac{1}{8 + 2\,x^2}, -\frac{\sqrt{3}}{8} + \frac{\pi}{6}\right\}$

In[9]:= $f[x\_, y\_] := \frac{y}{(x^2 + y^2)^2};$

$\text{Assuming}\left[\frac{\pi}{6} \le t \le \frac{5\pi}{6},\ \text{Simplify}\left[\left\{\int_{\frac{2}{\text{Sin}[t]}}^{4} r * f[r\,\text{Cos}[t], r\,\text{Sin}[t]]\,dr,\ \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\int_{\frac{2}{\text{Sin}[t]}}^{4} r * f[r\,\text{Cos}[t], r\,\text{Sin}[t]]\,dr\,dt\right\}\right]\right]$

Out[10]= $\left\{\frac{1}{4}\,\text{Sin}[t]\,(-1 + 2\,\text{Sin}[t]),\ -\frac{\sqrt{3}}{8} + \frac{\pi}{6}\right\}$

Numero complesso – 1

```
Reduce[(z)^3 - 27 I == 0 + 0 I, z]
```

$z == -3\,i \;||\; z == \frac{3}{2}\left(i - \sqrt{3}\right) \;||\; z == \frac{3}{2}\left(i + \sqrt{3}\right)$

Matrice, autovalori… – 1

$a = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 0 & -\sqrt{3} \end{pmatrix};$

```
b = a.Transpose[a]; MatrixForm[b]
```

$\begin{pmatrix} 5 & \sqrt{3} \\ \sqrt{3} & 3 \end{pmatrix}$

```
Eigenvalues[b]
```

{6, 2}

```
Eigenvectors[b]
```

$\left\{\left\{\sqrt{3}, 1\right\}, \left\{-\frac{1}{\sqrt{3}}, 1\right\}\right\}$

$v2 = \frac{\%[[1]]}{\text{Norm}[\%[[1]]]};\ v1 = \frac{\%[[2]]}{\text{Norm}[\%[[2]]]};$

```
m = Transpose[{v1, v2}]; MatrixForm[m]
```

$\begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

```
MatrixForm[Transpose[m].b.m]
```

$\begin{pmatrix} 2 & 0 \\ 0 & 6 \end{pmatrix}$