versione 0

Equazioni differenziali – 0

```
Simplify[DSolve[{
    y''[x] + 6 y'[x] + 10 y[x] == 39 Sin[x],
    y[0] == -2, y'[0] == 0
  }, y[x], x]]
```

$$\{\{y[x] \to -2\,\text{Cos}[x] + 3\,\left(1 - e^{-3\,x}\right)\,\text{Sin}[x]\}\}$$

Funzioni di due variabili, punti critici – 0

```
g[x_, y_] := (4 y + x^2) e^(x+y);
f[x_, y_] := g[x, y]
Expand[f[x, y]]
```

$$e^{x+y}\,x^2 + 4\,e^{x+y}\,y$$

```
grad = Simplify[{Together[D[f[x, y], x]], Together[D[f[x, y], y]]}]
```

$$\left\{e^{x+y}\,\left(2\,x + x^2 + 4\,y\right),\ e^{x+y}\,\left(4 + x^2 + 4\,y\right)\right\}$$

```
Solve[grad == {0, 0}, {x, y}]
```

Solve::ifun : Inverse functions are being used by Solve, so some
solutions may not be found; use Reduce for complete solution information. ≫

$$\{\{x \to 2,\ y \to -2\}\}$$

```
H[x_, y_] = {{D[f[x, y], x, x], D[f[x, y], x, y]}, {D[f[x, y], x, y], D[f[x, y], y, y]}};
Print[Simplify[MatrixForm[H[x, y]]]];
Print[{MatrixForm[H[2, -2]]}];
```

$$\begin{pmatrix} e^{x+y}\,\left(2 + 4\,x + x^2 + 4\,y\right) & e^{x+y}\,\left(4 + 2\,x + x^2 + 4\,y\right) \\ e^{x+y}\,\left(4 + 2\,x + x^2 + 4\,y\right) & e^{x+y}\,\left(8 + x^2 + 4\,y\right) \end{pmatrix}$$

$$\left\{\begin{pmatrix} 6 & 4 \\ 4 & 4 \end{pmatrix}\right\}$$

Integrale doppio – 0

```
f[x_, y_] := x^3/(1 + y^2);
Print[Expand[∫_√y^(y^(1/4)) f[x, y] dx]];
Print[∫_0^1 ∫_√y^(y^(1/4)) f[x, y] dx dy]
```

$$\frac{y}{4\,\left(1 + y^2\right)} - \frac{y^2}{4\,\left(1 + y^2\right)}$$

$$\frac{1}{16}\,\left(-4 + \pi + \text{Log}[4]\right)$$

Numeri complessi – 0

In[29]:=
```
a = Refine[Im[Expand[(x + y I)^2 - (x - y I)]], {x ∈ Reals, y ∈ Reals}];
Print[a]
b = Refine[Re[Expand[(x + y I)^2 - (x - y I)]], {x ∈ Reals, y ∈ Reals}];
Print[b]
Solve[{a == 0, b == 0}, {x, y}]
```

$y + 2 x y$

$-x + x^2 - y^2$

Out[33]= $\left\{\{x \to 0, y \to 0\}, \{x \to 1, y \to 0\}, \left\{y \to -\frac{\sqrt{3}}{2}, x \to -\frac{1}{2}\right\}, \left\{y \to \frac{\sqrt{3}}{2}, x \to -\frac{1}{2}\right\}\right\}$

Matrici, autovalori – 0

```
a = {{3, 0, 0}, {-1, 3, 6}, {k, 0, -3}}; Print[MatrixForm[a]];
Print[Eigenvalues[a]];
Print[Eigenvectors[a]]; Clear[b];
Print[MatrixForm[a - 3 IdentityMatrix[3]]];
```

$$\begin{pmatrix} 3 & 0 & 0 \\ -1 & 3 & 6 \\ k & 0 & -3 \end{pmatrix}$$

$\{-3, 3, 3\}$

$\{\{0, -1, 1\}, \{0, 1, 0\}, \{0, 0, 0\}\}$

$$\begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 6 \\ k & 0 & -6 \end{pmatrix}$$

versione 1

Equazioni differenziali – 1

```
Simplify[DSolve[{
    y''[x] + 4 y'[x] + 5 y[x] == 16 Cos[x],
    y[0] == -2, y'[0] == 0
  }, y[x], x]]
```

$\{\{y[x] \to 2 e^{-2x} \left(\left(-2 + e^{2x}\right) \text{Cos}[x] + \left(-5 + e^{2x}\right) \text{Sin}[x]\right)\}\}$

Funzioni di due variabili, punti critici – 1

```
g[x_, y_] := (4 y + x^2) e^(x+y);
f[x_, y_] := g[y, x]
Expand[f[x, y]]
```

$4 e^{x+y} x + e^{x+y} y^2$

```
grad = Simplify[{Together[D[f[x, y], x]], Together[D[f[x, y], y]]}]
```

$\left\{e^{x+y} \left(4 + 4 x + y^2\right), e^{x+y} (4 x + y (2 + y))\right\}$

```
Solve[grad == {0, 0}, {x, y}]
```

Solve::ifun : Inverse functions are being used by Solve, so some
solutions may not be found; use Reduce for complete solution information. >>

$\{\{x \to -2, y \to 2\}\}$

```
H[x_, y_] = {{D[f[x, y], x, x], D[f[x, y], x, y]}, {D[f[x, y], x, y], D[f[x, y], y, y]}};
Print[Simplify[MatrixForm[H[x, y]]]];
Print[{MatrixForm[H[-2, 2]]}];
```

$$\begin{pmatrix} e^{x+y}\left(8+4x+y^2\right) & e^{x+y}\left(4+4x+2y+y^2\right) \\ e^{x+y}\left(4+4x+2y+y^2\right) & e^{x+y}\left(2+4x+4y+y^2\right) \end{pmatrix}$$

$$\left\{\begin{pmatrix} 4 & 4 \\ 4 & 6 \end{pmatrix}\right\}$$

Integrale doppio – 1

```
f[x_, y_] := 64 x^3 / (1 + y^2);

Print[Expand[∫_(1/2 √y)^(∜y) f[x, y] dx]];

Print[∫_0^16 ∫_(1/2 √y)^(∜y) f[x, y] dx dy]
```

$$\frac{16\,y}{1+y^2}-\frac{y^2}{1+y^2}$$

```
ArcTan[16] + 8 (-2 + Log[257])
```

Numeri complessi – 1

In[44]:=
```
a = Refine[Im[Expand[(x + y I)^3 + (x - y I)]], {x ∈ Reals, y ∈ Reals}];
Print[a]
b = Refine[Re[Expand[(x + y I)^3 + (x - y I)]], {x ∈ Reals, y ∈ Reals}];
Print[b]
Solve[{a == 0, b == 0}, {x, y}]
```

$$-y+3x^2y-y^3$$

$$x+x^3-3xy^2$$

Out[48]=
$$\left\{\{x\to 0, y\to 0\}, \{x\to -i, y\to 0\}, \{x\to i, y\to 0\}, \left\{x\to -\frac{1}{\sqrt{2}}, y\to -\frac{1}{\sqrt{2}}\right\}, \left\{x\to -\frac{1}{\sqrt{2}}, y\to \frac{1}{\sqrt{2}}\right\}, \left\{x\to \frac{1}{\sqrt{2}}, y\to -\frac{1}{\sqrt{2}}\right\}, \left\{x\to \frac{1}{\sqrt{2}}, y\to \frac{1}{\sqrt{2}}\right\}, \{y\to -i, x\to 0\}, \{y\to i, x\to 0\}\right\}$$

Matrici, autovalori – 1

In[92]:=
```
a = {{2, 0, 0}, {-3, 2, 4}, {k, 0, -2}}; Print[MatrixForm[a]];
Print[Eigenvalues[a]];
Print[Eigenvectors[a]]; Clear[b];
Print[MatrixForm[a - 2 IdentityMatrix[3]]];
```

$$\begin{pmatrix} 2 & 0 & 0 \\ -3 & 2 & 4 \\ k & 0 & -2 \end{pmatrix}$$

```
{-2, 2, 2}
```

```
{{0, -1, 1}, {0, 1, 0}, {0, 0, 0}}
```

$$\begin{pmatrix} 0 & 0 & 0 \\ -3 & 0 & 4 \\ k & 0 & -4 \end{pmatrix}$$

versione 2

Equazioni differenziali – 2

```
Simplify[DSolve[{
    y''[x] - 4 y'[x] + 29 y[x] == 200 Sin[x],
    y[0] == -10, y'[0] == 10
   }, y[x], x]]
```

$$\{\{y[x] \to \text{Cos}[x] - 11\, e^{2x}\, \text{Cos}[5x] + 7\, \text{Sin}[x] + 5\, e^{2x}\, \text{Sin}[5x]\}\}$$

Funzioni di due variabili, punti critici – 2

```
g[x_, y_] := (4 y + x^2) e^(x+y);
f[x_, y_] := g[x, -y]
Expand[f[x, y]]
```

$$e^{x-y}\, x^2 - 4\, e^{x-y}\, y$$

```
grad = Simplify[{Together[D[f[x, y], x]], Together[D[f[x, y], y]]}]
```

$$\left\{e^{x-y}\left(2x + x^2 - 4y\right), -e^{x-y}\left(4 + x^2 - 4y\right)\right\}$$

```
Solve[grad == {0, 0}, {x, y}]
```

Solve::ifun : Inverse functions are being used by Solve, so some
solutions may not be found; use Reduce for complete solution information. >>

```
{{x → 2, y → 2}}
```

```
H[x_, y_] = {{D[f[x, y], x, x], D[f[x, y], x, y]}, {D[f[x, y], x, y], D[f[x, y], y, y]}};
Print[Simplify[MatrixForm[H[x, y]]]];
Print[{MatrixForm[H[2, 2]]}];
```

$$\begin{pmatrix} e^{x-y}\left(2 + 4x + x^2 - 4y\right) & -e^{x-y}\left(4 + 2x + x^2 - 4y\right) \\ -e^{x-y}\left(4 + 2x + x^2 - 4y\right) & e^{x-y}\left(8 + x^2 - 4y\right) \end{pmatrix}$$

$$\left\{\begin{pmatrix} 6 & -4 \\ -4 & 4 \end{pmatrix}\right\}$$

Integrale doppio – 2

```
f[x_, y_] := 8 x^7 / (1 + y^2);

Print[Expand[Integrate[f[x, y], {x, Sqrt[y], y^(1/4)}]]];

Print[Integrate[f[x, y], {x, Sqrt[y], y^(1/4)}, {y, 0, 1}]]
```

$$\frac{y^2}{1+y^2} - \frac{y^4}{1+y^2}$$

$$\frac{5}{3} - \frac{\pi}{2}$$

Numeri complessi – 2

In[49]:=
```
a = Refine[Im[Expand[(x + y I)^2 + (x - y I)]], {x ∈ Reals, y ∈ Reals}];
Print[a]
b = Refine[Re[Expand[(x + y I)^2 + (x - y I)]], {x ∈ Reals, y ∈ Reals}];
Print[b]
Solve[{a == 0, b == 0}, {x, y}]
```

$$-y + 2xy$$

$$x + x^2 - y^2$$

Out[53]= $\left\{\{x \to -1, y \to 0\}, \{x \to 0, y \to 0\}, \left\{y \to -\frac{\sqrt{3}}{2}, x \to \frac{1}{2}\right\}, \left\{y \to \frac{\sqrt{3}}{2}, x \to \frac{1}{2}\right\}\right\}$

Matrici, autovalori – 2

In[96]:=
```
a = {{4, 0, 0}, {-5, 4, 8}, {k, 0, -4}}; Print[MatrixForm[a]];
Print[Eigenvalues[a]];
Print[Eigenvectors[a]]; Clear[b];
Print[MatrixForm[a - 4 IdentityMatrix[3]]];
```

$$\begin{pmatrix} 4 & 0 & 0 \\ -5 & 4 & 8 \\ k & 0 & -4 \end{pmatrix}$$

$$\{-4, 4, 4\}$$

$$\{\{0, -1, 1\}, \{0, 1, 0\}, \{0, 0, 0\}\}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ -5 & 0 & 8 \\ k & 0 & -8 \end{pmatrix}$$

versione 3

Equazioni differenziali – 3

```
Simplify[DSolve[{
    9 y''[x] + 6 y'[x] + 2 y[x] == 85 Sin[x],
    y[0] == -2, y'[0] == 0
  }, y[x], x]]
```

$$\left\{\left\{y[x] \to 4\, e^{-x/3} \text{Cos}\left[\frac{x}{3}\right] - 6\, \text{Cos}[x] + 25\, e^{-x/3} \text{Sin}\left[\frac{x}{3}\right] - 7\, \text{Sin}[x]\right\}\right\}$$

Funzioni di due variabili, punti critici – 3

```
g[x_, y_] := (4 y + x^2) e^(x+y);
f[x_, y_] := -g[y, -x]
Expand[f[x, y]]
```

$$4\, e^{-x+y}\, x - e^{-x+y}\, y^2$$

```
grad = Simplify[{Together[D[f[x, y], x]], Together[D[f[x, y], y]]}]
```

$$\left\{e^{-x+y}\left(4 - 4\, x + y^2\right),\ e^{-x+y}\,(4\, x - y\,(2 + y))\right\}$$

```
Solve[grad == {0, 0}, {x, y}]
```

Solve::ifun : Inverse functions are being used by Solve, so some
solutions may not be found; use Reduce for complete solution information. >>

$$\{\{x \to 2,\ y \to 2\}\}$$

```
H[x_, y_] = {{D[f[x, y], x, x], D[f[x, y], x, y]}, {D[f[x, y], x, y], D[f[x, y], y, y]}};
Print[Simplify[MatrixForm[H[x, y]]]];
Print[{MatrixForm[H[2, 2]]}];
```

$$\begin{pmatrix} e^{-x+y}\left(-8 + 4\, x - y^2\right) & e^{-x+y}\left(4 - 4\, x + 2\, y + y^2\right) \\ e^{-x+y}\left(4 - 4\, x + 2\, y + y^2\right) & e^{-x+y}\left(-2 + 4\, x - 4\, y - y^2\right) \end{pmatrix}$$

$$\left\{\begin{pmatrix} -4 & 4 \\ 4 & -6 \end{pmatrix}\right\}$$

Integrale doppio – 3

```
f[x_, y_] := 6 x^5 / (1 + y^2);
Print[Expand[∫_(y^(1/3))^(y^(1/6)) f[x, y] dx]];
Print[∫_0^1 ∫_(y^(1/3))^(y^(1/6)) f[x, y] dx dy]
```

$$\frac{y}{1 + y^2} - \frac{y^2}{1 + y^2}$$

$$\frac{1}{4}\,(-4 + \pi + \text{Log}[4])$$

Numeri complessi – 3

In[54]:=
```
a = Refine[Im[Expand[(x + y I) ^ 3 - (x - y I)]], {x ∈ Reals, y ∈ Reals}];
Print[a]
b = Refine[Re[Expand[(x + y I) ^ 3 - (x - y I)]], {x ∈ Reals, y ∈ Reals}];
Print[b]
Solve[{a == 0, b == 0}, {x, y}]
```

$y + 3 x^2 y - y^3$

$-x + x^3 - 3 x y^2$

Out[58]= $\left\{\{x \to -1, y \to 0\}, \{x \to 0, y \to 0\}, \{x \to 1, y \to 0\}, \left\{x \to -\frac{i}{\sqrt{2}}, y \to -\frac{i}{\sqrt{2}}\right\}, \left\{x \to -\frac{i}{\sqrt{2}}, y \to \frac{i}{\sqrt{2}}\right\}, \left\{x \to \frac{i}{\sqrt{2}}, y \to -\frac{i}{\sqrt{2}}\right\}, \left\{x \to \frac{i}{\sqrt{2}}, y \to \frac{i}{\sqrt{2}}\right\}, \{y \to -1, x \to 0\}, \{y \to 1, x \to 0\}\right\}$

Matrici, autovalori – 3

```
a = {{1, 0, 0}, {-7, 1, 2}, {k, 0, -1}}; Print[MatrixForm[a]];
Print[Eigenvalues[a]];
Print[Eigenvectors[a]]; Clear[b];
Print[MatrixForm[a - 1 IdentityMatrix[3]]];
```

$$\begin{pmatrix} 1 & 0 & 0 \\ -7 & 1 & 2 \\ k & 0 & -1 \end{pmatrix}$$

$\{-1, 1, 1\}$

$\{\{0, -1, 1\}, \{0, 1, 0\}, \{0, 0, 0\}\}$

$$\begin{pmatrix} 0 & 0 & 0 \\ -7 & 0 & 2 \\ k & 0 & -2 \end{pmatrix}$$