Probabilità e Statistica Matematica (C.d.L. in Chimica Industriale)
Soluzioni 12. 12. 2005

1. (A)

   (a) $\binom{5}{1}\frac{1}{4}(\frac{3}{4})^4 = \frac{405}{1024} \approx 0,6328 - 0,2373 = 0,3955.$

   (b) $1 - \binom{5}{0}(\frac{3}{4})^5 - \binom{5}{1}\frac{1}{4}(\frac{3}{4})^4 = \frac{47}{128} \approx 1 - 0,6328 = 0,3672.$

   (c) $E(X) = np = \frac{5}{4} = 1,25, \quad \mathrm{Var}(X) = npq = \frac{15}{16} = 0,9375.$

   (d) 1.

   (B)

   (a) $P(1 \text{ pallina rossa}) = 5 \cdot \frac{2}{8} \cdot \frac{6}{7} \cdot \frac{5}{6} \cdot \frac{4}{5} \cdot \frac{3}{4} = \frac{15}{28} \approx 0,5357.$

   (b) $P(2 \text{ palline rosse}) = \binom{5}{2} \cdot \frac{1}{4} \cdot \frac{1}{7} = \frac{10}{28} \approx 0,3571.$

   (c) $P(0 \text{ palline rosse}) = \frac{6}{8} \cdot \frac{5}{7} \cdot \frac{4}{6} \cdot \frac{3}{5} \cdot \frac{2}{4} = \frac{3}{28} \approx 0,1071,$
   $E(X) = \frac{35}{28} = \frac{5}{4} = 1,25, \quad \mathrm{Var}(X) = \frac{55}{28} - \frac{25}{16} = \frac{45}{112} = \approx 0,4018.$

   (d) 1.

2. (a) $P(D) = P(D|A) \cdot P(A) + P(D|B) \cdot P(B) + P(D|C) \cdot P(C)$
   $= \frac{10}{100} \cdot \frac{3}{10} + \frac{5}{100} \cdot \frac{2}{10} + \frac{2}{100} \cdot \frac{5}{10} = \frac{50}{1000} = \frac{1}{20} = 0,05.$

   (b) $1 - P(C|D) = 1 - \frac{P(C \cap D)}{P(D)} = 1 - \frac{1}{5} = \frac{4}{5} = 0,8.$

3. (a) $(\frac{35}{36})^7 \cdot \frac{1}{36} \approx 0,0228.$

   (b) $1 - (\frac{35}{36})^n > 0,5, \text{ cioè } n > \frac{\log 0,5}{\log(\frac{35}{36})} \approx 24,6.$

4. (a) $\frac{2}{10} = \frac{1}{5} = 0,2.$

   (b) $P(X^2 < \frac{1}{4}) = P(X < \frac{1}{2}) = 0,5.$

   (c) $F(x) = P(X^2 \le x) = P(X \le \sqrt{x}) = \begin{cases} 0 & \text{per } x < 0, \\ \sqrt{x} & \text{per } 0 \le x \le 1, \\ 1 & \text{per } x > 1. \end{cases}$

   $f(x) = \begin{cases} \frac{1}{2\sqrt{x}} & \text{per } 0 \le x \le 1, \\ 0 & \text{altrimenti.} \end{cases}.$

   (d) $E(X^2) = \int_0^1 x^2\,dx = \frac{1}{3}, \quad \mathrm{Var}(X^2) = \int_0^1 x^4\,dx - \frac{1}{9} = \frac{1}{5} - \frac{1}{9} = \frac{4}{45}.$