versione 0

Equazioni differenziali – 0

```
Simplify[DSolve[{
    y'[x] == (2 x^2 + 3)/x y[x] + 6 x^4,
    y[1] == -2
  }, y[x], x]]
```

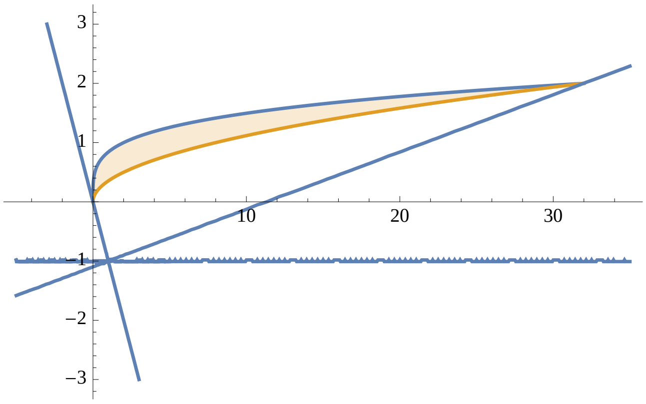
$$\left\{\left\{y[x] \to \frac{\left(-3\,e + e^{x^2}\right) x^3}{e}\right\}\right\}$$

Funzioni di due variabili, punti critici – 0

```
g[x_, y_] := (x + y)/(1 + y);
f[x_, y_] := g[x, y]
Together[f[x, y]]
```

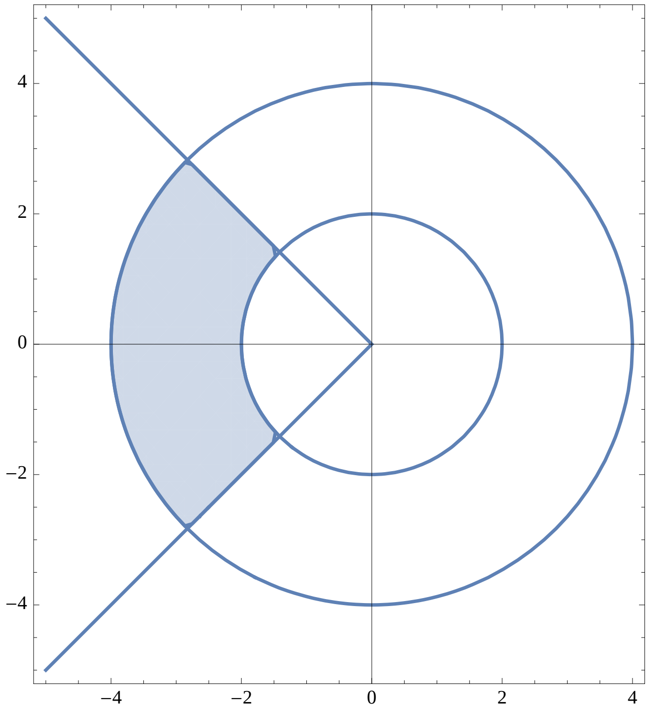
$$\frac{x + y}{1 + y}$$

```
aa = Plot[{(x/2)^(1/4), Sqrt[x/8]}, {x, 0, 32}, Filling → {2 → {1}}];

ab = ContourPlot[f[x, y] == 0, {x, -5, 5}, {y, -3, 3}];

ac = ContourPlot[f[x, y] == f[32, 2], {x, -5, 35}, {y, -3, 3}];
Show[aa, ab, ac, PlotRange → All]
```



Integrale doppio – 0

```
f[x_, y_] := (x + 1)/(x^2 + y^2)^2;
aa = RegionPlot[{4 < x^2 + y^2 < 16 && x + Abs[y] < 0}, {x, -5, 4}, {y, -5, 5}];
ab = ContourPlot[{4 == x^2 + y^2}, {x, -5, 5}, {y, -5, 5}];
ac = ContourPlot[{16 == x^2 + y^2}, {x, -5, 5}, {y, -5, 5}];
ad = ContourPlot[{x + Abs[y] == 0}, {x, -5, 5}, {y, -5, 5}];
Show[aa, ab, ac, ad, AspectRatio → Automatic, Axes → True]
```




```
Simplify[f[r Cos[t], r Sin[t]] * r]
```

$$\frac{1 + r\,\mathrm{Cos}[t]}{r^3}$$

$$\int_2^4 \int_{\frac{3\pi}{4}}^{\frac{5\pi}{4}} f[r\,\mathrm{Cos}[t], r\,\mathrm{Sin}[t]] * r \, dt \, dr$$

$$\frac{1}{64}\left(-16\sqrt{2} + 3\pi\right)$$

Numeri complessi – 0

z^3

=

$1 + 2\, e^{\frac{i\pi}{3}}$

<u>svolgimento</u>

w=

$2 + i\sqrt{3}$

|w| =

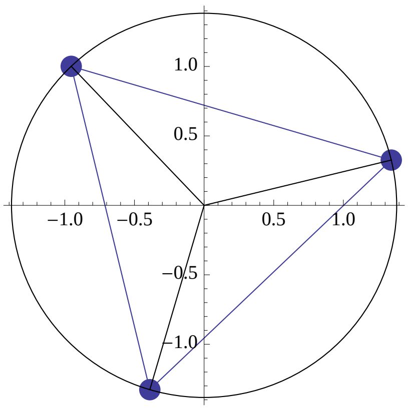
$\sqrt{7}$

Un argomento di w è

$\text{ArcTan}\left[\frac{\sqrt{3}}{2}\right]$

le soluzioni sono

$$\left\{7^{1/6}\left(\text{Cos}\left[\frac{1}{3}\text{ArcTan}\left[\frac{\sqrt{3}}{2}\right]\right] + i\,\text{Sin}\left[\frac{1}{3}\text{ArcTan}\left[\frac{\sqrt{3}}{2}\right]\right]\right),\right.$$
$$7^{1/6}\left(i\,\text{Cos}\left[\frac{\pi}{6} + \frac{1}{3}\text{ArcTan}\left[\frac{\sqrt{3}}{2}\right]\right] - \text{Sin}\left[\frac{\pi}{6} + \frac{1}{3}\text{ArcTan}\left[\frac{\sqrt{3}}{2}\right]\right]\right),$$
$$\left.7^{1/6}\left(-i\,\text{Cos}\left[\frac{\pi}{6} - \frac{1}{3}\text{ArcTan}\left[\frac{\sqrt{3}}{2}\right]\right] - \text{Sin}\left[\frac{\pi}{6} - \frac{1}{3}\text{ArcTan}\left[\frac{\sqrt{3}}{2}\right]\right]\right)\right\}$$



■ Altra versione con dati più semplici

z^3

=

$-1 + 2\, e^{\frac{i\pi}{3}}$

<u>svolgimento</u>

w=

$i\sqrt{3}$

|w| =

$\sqrt{3}$

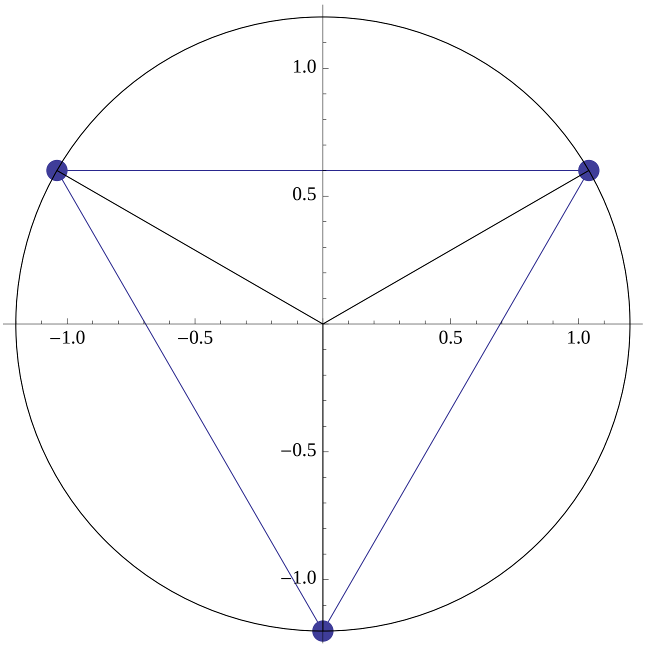
Un argomento di w è

$\frac{\pi}{2}$

le soluzioni sono

$\left\{3^{1/6}\left(\frac{i}{2}+\frac{\sqrt{3}}{2}\right), 3^{1/6}\left(\frac{i}{2}-\frac{\sqrt{3}}{2}\right), -i\, 3^{1/6}\right\}$

Out[36]=



Matrici, autovalori – 0

```
a = (-3 -1 1; -1 -3 1; 1 1 k); MatrixForm[a]
```

$$\begin{pmatrix} -3 & -1 & 1 \\ -1 & -3 & 1 \\ 1 & 1 & k \end{pmatrix}$$

```
Eigenvalues[a]
```

$$\left\{-2, \frac{1}{2}\left(-4+k-\sqrt{24+8k+k^2}\right), \frac{1}{2}\left(-4+k+\sqrt{24+8k+k^2}\right)\right\}$$

```
p[x_] = CharacteristicPolynomial[a, x]
```

$4+8k-6x+6kx-6x^2+kx^2-x^3$

```
Factor[p[x]]
```

$(2+x)\left(2+4k-4x+kx-x^2\right)$

```
Solve[{p[-2] == 0, p'[-2] == 0}, k]
```

$\{\{k \to -3\}\}$

```
k = -3;
Print[Eigenvalues[a]]; Print[Orthogonalize[Eigenvectors[a]]]
```

{-5, -2, -2}

$$\left\{\left\{-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}, \left\{\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right\}, \left\{-\frac{1}{\sqrt{6}}, \sqrt{\frac{2}{3}}, \frac{1}{\sqrt{6}}\right\}\right\}$$

versione 1

Equazioni differenziali – 1

```
Simplify[DSolve[{
    y'[x] == (2 x^2 - 3)/x y[x] + 6 x^-2,
    y[1] == -2
  }, y[x], x]]
```

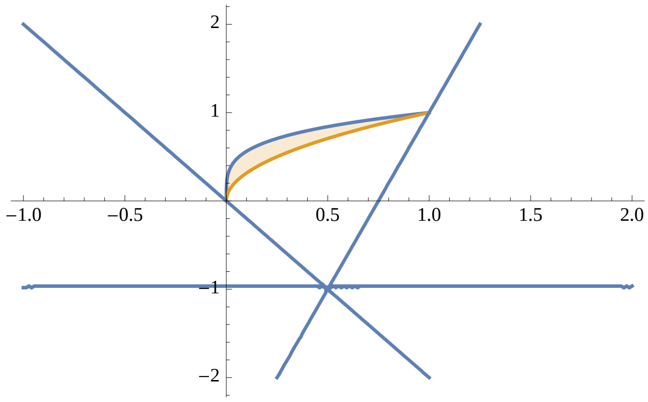
$$\left\{\left\{y[x] \to \frac{-3\,e + e^{x^2}}{e\,x^3}\right\}\right\}$$

Funzioni di due variabili, punti critici – 1

```
g[x_, y_] := (x + y)/(1 + y);
f[x_, y_] := g[2 x, y]
Together[f[x, y]]
```

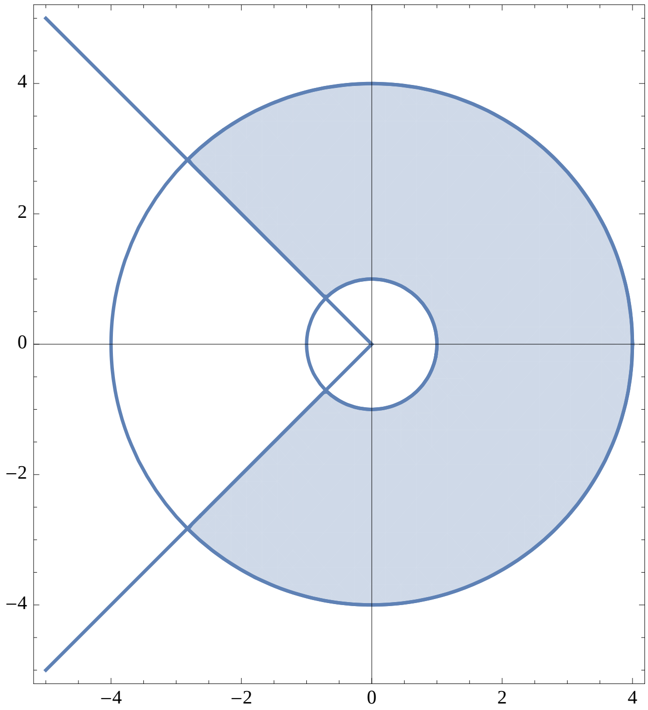
$$\frac{2x + y}{1 + y}$$

```
aa = Plot[{x^(1/4), Sqrt[x]}, {x, 0, 1}, Filling → {2 → {1}}];
ab = ContourPlot[f[x, y] == 0, {x, -1, 2}, {y, -2, 2}];

ac = ContourPlot[f[x, y] == f[1, 1], {x, -1, 2}, {y, -2, 2}]; Show[aa, ab, ac, PlotRange → All]
```



Integrale doppio – 0

```
f[x_, y_] := (x + 2)/(x^2 + y^2)^3;
aa = RegionPlot[{1 < x^2 + y^2 < 16 && x + Abs[y] > 0}, {x, -5, 4}, {y, -5, 5}];
ab = ContourPlot[{1 == x^2 + y^2}, {x, -5, 5}, {y, -5, 5}];
ac = ContourPlot[{16 == x^2 + y^2}, {x, -5, 5}, {y, -5, 5}];
ad = ContourPlot[{x + Abs[y] == 0}, {x, -5, 5}, {y, -5, 5}];
Show[aa, ab, ac, ad, AspectRatio → Automatic, Axes → True]
```




```
Simplify[f[r Cos[t], r Sin[t]] * r]
```

$$\frac{2 + r\,\mathrm{Cos}[t]}{r^5}$$

$$\int_1^4 \int_{\frac{-3\pi}{4}}^{\frac{3\pi}{4}} f[r\,\mathrm{Cos}[t], r\,\mathrm{Sin}[t]] * r \, dt \, dr$$

$$\frac{3\left(112\sqrt{2} + 255\pi\right)}{1024}$$

Numeri complessi – 1

z^3

=

$-1 + 2\, e^{-\frac{2 i \pi}{3}}$

svolgimento

w=

$-2 - i\sqrt{3}$

|w|=

$\sqrt{7}$

Un argomento di w è

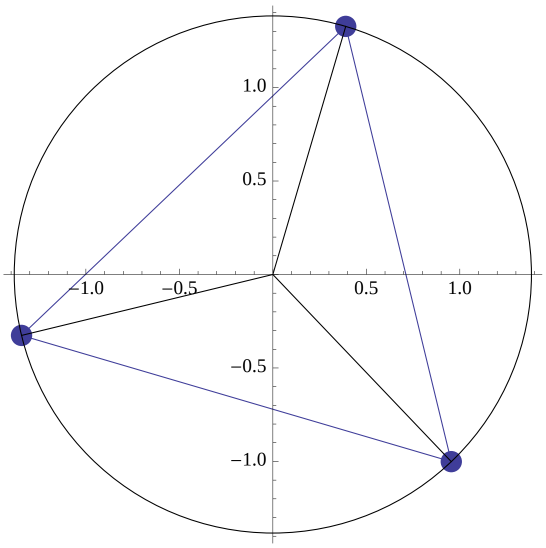
$-\pi + \text{ArcTan}\left[\frac{\sqrt{3}}{2}\right]$

le soluzioni sono

$$\left\{7^{1/6}\left(\text{Cos}\left[\frac{1}{3}\left(-\pi + \text{ArcTan}\left[\frac{\sqrt{3}}{2}\right]\right)\right] + i\,\text{Sin}\left[\frac{1}{3}\left(-\pi + \text{ArcTan}\left[\frac{\sqrt{3}}{2}\right]\right)\right]\right),\right.$$

$$7^{1/6}\left(i\,\text{Cos}\left[\frac{\pi}{6} + \frac{1}{3}\left(-\pi + \text{ArcTan}\left[\frac{\sqrt{3}}{2}\right]\right)\right] - \text{Sin}\left[\frac{\pi}{6} + \frac{1}{3}\left(-\pi + \text{ArcTan}\left[\frac{\sqrt{3}}{2}\right]\right)\right]\right),$$

$$\left.7^{1/6}\left(-i\,\text{Cos}\left[\frac{\pi}{6} + \frac{1}{3}\left(\pi - \text{ArcTan}\left[\frac{\sqrt{3}}{2}\right]\right)\right] - \text{Sin}\left[\frac{\pi}{6} + \frac{1}{3}\left(\pi - \text{ArcTan}\left[\frac{\sqrt{3}}{2}\right]\right)\right]\right)\right\}$$



- **Altra versione con dati più semplici**

z^3

=

$1 + 2\, e^{-\frac{2 i \pi}{3}}$

svolgimento

w=

$-i\sqrt{3}$

|w|=

$\sqrt{3}$

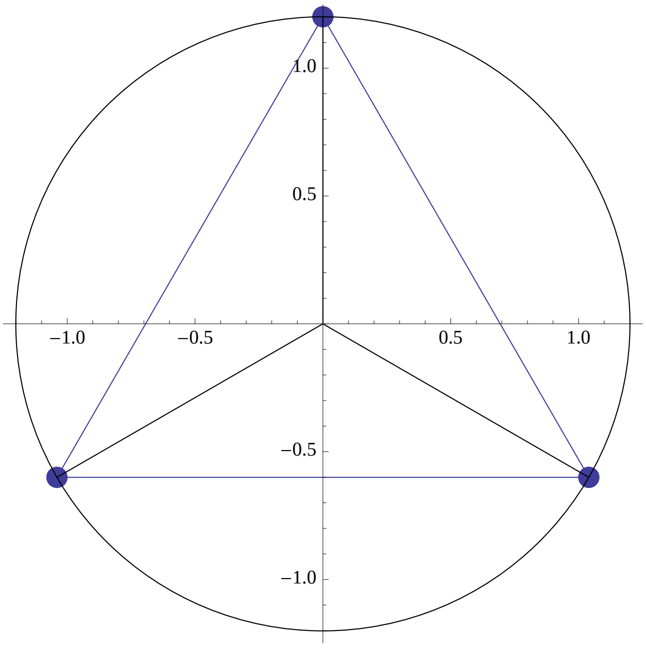
Un argomento di w è

$-\frac{\pi}{2}$

le soluzioni sono

$\left\{3^{1/6}\left(-\frac{i}{2}+\frac{\sqrt{3}}{2}\right), i\,3^{1/6}, 3^{1/6}\left(-\frac{i}{2}-\frac{\sqrt{3}}{2}\right)\right\}$

Out[18]=



Matrici, autovalori – 1

```
Clear["Global`*"]; a = {{3, 1, -1}, {1, 3, 1}, {-1, 1, k}}; MatrixForm[a]
```

$$\begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & k \end{pmatrix}$$

```
Eigenvalues[a]
```

$$\left\{4, \frac{1}{2}\left(2+k-\sqrt{12-4k+k^2}\right), \frac{1}{2}\left(2+k+\sqrt{12-4k+k^2}\right)\right\}$$

```
p[x_] = CharacteristicPolynomial[a, x]
```

$$-8+8k-6x-6kx+6x^2+kx^2-x^3$$

```
Factor[p[x]]
```

$$(-4+x)\left(2-2k+2x+kx-x^2\right)$$

```
Solve[{p[4] == 0, p'[4] == 0}, k]
```

{{k → 3}}

```
k = 3;
Print[Eigenvalues[a]]; Print[Orthogonalize[Eigenvectors[a]]]
```

{4, 4, 1}

$$\left\{\left\{-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{6}}, \sqrt{\frac{2}{3}}, \frac{1}{\sqrt{6}}\right\}, \left\{\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}\right\}$$