versione 0

Equazioni differenziali – 0

```
Simplify[DSolve[{
    y'[x] == (ⅇ^x y[x]^-3)/4, y[Log[24]] == -2
  }, y[x], x]]
```

DSolve::bvnul : For some branches of the general solution, the given boundary conditions lead to an empty solution. ≫

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General::stop : Further output of DSolve::bvnul will be suppressed during this calculation. ≫

$$\{\{y[x] \to -(-8 + e^x)^{1/4}\}\}$$

```
Reduce[-8 + ⅇ^x > 0, x, Reals]
```

$$x > 3\,\text{Log}[2]$$

Funzioni di due variabili, punti critici – 0

```
g[x_, y_] := Log[x^2 - y^2] + 2 y + 4 x;
f[x_, y_] := g[x, y]; Print[f[x, y]];
grad = Simplify[{Together[D[f[x, y], x]], Together[D[f[x, y], y]]}]
```

$$4x + 2y + \text{Log}[x^2 - y^2]$$

$$\left\{\frac{2\left(x + 2x^2 - 2y^2\right)}{x^2 - y^2}, \frac{2\left(-x^2 + y + y^2\right)}{-x^2 + y^2}\right\}$$

```
Reduce[grad == {0, 0}, {x, y}]
```

$$x == -\frac{2}{3} \;\&\&\; y == \frac{1}{3}$$

```
H[x_, y_] = {{D[f[x, y], x, x], D[f[x, y], x, y]}, {D[f[x, y], x, y], D[f[x, y], y, y]}};
H[x, y];
Print[MatrixForm[H[x, y]]];
```

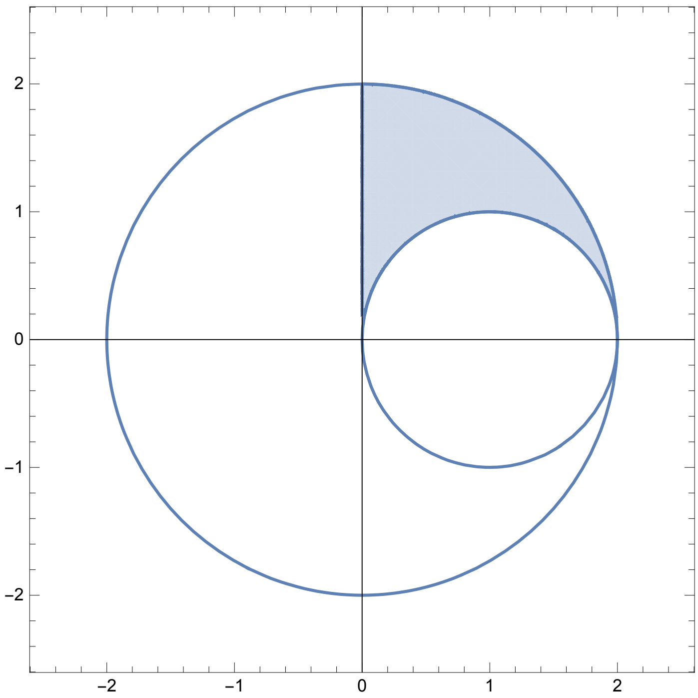
$$\begin{pmatrix} -\frac{4x^2}{(x^2-y^2)^2} + \frac{2}{x^2-y^2} & \frac{4xy}{(x^2-y^2)^2} \\ \frac{4xy}{(x^2-y^2)^2} & -\frac{4y^2}{(x^2-y^2)^2} - \frac{2}{x^2-y^2} \end{pmatrix}$$

```
Print[{MatrixForm[H[-2/3, 1/3]]}];
```

$$\left\{\begin{pmatrix} -10 & -8 \\ -8 & -10 \end{pmatrix}\right\}$$

Integrale doppio – 0

```
f[x_, y_] := x^2 y + y^3;
aa = RegionPlot[{x^2 + y^2 < 4 && x^2 + y^2 - 2 x > 0 && x > 0 && y > 0},
    {x, -2.5, 2.5}, {y, -2.5, 2.5}, MaxRecursion → 10];
ab = ContourPlot[{4 == x^2 + y^2}, {x, -2.5, 2.5}, {y, -2.5, 2.5}];
ac = ContourPlot[{x^2 + y^2 - 2 x == 0}, {x, -2.5, 2.5}, {y, -2.5, 2.5}];
Show[aa, ab, ac, AspectRatio → Automatic, Axes → True,
 Ticks → {{-2, 1, 2}, {-2, 2}}]
```




```
Simplify[f[r Cos[t], r Sin[t]] * r]
```

$r^4 \sin[t]$

$\text{Simplify}\left[\int_{2\cos[t]}^{2} f[r\cos[t], r\sin[t]] * r \, dr\right]$

$-\frac{32}{5}\left(-1+\cos[t]^5\right)\sin[t]$

$\int_0^{\frac{\pi}{2}}\int_{2\cos[t]}^{2} f[r\cos[t], r\sin[t]] * r \, dr \, dt$

$\frac{16}{3}$

$\int_{\sqrt{2x-x^2}}^{\sqrt{4-x^2}} f[x, y] \, dy$

$4 - x^2$

$\int_0^2\int_{\sqrt{2x-x^2}}^{\sqrt{4-x^2}} f[x, y] \, dy \, dx$

$\frac{16}{3}$

Numeri complessi – 0

$\{\text{Abs}[(1 - i\sqrt{3})], \text{Arg}[(1 - i\sqrt{3})]\}$

$\{2, -\frac{\pi}{3}\}$

$\{\text{Abs}[(1 - i\sqrt{3})^{-5}], \text{Arg}[(1 - i\sqrt{3})^{-5}]\}$

$\{\frac{1}{32}, -\frac{\pi}{3}\}$

$\{\text{Re}[(1 - i\sqrt{3})^{-5}], \text{Im}[(1 - i\sqrt{3})^{-5}]\}$

$\{\frac{1}{64}, -\frac{\sqrt{3}}{64}\}$

Matrici, autovalori – 0

$A = \begin{pmatrix} 1 & -1 & a \\ 0 & 2 & b \\ -1 & -1 & c \end{pmatrix}; v = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix};$

```
Solve[A.v == v, {a, b, c}]
```

{{a → 0, b → 0, c → 2}}

$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & 0 \\ -1 & -1 & 2 \end{pmatrix};$

```
Print[Eigenvalues[A]]; Print[Eigenvectors[A]]
```

{2, 2, 1}

{{0, 0, 1}, {-1, 1, 0}, {1, 0, 1}}

versione I

Equazioni differenziali – I

```
Simplify[DSolve[{
    y'[x] == e^-x y[x]^-4 / 5,
    y[-Log[24]] == 1
  }, y[x], x]]
```

$$y'[x] == \frac{e^{-x}\, y[x]^{-4}}{5}$$

DSolve::bvnul : For some branches of the general solution, the given boundary conditions lead to an empty solution. ≫

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DSolve::bvnul : For some branches of the general solution, the given boundary conditions lead to an empty solution. ≫

General::stop : Further output of DSolve::bvnul will be suppressed during this calculation. ≫

$\{\{y[x] \to (25 - e^{-x})^{1/5}\}\}$

```
Reduce[25 - e^-x > 0, x, Reals]
```

x > -2 Log[5]

Funzioni di due variabili, punti critici – I

```
g[x_, y_] := Log[x^2 - y^2] + 2 y + 4 x;
f[x_, y_] := g[y, -x]; Print[Simplify[f[x, y]]];
grad = Simplify[{Together[D[f[x, y], x]], Together[D[f[x, y], y]]}]
```

$-2x + 4y + \text{Log}[-x^2 + y^2]$

$\{\frac{2(x - x^2 + y^2)}{x^2 - y^2}, \frac{2(-2x^2 + y + 2y^2)}{-x^2 + y^2}\}$

```
Reduce[grad == {0, 0}, {x, y}]
```

$x == -\frac{1}{3} \,\&\&\, y == -\frac{2}{3}$

```
H[x_, y_] = {{D[f[x, y], x, x], D[f[x, y], x, y]}, {D[f[x, y], x, y], D[f[x, y], y, y]}};
H[x, y];
Print[MatrixForm[H[x, y]]];
```

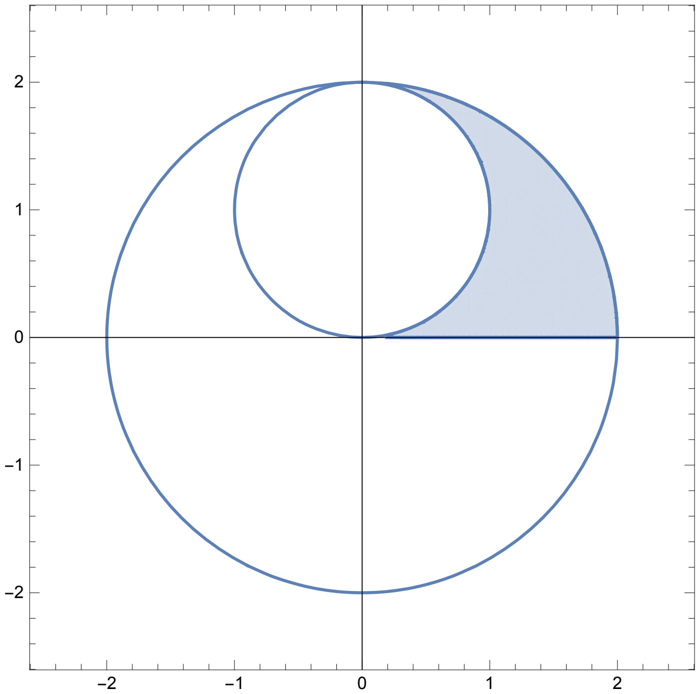
$$\begin{pmatrix} -\frac{4x^2}{(-x^2+y^2)^2} - \frac{2}{-x^2+y^2} & \frac{4xy}{(-x^2+y^2)^2} \\ \frac{4xy}{(-x^2+y^2)^2} & -\frac{4y^2}{(-x^2+y^2)^2} + \frac{2}{-x^2+y^2} \end{pmatrix}$$

```
Print[{MatrixForm[H[-1/3, -2/3]]}];
```

$$\left\{\begin{pmatrix} -10 & 8 \\ 8 & -10 \end{pmatrix}\right\}$$

Integrale doppio – I

```
f[x_, y_] := x^3 + x y^2;
aa = RegionPlot[{x^2 + y^2 < 4 && x^2 + y^2 - 2 y > 0 && x > 0 && y > 0},
   {x, -2.5, 2.5}, {y, -2.5, 2.5}, MaxRecursion → 10];
ab = ContourPlot[{4 == x^2 + y^2}, {x, -2.5, 2.5}, {y, -2.5, 2.5}];
ac = ContourPlot[{x^2 + y^2 - 2 y == 0}, {x, -2.5, 2.5}, {y, -2.5, 2.5}];
Show[aa, ab, ac, AspectRatio → Automatic, Axes → True,
 Ticks → {{-2, 1, 2}, {-2, 2}}]
```




```
Simplify[f[r Cos[t], r Sin[t]] * r]
```

$r^4 \operatorname{Cos}[t]$

```
Simplify[∫_{2 Sin[t]}^{2} f[r Cos[t], r Sin[t]] * r ⅆr]
```

$-\frac{32}{5} \operatorname{Cos}[t] \left(-1 + \operatorname{Sin}[t]^5\right)$

```
∫_0^{π/2} ∫_{2 Sin[t]}^{2} f[r Cos[t], r Sin[t]] * r ⅆr ⅆt
```

$\frac{16}{3}$

```
∫_{Sqrt[2 y - y^2]}^{Sqrt[4 - y^2]} f[x, y] ⅆx
```

$4 - y^2$

$$\int_0^2 \int_{\sqrt{2y-y^2}}^{\sqrt{4-y^2}} f[x, y]\, dx\, dy$$

$$\frac{16}{3}$$

Numeri complessi – I

```
{Abs[(1 - ⅈ)], Arg[(1 - ⅈ)]}
```

$$\left\{\sqrt{2},\ -\frac{\pi}{4}\right\}$$

```
{Abs[(1 - ⅈ)^-7], Arg[(1 - ⅈ)^-7]}
```

$$\left\{\frac{1}{8\sqrt{2}},\ -\frac{\pi}{4}\right\}$$

```
{Re[(1 - ⅈ)^-7], Im[(1 - ⅈ)^-7]}
```

$$\left\{\frac{1}{16},\ -\frac{1}{16}\right\}$$

Matrici, autovalori – I

$$A = \begin{pmatrix} 2 & -1 & a \\ 0 & 3 & b \\ -1 & -1 & c \end{pmatrix};\ v = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix};$$

```
Solve[A.v == 2 v, {a, b, c}]
```

{{a → 0, b → 0, c → 3}}

$$A = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 3 & 0 \\ -1 & -1 & 3 \end{pmatrix};$$

```
Print[Eigenvalues[A]]; Print[Eigenvectors[A]]
```

{3, 3, 2}

{{0, 0, 1}, {-1, 1, 0}, {1, 0, 1}}

versione 2

Equazioni differenziali – 2

```
Simplify[DSolve[{
    y'[x] == ⅇ^(-2 x) y[x]^-1 / 4,  y[-Log[2]] == -1
  }, y[x], x]]
```

DSolve::bvnul : For some branches of the general solution, the given boundary conditions lead to an empty solution. ≫

$$\left\{\left\{y[x] \to -\frac{1}{2}\sqrt{8 - e^{-2x}}\right\}\right\}$$

```
Reduce[8 - ⅇ^(-2 x) > 0, x, Reals]
```

$$x > -\frac{3\,\mathrm{Log}[2]}{2}$$

Funzioni di due variabili, punti critici – 2

```
g[x_, y_] := Log[x^2 - y^2] + 2 y + 4 x;
f[x_, y_] := 3 g[x/3, y/6]; Print[Simplify[f[x, y]]];
grad = Simplify[{Together[D[f[x, y], x]], Together[D[f[x, y], y]]}]
```

$4\,x + y + 3\,\mathrm{Log}\left[\frac{1}{36}\,(4\,x^2 - y^2)\right]$

$\left\{\frac{4\,(6\,x + 4\,x^2 - y^2)}{4\,x^2 - y^2}, \frac{-4\,x^2 + y\,(6 + y)}{-4\,x^2 + y^2}\right\}$

```
Reduce[grad == {0, 0}, {x, y}]
```

x == -2 && y == 2

```
H[x_, y_] = {{D[f[x, y], x, x], D[f[x, y], x, y]}, {D[f[x, y], x, y], D[f[x, y], y, y]}};
H[x, y];
Print[Simplify[MatrixForm[H[x, y]]]];
```

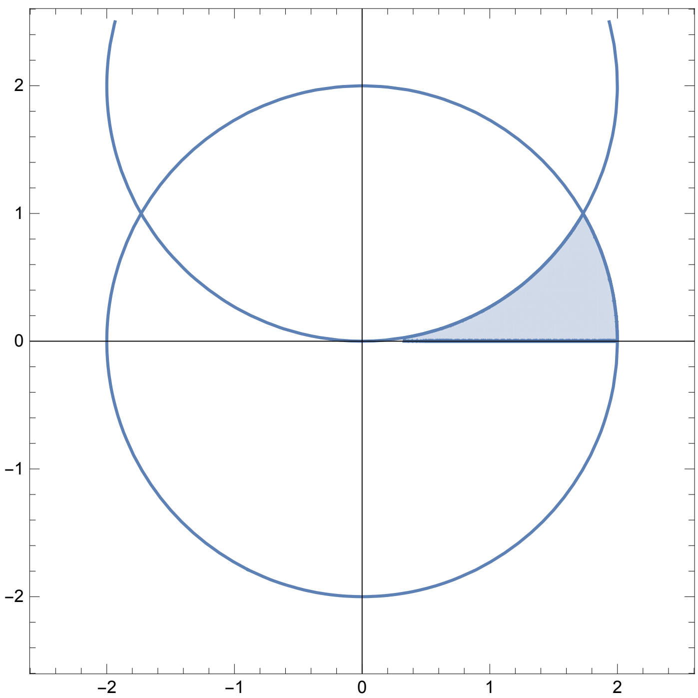
$$\begin{pmatrix} -\frac{24\,(4\,x^2+y^2)}{(-4\,x^2+y^2)^2} & \frac{48\,x\,y}{(-4\,x^2+y^2)^2} \\ \frac{48\,x\,y}{(-4\,x^2+y^2)^2} & -\frac{6\,(4\,x^2+y^2)}{(-4\,x^2+y^2)^2} \end{pmatrix}$$

```
Print[{MatrixForm[H[-2, 2]]}];
```

$$\left\{\begin{pmatrix} -\frac{10}{3} & -\frac{4}{3} \\ -\frac{4}{3} & -\frac{5}{6} \end{pmatrix}\right\}$$

Integrale doppio – 2

```
f[x_, y_] := x^3 + x y^2;
aa = RegionPlot[{x^2 + y^2 < 4 && x^2 + y^2 - 4 y > 0 && x > 0 && y > 0},
    {x, -2.5, 2.5}, {y, -2.5, 2.5}, MaxRecursion → 10];
ab = ContourPlot[{4 == x^2 + y^2}, {x, -2.5, 2.5}, {y, -2.5, 2.5}];
ac = ContourPlot[{x^2 + y^2 - 4 y == 0}, {x, -2.5, 2.5}, {y, -2.5, 2.5}];
Show[aa, ab, ac, AspectRatio → Automatic, Axes → True,
 Ticks → {{-2, 1, 2}, {-2, 2}}]
```




```
Simplify[f[r Cos[t], r Sin[t]] * r]
```

$r^4\,\mathrm{Cos}[t]$

$$\text{Simplify}\left[\int_{4\,\text{Sin}[t]}^{2} f[r\,\text{Cos}[t],\, r\,\text{Sin}[t]] * r\, dr\right]$$

$$\frac{32}{5}\,\text{Cos}[t]\,\left(1 - 32\,\text{Sin}[t]^5\right)$$

$$\int_0^{\frac{\pi}{6}}\int_{4\,\text{Sin}[t]}^{2} f[r\,\text{Cos}[t],\, r\,\text{Sin}[t]] * r\, dr\, dt$$

$$\frac{8}{3}$$

$$\int_{\sqrt{4\,y-y^2}}^{\sqrt{4-y^2}} f[x,\, y]\, dx$$

$$4 - 4\,y^2$$

$$\int_0^1\int_{\sqrt{4\,y-y^2}}^{\sqrt{4-y^2}} f[x,\, y]\, dx\, dy$$

$$\frac{8}{3}$$

Numeri complessi – 2

$$\left\{\text{Abs}\left[\left(1 + i\,\sqrt{3}\right)\right],\, \text{Arg}\left[\left(1 + i\,\sqrt{3}\right)\right]\right\}$$

$$\left\{2,\, \frac{\pi}{3}\right\}$$

$$\left\{\text{Abs}\left[\left(1 + i\,\sqrt{3}\right)^{-4}\right],\, \text{Arg}\left[\left(1 + i\,\sqrt{3}\right)^{-4}\right]\right\}$$

$$\left\{\frac{1}{16},\, \frac{2\,\pi}{3}\right\}$$

$$\left\{\text{Re}\left[\left(1 + i\,\sqrt{3}\right)^{-4}\right],\, \text{Im}\left[\left(1 + i\,\sqrt{3}\right)^{-4}\right]\right\}$$

$$\left\{-\frac{1}{32},\, \frac{\sqrt{3}}{32}\right\}$$

Matrici, autovalori – 2

$$A = \begin{pmatrix} 3 & -1 & a \\ 0 & 4 & b \\ -1 & -1 & c \end{pmatrix};\ v = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix};$$

```
Solve[A.v == 3 v, {a, b, c}]
```

```
{{a → 0, b → 0, c → 4}}
```

$$A = \begin{pmatrix} 3 & -1 & 0 \\ 0 & 4 & 0 \\ -1 & -1 & 4 \end{pmatrix};$$

```
Print[Eigenvalues[A]]; Print[Eigenvectors[A]]
```

```
{4, 4, 3}
{{0, 0, 1}, {-1, 1, 0}, {1, 0, 1}}
```

versione 3

Equazioni differenziali – 3

```
Simplify[DSolve[{
    y'[x] == (e^(2 x) y[x]^3)/2,  y[Log[2]] == 2
  }, y[x], x]]
```

$\{\{y[x] \to \frac{2}{\sqrt{9-2\,e^{2x}}}\}\}$

```
Reduce[9 - 2 e^(2 x) > 0, x, Reals]
```

$x < \frac{1}{2}\,(-\text{Log}[2] + 2\,\text{Log}[3])$

Funzioni di due variabili, punti critici – 3

```
g[x_, y_] := Log[x^2 - y^2]  + 2 y + 4 x;
f[x_, y_] := 3 g[x/6, -y/3]; Print[Simplify[f[x, y]]];
grad = Simplify[{Together[D[f[x, y], x]], Together[D[f[x, y], y]]}]
```

$2x - 2y + 3\,\text{Log}\left[\frac{1}{36}\,(x^2 - 4y^2)\right]$

$\left\{\frac{2\,(3x + x^2 - 4y^2)}{x^2 - 4y^2},\; -\frac{2\,(x^2 - 4\,(-3 + y)\,y)}{x^2 - 4y^2}\right\}$

```
Reduce[grad == {0, 0}, {x, y}]
```

$x == -4\ \&\&\ y == -1$

```
H[x_, y_] = {{D[f[x, y], x, x], D[f[x, y], x, y]}, {D[f[x, y], x, y], D[f[x, y], y, y]}};
H[x, y];
Print[MatrixForm[Simplify[H[x, y]]]];
```

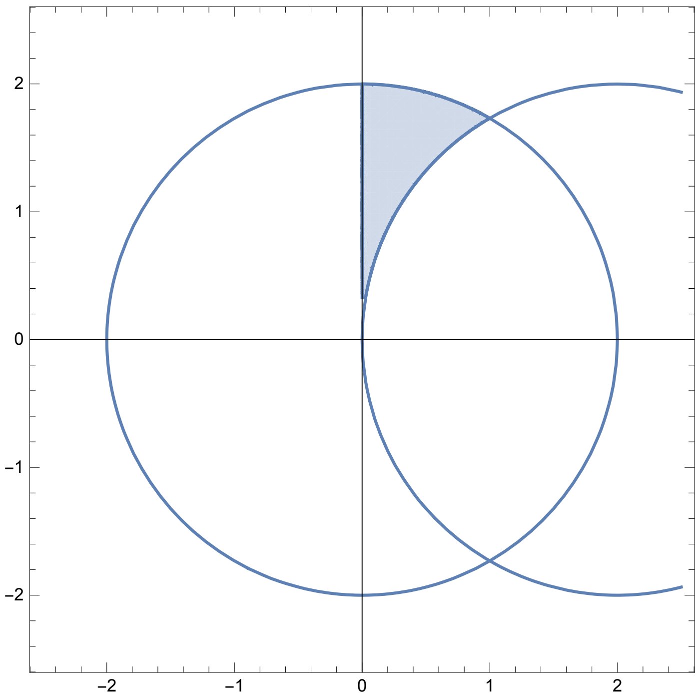
$$\begin{pmatrix} -\frac{6\,(x^2+4y^2)}{(x^2-4y^2)^2} & \frac{48\,x\,y}{(x^2-4y^2)^2} \\ \frac{48\,x\,y}{(x^2-4y^2)^2} & -\frac{24\,(x^2+4y^2)}{(x^2-4y^2)^2} \end{pmatrix}$$

```
Print[{MatrixForm[H[-4, -1]]}];
```

$$\left\{\begin{pmatrix} -\frac{5}{6} & \frac{4}{3} \\ \frac{4}{3} & -\frac{10}{3} \end{pmatrix}\right\}$$

Integrale doppio – 3

```
f[x_, y_] := x^2 y + y^3;
aa = RegionPlot[{x^2 + y^2 < 4 && x^2 + y^2 - 4 x > 0 && x > 0 && y > 0},
    {x, -2.5, 2.5}, {y, -2.5, 2.5}, MaxRecursion → 10];
ab = ContourPlot[{4 == x^2 + y^2}, {x, -2.5, 2.5}, {y, -2.5, 2.5}];
ac = ContourPlot[{x^2 + y^2 - 4 x == 0}, {x, -2.5, 2.5}, {y, -2.5, 2.5}];
Show[aa, ab, ac, AspectRatio → Automatic, Axes → True,
 Ticks → {{-2, 1, 2}, {-2, 2}}]
```




```
Simplify[f[r Cos[t], r Sin[t]] * r]
```

$r^4 \, \mathrm{Sin}[t]$

$$\mathrm{Simplify}\left[\int_{4\,\mathrm{Cos}[t]}^{2} f[r\,\mathrm{Cos}[t],\, r\,\mathrm{Sin}[t]] * r \, dr\right]$$

$$-\frac{32}{5}\left(-1 + 32\,\mathrm{Cos}[t]^5\right)\mathrm{Sin}[t]$$

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\int_{4\,\mathrm{Cos}[t]}^{2} f[r\,\mathrm{Cos}[t],\, r\,\mathrm{Sin}[t]] * r \, dr \, dt$$

$$\frac{8}{3}$$

$$\int_{\sqrt{4x-x^2}}^{\sqrt{4-x^2}} f[x, y] \, dy$$

$$4 - 4\,x^2$$

$$\int_{0}^{1}\int_{\sqrt{4x-x^2}}^{\sqrt{4-x^2}} f[x, y] \, dy \, dx$$

$$\frac{8}{3}$$

Numeri complessi – 3

$$\{\mathrm{Abs}[(1 + i)],\, \mathrm{Arg}[(1 + i)]\}$$

$$\left\{\sqrt{2},\, \frac{\pi}{4}\right\}$$

```
{Abs[(1 + ⅈ)^-5], Arg[(1 + ⅈ)^-5]}
```

$$\left\{\frac{1}{4\sqrt{2}}, \frac{3\pi}{4}\right\}$$

```
{Re[(1 + ⅈ)^-5], Im[(1 + ⅈ)^-5]}
```

$$\left\{-\frac{1}{8}, \frac{1}{8}\right\}$$

Matrici, autovalori – 3

$$A = \begin{pmatrix} 4 & -1 & a \\ 0 & 5 & b \\ -1 & -1 & c \end{pmatrix};\ v = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix};$$

```
Solve[A.v == 4 v, {a, b, c}]
```

```
{{a → 0, b → 0, c → 5}}
```

$$A = \begin{pmatrix} 4 & -1 & 0 \\ 0 & 5 & 0 \\ -1 & -1 & 5 \end{pmatrix};$$

```
Print[Eigenvalues[A]]; Print[Eigenvectors[A]]
```

```
{5, 5, 4}
{{0, 0, 1}, {-1, 1, 0}, {1, 0, 1}}
```