versione 0

Equazioni differenziali – 0

```
Expand[DSolve[{y''[x] + 3 y'[x] == 12 e^x + 18 x + 21,
    y[0] == 1, y'[0] == 5},
  y[x], x]]
```

$$\{\{y[x] \to -3 + e^{-3x} + 3e^{x} + 5x + 3x^{2}\}\}$$

Funzioni di due variabili, punti critici – 0

```
f[x_, y_] := (x^2 - y^2) Log[x]
Expand[f[x, y]]
```

$$x^2 \text{Log}[x] - y^2 \text{Log}[x]$$

```
grad = Simplify[{Together[D[f[x, y], x]], Together[D[f[x, y], y]]}]
```

$$\left\{x - \frac{y^2}{x} + 2x \text{Log}[x],\ -2y \text{Log}[x]\right\}$$

```
Solve[grad == {0, 0}, {x, y}]
```

Solve::ifun : Inverse functions are being used by Solve, so some
solutions may not be found; use Reduce for complete solution information. >>

$$\{\{x \to 1, y \to -1\}, \{x \to 1, y \to 1\}\}$$

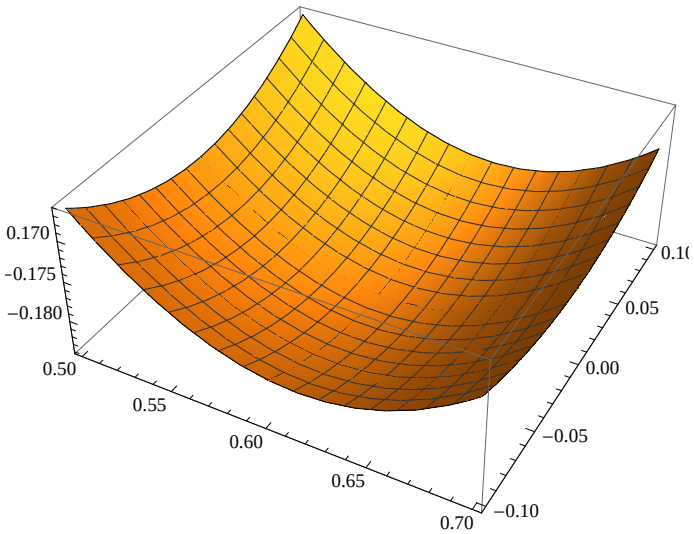
```
H[x_, y_] = {{D[f[x, y], x, x], D[f[x, y], x, y]}, {D[f[x, y], x, y], D[f[x, y], y, y]}};
H[x, y];
Print[MatrixForm[H[x, y]]];
```

$$\begin{pmatrix} 4 - \frac{x^2 - y^2}{x^2} + 2 \text{Log}[x] & -\frac{2y}{x} \\ -\frac{2y}{x} & -2 \text{Log}[x] \end{pmatrix}$$

```
Print[MatrixForm[H[e^(-1/2), 0]]];
```

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

```
Plot3D[f[x, y], {x, .5, .7}, {y, -.1, .1}]
```



Integrale doppio – 0

```
f[x_, y_] := (x^2 - y^2)/(x^2 + y^2);
g[r_, t_] := r f[r Cos[t], r Sin[t]];
Print[TrigReduce[g[r, t]]];
Print[∫_(2/(Cos[t]+Sin[t]))^2 g[r, t] dr];
∫_(2/(Cos[t]+Sin[t]))^2 g[r, t] dr;
Print[∫_(π/4)^(π/2) ∫_(2/(Cos[t]+Sin[t]))^2 g[r, t] dr dt]
```

```
r Cos[2 t]
```

$$2\cos[2t] - \frac{2\cos[2t]}{(\cos[t]+\sin[t])^2}$$

```
-1 + Log[2]
```

Numeri complessi – 0

$$z = \frac{\left(1 + i\sqrt{3}\right)^7}{\sqrt{3} + i};$$ **Print[{Re[z], Im[z], Abs[z], Arg[z]}]**

$$\left\{32\sqrt{3},\ 32,\ 64,\ \frac{\pi}{6}\right\}$$

Matrici, autovalori – 0

In[1]:=
$$\mathbf{a} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 7 & \sqrt{3} \\ 0 & \sqrt{3} & 9 \end{pmatrix};$$ **Eigenvalues[a]**

Out[1]= {10, 6, 2}

In[2]:= **Eigenvectors[a]**

$$\left\{\{1, 0, 0\},\ \left\{0,\ \frac{1}{\sqrt{3}},\ 1\right\},\ \left\{0,\ -\sqrt{3},\ 1\right\}\right\}$$

In[13]:=
$$\mathbf{m} = \mathbf{Orthogonalize}\left[\left\{\{1, 0, 0\},\ \left\{0,\ \frac{1}{\sqrt{3}},\ 1\right\},\ \left\{0,\ -\sqrt{3},\ 1\right\}\right\}\right];$$ **MatrixForm[m]**

Out[13]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

```
In[16]:= {Re[Eigenvalues[m]],
          Im[Eigenvalues[m]],
          Arg[Eigenvalues[m]]}
```

Out[16]= $\left\{\left\{1, \frac{1}{2}, \frac{1}{2}\right\}, \left\{0, \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}\right\}, \left\{0, \frac{\pi}{3}, -\frac{\pi}{3}\right\}\right\}$

In[17]:=
```
m1 = Transpose[m]; MatrixForm[m1]
```

Out[17]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

In[18]:=
```
MatrixForm[m.a.m1]
```

Out[18]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

versione 1

Equazioni differenziali – 1

```
Expand[DSolve[{y''[x] + 2 y'[x] == 18 e^x + 12 x + 8,
    y[0] == 1, y'[0] == 5},
  y[x], x]]
```

$$\left\{\left\{y[x] \to -6 + e^{-2x} + 6\,e^{x} + x + 3\,x^{2}\right\}\right\}$$

Funzioni di due variabili, punti critici – 1

```
f[x_, y_] := (x^2 - y^2) Log[y]
Expand[f[x, y]]
```

$$x^2 \,\text{Log}[y] - y^2 \,\text{Log}[y]$$

```
grad = Simplify[{Together[D[f[x, y], x]], Together[D[f[x, y], y]]}]
```

$$\left\{2\,x\,\text{Log}[y],\ \frac{x^2}{y} - y - 2\,y\,\text{Log}[y]\right\}$$

```
Solve[grad == {0, 0}, {x, y}]
```

Solve::incnst :
Inconsistent or redundant transcendental equation. After reduction, the bad equation is 1 – y == 0. ≫

$$\left\{\{x \to -1,\ y \to 1\},\ \left\{x \to 0,\ y \to \frac{1}{\sqrt{e}}\right\},\ \{x \to 1,\ y \to 1\}\right\}$$

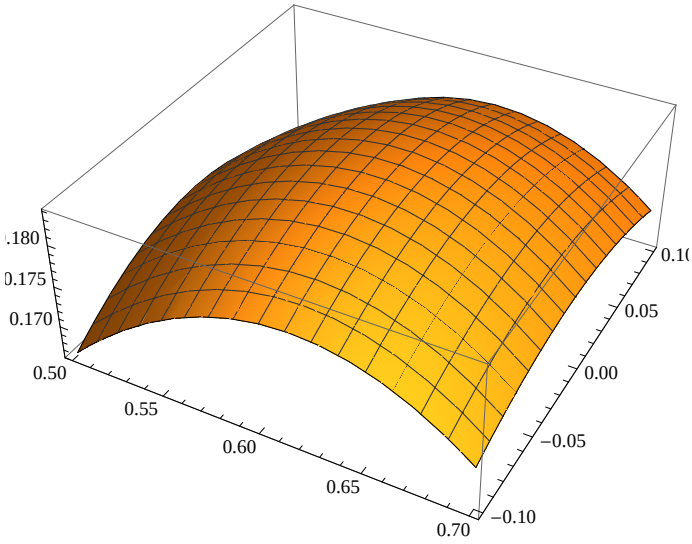
```
H[x_, y_] = {{D[f[x, y], x, x], D[f[x, y], x, y]}, {D[f[x, y], x, y], D[f[x, y], y, y]}};
H[x, y];
Print[MatrixForm[H[x, y]]];
```

$$\begin{pmatrix} 2\,\text{Log}[y] & \frac{2x}{y} \\ \frac{2x}{y} & -4 - \frac{x^2 - y^2}{y^2} - 2\,\text{Log}[y] \end{pmatrix}$$

```
Print[MatrixForm[H[0, e^(-1/2)]]];
```

$$\begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$$

```
Plot3D[f[x, y], {y, .5, .7}, {x, -.1, .1}]
```



Integrale doppio – 1

```
f[x_, y_] := (x^2 - y^2)/(x^2 + y^2);
g[r_, t_] := r f[r Cos[t], r Sin[t]];
Print[TrigReduce[g[r, t]]];
Print[∫_(3/(Cos[t]+Sin[t]))^3 g[r, t] dr];
∫_(3/(Cos[t]+Sin[t]))^3 g[r, t] dr;
Print[∫_(π/4)^(π/2) ∫_(2/(Cos[t]+Sin[t]))^3 g[r, t] dr dt]
```

$$r \cos[2t]$$

$$\frac{9 \cos[t] \cos[2t] \sin[t]}{(\cos[t] + \sin[t])^2}$$

$$-\frac{9}{4} + \log[2]$$

Numeri complessi – 0

```
In[32]:= z = (Sqrt[3] + i)^8/(-1 + i Sqrt[3]); Print[{Re[z], Im[z], Abs[z], Arg[z]}]
```

$$\left\{-64,\ 64\sqrt{3},\ 128,\ \frac{2\pi}{3}\right\}$$

Matrici, autovalori – 1

```
In[19]:= a = {{5, 0, -2}, {0, 5, 0}, {-2, 0, 5}}; Eigenvalues[a]
Out[19]= {7, 5, 3}
```

In[20]:=
```
Eigenvectors[a]
```

Out[20]=
```
{{-1, 0, 1}, {0, 1, 0}, {1, 0, 1}}
```

In[27]:=
```
m = Orthogonalize[{{1, 0, -1}, {0, 1, 0}, {1, 0, 1}}]; MatrixForm[m]
```

Out[27]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

In[28]:=
```
{Re[Eigenvalues[m]],
 Im[Eigenvalues[m]],
 Arg[Eigenvalues[m]]}
```

Out[28]=
$$\left\{\left\{1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}, \left\{0, \frac{\pi}{4}, -\frac{\pi}{4}\right\}\right\}$$

In[29]:=
```
m1 = Transpose[m]; MatrixForm[m1]
```

Out[29]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

In[31]:=
```
Simplify[MatrixForm[m.a.m1]]
```

Out[31]//MatrixForm=

$$\begin{pmatrix} 7 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

versione 2

Equazioni differenziali – 2

```
Simplify[DSolve[{y''[x] + 4 y[x] == 10 ℯ^x + 8 x + 8,
    y[0] == 1, y'[0] == 5},
  y[x], x]]
```

$$\left\{\left\{y[x] \to \frac{1}{2}\left(4\left(1 + e^{x} + x\right) - 6\,\text{Cos}[2x] + \text{Sin}[2x]\right)\right\}\right\}$$

Funzioni di due variabili, punti critici – 2

```
f[x_, y_] := (x^2 - 4 y^2) Log[y]
Expand[f[x, y]]
```

$$x^2\,\text{Log}[y] - 4y^2\,\text{Log}[y]$$

```
grad = Simplify[{Together[D[f[x, y], x]], Together[D[f[x, y], y]]}]
```

$$\left\{2x\,\text{Log}[y], \frac{x^2}{y} - 4y - 8y\,\text{Log}[y]\right\}$$

```
Solve[grad == {0, 0}, {x, y}]
```

Solve::incnst :
Inconsistent or redundant transcendental equation. After reduction, the bad equation is 1 – y == 0. ≫

$$\left\{\{x \to -2, y \to 1\}, \left\{x \to 0, y \to \frac{1}{\sqrt{e}}\right\}, \{x \to 2, y \to 1\}\right\}$$

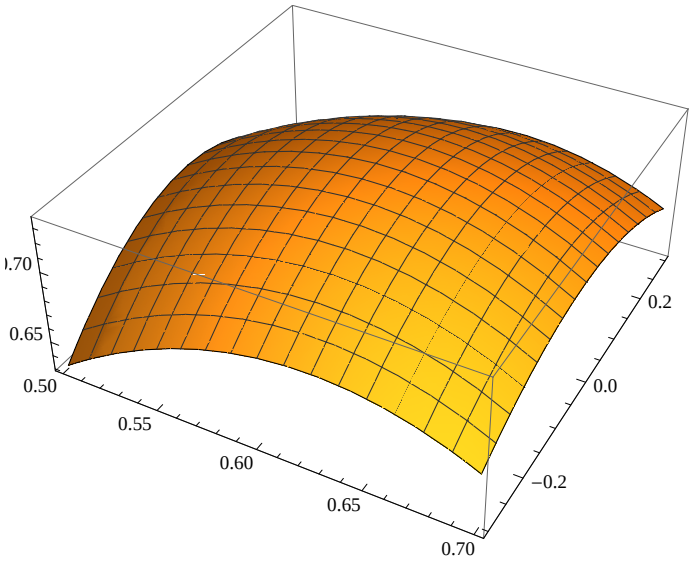
```
H[x_, y_] = {{D[f[x, y], x, x], D[f[x, y], x, y]}, {D[f[x, y], x, y], D[f[x, y], y, y]}};
H[x, y];
Print[MatrixForm[H[x, y]]];
```

$$\begin{pmatrix} 2\,\text{Log}[y] & \frac{2x}{y} \\ \frac{2x}{y} & -4-\frac{x^2-y^2}{y^2}-2\,\text{Log}[y] \end{pmatrix}$$

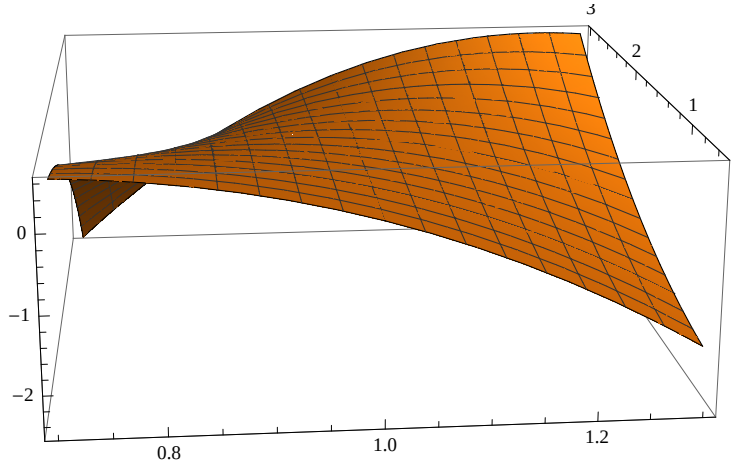
```
Print[MatrixForm[H[0, e^(-1/2)]]];
```

$$\begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$$

```
Plot3D[f[x, y], {y, .5, .7}, {x, -.3, .3}]
```




```
Plot3D[f[x, y], {y, .7, 1.3}, {x, .5, 3}]
```



Integrale doppio – 2

```
f[x_, y_] := (x - y)/(x^2 + y^2);
g[r_, t_] := Simplify[r f[r Cos[t], r Sin[t]]];
Print[g[r, t]];
Print[Simplify[∫_(1/(Cos[t]+Sin[t]))^3 g[r, t] dr]];
Print[Simplify[∫_(π/4)^(π/2) ∫_(1/(Cos[t]+Sin[t]))^3 g[r, t] dr dt]]
```

$\text{Cos}[t] - \text{Sin}[t]$

$(\text{Cos}[t] - \text{Sin}[t])\left(3 - \frac{1}{\text{Cos}[t] + \text{Sin}[t]}\right)$

$\frac{1}{2}\left(6 - 6\sqrt{2} + \text{Log}[2]\right)$

Numeri complessi – 2

$z = \frac{\left(\sqrt{3} - i\right)^{11}}{1 + i\sqrt{3}}$; Print[{Re[z], Im[z], Abs[z], Arg[z]}]

$\left\{512\sqrt{3}, -512, 1024, -\frac{\pi}{6}\right\}$

Matrici, autovalori – 2

In[33]:=
```
a = {{7, 0, -Sqrt[3]}, {0, 6, 0}, {-Sqrt[3], 0, 5}}; Eigenvalues[a]
```

Out[33]= $\{8, 6, 4\}$

In[34]:=
```
Eigenvectors[a]
```

Out[34]= $\left\{\left\{-\sqrt{3}, 0, 1\right\}, \{0, 1, 0\}, \left\{\frac{1}{\sqrt{3}}, 0, 1\right\}\right\}$

In[36]:=
```
m = Orthogonalize[{{Sqrt[3], 0, -1}, {0, 1, 0}, {1/Sqrt[3], 0, 1}}]; MatrixForm[m]
```

Out[36]//MatrixForm=

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{pmatrix}$$

In[37]:=
```
{Re[Eigenvalues[m]],
 Im[Eigenvalues[m]],
 Arg[Eigenvalues[m]]}
```

Out[37]= $\left\{\left\{1, \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right\}, \left\{0, \frac{1}{2}, -\frac{1}{2}\right\}, \left\{0, \frac{\pi}{6}, -\frac{\pi}{6}\right\}\right\}$

In[38]:=
```
m1 = Transpose[m]; MatrixForm[m1]
```

Out[38]//MatrixForm=

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{pmatrix}$$

In[39]:=
```
Simplify[MatrixForm[m.a.m1]]
```

Out[39]//MatrixForm=

$$\begin{pmatrix} 8 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

versione 3

Equazioni differenziali – 3

```
Simplify[DSolve[{y''[x] + 9 y[x] == 10 e^-x + 27 x + 18,
    y[0] == 10, y'[0] == 0},
  y[x], x]]
```

$$\left\{\left\{y[x] \to 2 + e^{-x} + 3x + 7\,\text{Cos}[3x] - \frac{2}{3}\,\text{Sin}[3x]\right\}\right\}$$

Funzioni di due variabili, punti critici – 3

```
f[x_, y_] := (x^2 - 4 y^2) Log[x]
Expand[f[x, y]]
```

$$x^2\,\text{Log}[x] - 4y^2\,\text{Log}[x]$$

```
grad = Simplify[{Together[D[f[x, y], x]], Together[D[f[x, y], y]]}]
```

$$\left\{x - \frac{4y^2}{x} + 2x\,\text{Log}[x],\ -8y\,\text{Log}[x]\right\}$$

```
Solve[grad == {0, 0}, {x, y}, Reals]
```

$$\left\{\left\{x \to 1,\ y \to -\frac{1}{2}\right\},\ \left\{x \to 1,\ y \to \frac{1}{2}\right\},\ \left\{x \to \frac{1}{\sqrt{e}},\ y \to 0\right\}\right\}$$

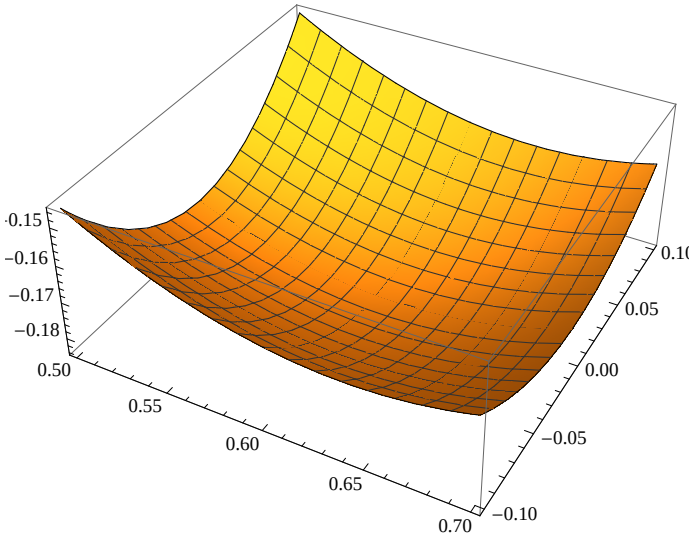
```
H[x_, y_] = {{D[f[x, y], x, x], D[f[x, y], x, y]}, {D[f[x, y], x, y], D[f[x, y], y, y]}};
H[x, y];
Print[MatrixForm[H[x, y]]];
```

$$\begin{pmatrix} 4 - \frac{x^2 - 4y^2}{x^2} + 2\,\text{Log}[x] & -\frac{8y}{x} \\ -\frac{8y}{x} & -8\,\text{Log}[x] \end{pmatrix}$$

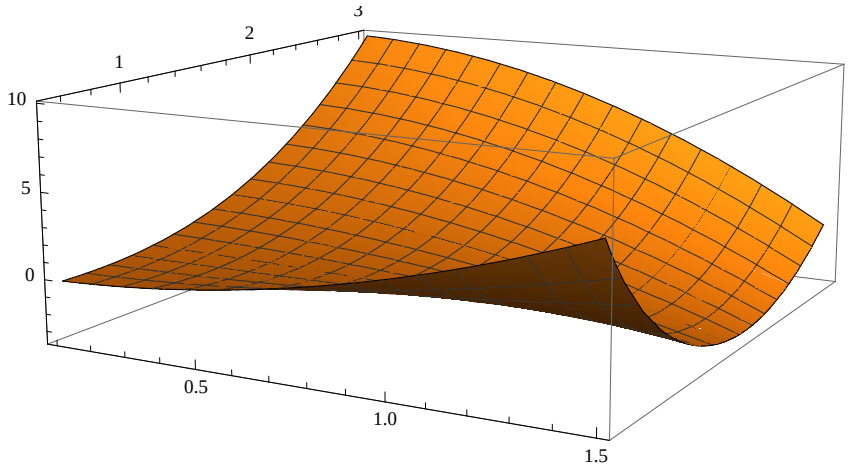
```
Print[MatrixForm[H[e^(-1/2), 0]]];
```

$$\begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$$

```
Plot3D[f[x, y], {x, .5, .7}, {y, -.1, .1}]
```




```
Plot3D[f[x, y], {y, .1, 1.5}, {x, .5, 3}]
```



Integrale doppio – 3

$$f[x\_, y\_] := \frac{x - y}{x^2 + y^2};$$

```
g[r_, t_] := Simplify[r f[r Cos[t], r Sin[t]]];
Print[g[r, t]];
```

$$\text{Print}\left[\text{Simplify}\left[\int_{\frac{1}{\text{Cos}[t]+\text{Sin}[t]}}^{2} g[r, t]\, dr\right]\right];$$

$$\text{Print}\left[\text{Simplify}\left[\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_{\frac{1}{\text{Cos}[t]+\text{Sin}[t]}}^{2} g[r, t]\, dr\, dt\right]\right]$$

$$\text{Cos}[t] - \text{Sin}[t]$$

$$(\text{Cos}[t] - \text{Sin}[t])\left(2 - \frac{1}{\text{Cos}[t] + \text{Sin}[t]}\right)$$

$$1 - \sqrt{3} - \frac{1}{2}\text{Log}\left[4 - 2\sqrt{3}\right]$$

Numeri complessi – 3

$$z = \frac{\left(1 - i\sqrt{3}\right)^9}{\sqrt{3} - i}; \text{Print}[\{\text{Re}[z], \text{Im}[z], \text{Abs}[z], \text{Arg}[z]\}]$$

$$\left\{-128\sqrt{3}, -128, 256, -\frac{5\pi}{6}\right\}$$

Matrici, autovalori – 3

In[40]:=
```
a = {{2, 1, 0}, {1, 2, 0}, {0, 0, 9}}; Eigenvalues[a]
```

Out[40]=
```
{9, 3, 1}
```

In[41]:=
```
Eigenvectors[a]
```

Out[41]=
```
{{0, 0, 1}, {1, 1, 0}, {-1, 1, 0}}
```

In[43]:=
```
m = Orthogonalize[{{1, 1, 0}, {-1, 1, 0}, {0, 0, 1}}]; MatrixForm[m]
```

Out[43]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[44]:=
```
{Re[Eigenvalues[m]],
 Im[Eigenvalues[m]],
 Arg[Eigenvalues[m]]}
```

Out[44]=
$$\left\{\left\{1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\}, \left\{0, \frac{\pi}{4}, -\frac{\pi}{4}\right\}\right\}$$

In[45]:=
```
m1 = Transpose[m]; MatrixForm[m1]
```

Out[45]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[46]:=
```
Simplify[MatrixForm[m.a.m1]]
```

Out[46]//MatrixForm=

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$