Are European students sufficiently prepared in mathematics? – Questionnaire results

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Acknowledgements. This article is partly the result of a collective effort. A first draft was written together with James Blowey (Durham), Hubert Flenner (Bochum), Laura Guidotti (Bologna), Salomon Ofman (Paris 7), Mirella Manaresi (Bologna), Evangelia Samiou (Cyprus), Brian Straughan (Durham), and Alekos Vidras (Cyprus). Gratefully acknowledged are also the contributions of Peter Eichelsbacher (Bochum), Giuseppe Mulone (Catania), Stefan Schröer (Düsseldorf, formerly Bayreuth), Bernd Siebert (Freiburg), Jean-Jacques Szczeciniarz (Paris 7, formerly Bordeaux 3). We have to thank many other colleagues who were involved in organising the test and distributing the questionnaire.

Introduction

A number of changes have taken place in recent years which have profoundly affected the teaching of mathematics at the university level. On the one hand, in many countries, significantly more students are now entering university and more curricula provide at least one compulsory mathematics course. On the other hand, a decreasing percentage of students appears to be opting for mathematics or for studies which require substantial amounts of mathematics. A general aversion to mathematics and 'hard' sciences is perceived.

Are European students well prepared to meet the challenges of the use of mathematics in their studies at university level? Despite the growing importance of mathematics in many sciences and in almost every aspect of human life, university teachers often have the impression that students are not familiar with the mathematics they need and that there is even a trend that students perform more badly. Is this only a subjective perception? Are there significant differences from country to country?

Since the mathematical and scientific literacy of students have a great impact on the economical competitiveness of the countries, in recent years there have been conducted several international comparative studies on the mathematical literacy of students, studies which are interesting not only for pedagogical considerations but also for political and economical reasons.

In the framework of the European Socrates project "Diffusion and improvement of mathematical knowledge in Europe" (with short title "Mathematics in Europe") coordinated by Professor M. Manaresi of the Department of Mathematics of the University of Bologna and in which participated the universities of Bochum (responsible Prof. H. Flenner), Cyprus (responsible Prof. A. Vidras), Durham (responsible Prof. B. Straughan), Paris 7 (responsible Prof. S. Ofman), we conducted a multiple choice test in order to try to answer the questions above. Here we shall report on the results of the test which was given to 3441 first-year students (whose curricula required at least one compulsory mathematics course) in September and October 2003 during the first week of university.

It is interesting to compare our results with those obtained by other completely different designed tests given to a much larger number of students. Here we shall refer only to two major comparative studies to which we shall compare our results: the "Trends in International Mathematics and Science Study" (TIMSS) and the "OECD Programme for International Student Assessment" (PISA); see [13] and [10], respectively, for the complete results of these studies.

In 1999 the ministers of university instruction from 29 European countries signed a document in Bologna (the "Declaration of Bologna") which initiated an important process to harmonise the various European systems of higher education: the *Bologna Process*. The results of the test we report on show that similar efforts are needed to harmonise also the European education at secondary school level.

The project and its partners

The idea of the test came up during the realisation of the project "Diffusion and improvement of mathematical knowledge in Europe" a project carried out in the framework of the European Community Programme 'Socrates'. To have a better understanding of the different starting backgrounds and of the students from which the sample of the test was drawn, let us briefly introduce the partner universities of the project.

University of Bologna. Having more than 100,000 enrolled students, the University of Bologna is one of the largest Italian universities. Founded in 1088, it is certainly the oldest in Europe. Due to its long tradition, its high reputation and, last not least, the cultural life offered in the beautiful historic city centre of Bologna, the university attracts students from all parts of Italy, but mainly from the adriatic regions, that is, from Emilia-Romagna, Marches, Abruzzo, Apulia, and Veneto. The students are enrolled in the various courses after having obtained the diploma of the higher middle school, in general without examinations for admission. One of the very few exceptions is Biotechnology which has a restricted and competitive entrance. In recent years the University of Bologna has also set up faculties and courses at Cesena, Faenza, Forlì, Imola, Ravenna, Reggio Emilia and Rimini.

University of Paris 7 - Denis Diderot. The University of Paris 7 is located in the densest university area of France, the region Île de France. Founded in 1970 as the 13th of the 17 universities of Île de France, Paris 7 took in 1994 the name of Denis Diderot. Its great reputation is based on an excellent research in many fields of science. Paris 7 has about 27,000 students and is unique within the 8 Parisian universities because of its broad multidisciplinarity. Students who wish to study mathematics must have the Baccalauréat Scientifique (Bac S). In the academic year 2004/2005 the University Paris 7 started to adapt the European system of education: Licence-Masters-Ecoles doctorales.

Ruhr-University Bochum. The Ruhr-University Bochum, a campus university, is the first university which was founded in the post-war Federal Republic of Germany. Opened in 1965, the Ruhr-University of Bochum has grown quickly and now has about 32,000 students coming mainly from the Ruhr region, once a heavy industrial region which changed its character primarily under the influence of the new university. The Ruhr-University offers the whole spectrum of the natural and social sciences, medicine, engineering, and the humanities, and it was among the first universities in Germany which introduced the Bachelor's and Master's system.

University of Cyprus. In Cyprus higher education, that is, education beyond secondary school level, was mainly developed in the years after the country became independent in 1960. The University of Cyprus was founded in 1991 and is the only state university in the Republic of Cyprus. Approximately 550 students are admitted to the University of Cyprus each year. The great majority must satisfactorily pass the entrance examinations set by the Ministry of Education and Culture. It should be noted that the University alone decides on the number of students that enter its faculties, so that usually students compete for a placement at a scientific field of their choice.

The School of Engineering enrolled students for first time in the academic year 2003/2004. Admission to that School requires a minimum grade in the entrance exam. For the School of Sciences there is no such requirement.

A particular fact of the University of Cyprus is that male students enter the University after a 26-month military service in the National Guard. However, the female students begin their studies at the university immediately after graduating from high school.

University of Durham. The University of Durham was founded in 1832 and is the third oldest in England. The University of Durham is collegiate and is located in the city of Durham and the nearby town of Stockton.

Entrance to English universities is competitive and offers to students are normally made subject to sufficiently good A-levels or Scottish Highers. There are three examination boards in England (see their websites [2], [3], [4]) whose Advanced level/Advanced Subsidiary level (A-level/AS-level) specifications are approved by the Qualifications and Curriculum Authority (QCA). The grades A, B, C, D, E at A-level, where A is the highest one, count as a pass and correspond to 120, 100, 80, 60, 40 points, respectively. AS-level points count half. The assessment of an A-level in mathematics is based on six modules taken from 7 modules of Pure Mathematics, 6 of Mechanics, 6 of Statistics and 2 of Discrete Mathematics. There are many possible combinations but there is a common core of about 50% which are incorporated into the Pure Mathematics modules. It is interesting to remark that the core topics include differentiation, integration and simple differential equations, see [7].

Application for entry to an English University is made through the Universities & Colleges Admissions Service (UCAS), see http://www.ucas.ac.uk. Students select prospective universities which, based on their application and interview, may make a conditional offer to the student stipulating that they achieve minimum grades (or tariff) in their A-levels. The standard offer at Durham is with AAB= 340 points among the highest tariff scores in England, see Fig. 1 that plots the number of English universities against their standard stipulated tariff scores for entry degree in mathematics. If students subse-

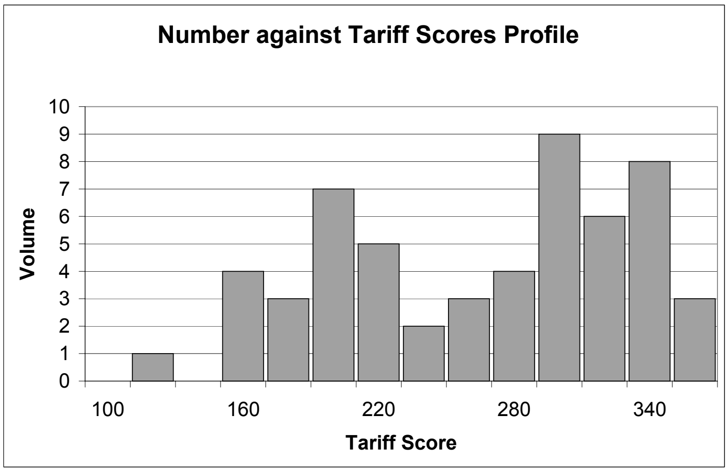

Fig. 1. Number of English universities against their tariff scores

quently fail to achieve their grade they enter into clearing whereby students and university places are matched.

Statistics on the numbers of applications which includes some relevant data (age, male/female, grades achieved etc.) may be found at the UCAS website http://www.ucas.ac.uk/figures/, from which also Fig. 1 has been taken.

Other universities which participated in the test. Some of the colleagues involved in the project used their personal contacts to include in the test groups of students of other than the partner universities: in Germany from the University of Bayreuth (founded in 1975 and youngest German university with more than 9,000 students from all parts of Germany) and the Albert-Ludwig University of Freiburg (which was founded in 1457, and has more than 21,500 students), in France from the University of Bordeaux 3 – Michel de Montaigne (which, founded in 1441 and reformed in 1968-70, has about 15,000 students, essentially in social sciences and humanities) and in Italy from the University of Catania (which was founded in 1434 and has about 14,000 students mainly from Sicily).

Basic differences of the educational systems in the five partner countries

The educational systems and especially the high school systems in participating countries are very different. For a detailed information on the European education systems, the interested reader may consult the EURYDICE data base, EURYBASE [1].

The number of hours in mathematics in the last three years of high school and the curriculum in mathematics are of particular importance in our context. There are already many significant differences in this with respect to the different types of schools in countries (e.g. Italy and France). Table 1 gives an overview of entrance ages, number of years of study and whether entrance is competitive or not in each of participating countries. The third column refers to the participating universities only and we note that other universities may be regulated differently.

Table 1. Educational systems

Country	Minimum entrance age	Number of school years	Competitive entrance
Italy	19	13	no
Germany	19	13	no
England	18	13	yes
Cyprus	18	12	yes
France	18	12	no

The actual average age of students entering university is sometimes higher (e.g. military service, repeating years, mature students). Note that in the UK children enter school in the academic year in which they are five.

In almost every EU partner, students have the possibility of choosing a curriculum with an advanced level of mathematical training, e.g. by choosing an appropriate school or an advanced level course at the school. Thus pupils of a French "lycée d'enseignement général et technologique", on completion of the second class common to all pupils, choose the type of baccalaureate they intend to work towards: this may be one of the three general categories (economic and social "Bac ES", literary "Bac L", or scientific "Bac S") or one of seven technological categories. In Germany, the curriculum varies in accordance with the type of upper secondary education and training. Pupils in the "Gymnasiale Oberstufe" must study subjects from three groups (languages/literature/the arts; social sciences; and mathematics/natural sciences/technology) and can choose an advanced course of mathematics. In England, there are no compulsory subjects at upper secondary level of education. Students choose courses of study from the range offered by the school or further education institution depending upon the qualification they seek. In Italy, however, the mathematical training of the students depends much on the type of school they attend and that they have chosen at the age of 14. There are several types of upper secondary education institutions: Licei (upper secondary schools) of various types (classical, scientific, linguistic, artistic, psycho-pedagogical), Istituti Tecnici (technical schools), Istituti professionali (vocational schools). Though the extent of mathematical training varies widely, any Italian upper secondary school leaving certificate (esame di stato) entitles the holder to study all subjects at university.

The scope and the design of the questionnaire

The objective of the test was to assess the actual level of mathematical knowledge attained by the students of the participating universities and to make a meaningful comparison between them in order to:

- stimulate the discussion on the high school curricula in every participating member state;
- help devise policy of teaching students in their first-year of study at the university;
- facilitate mobility by giving an insight into guiding students and educators where students desire to switch countries.

In view of the differences of the educational systems it was difficult to design a test which reflected the intersection of knowledge in mathematics in the different member countries. The partners agreed upon a test based on the background and skills of mathematics which one should expect from a beginner at university in mathematics, science or engineering. These contain:

- understanding of the basic properties of common mathematical functions such as logarithms, exponentials, trigonometric functions, including the ability to differentiate and integrate functions involving combinations of the above;
- basic logic and understanding of logical operations like negation, implication, equivalence etc.;
- ability to manipulate formulae and inequalities with the standard tools taught in algebra;
- understanding of elementary geometric techniques and solving elementary geometric problems in the plane.

We designed Test A consisting of fourteen questions focussed on seven areas with two questions per theme, see below. The questions 1, 2, 3, 4, 5, 8, and 9 were taken from the students' guide of the Italian Mathematical Union [11] and other questions were based on Durham's diagnostic test for engineering students. The questions grouped by arguments had been:

Logarithms and exponentials (questions 2 and 9)

2. The solution of the equation $\log_2(\log_3 x) = 3$ is
   a) $x = 3$ b) $x = 34$ c) $x = 36$ d) $x = 38$
   e) none of the above answers is correct
9. The number $\sqrt{0.9}$ is equal to
   a) 0.3 b) 0.81 c) a number between 0.81 and 0.9
   d) a number between 0.9 and 1 e) none of the above answers is correct

Equations and inequalities (questions 4 and 11)

4. The inequality $\frac{x^2-1}{x} > 0$ holds
   a) for each $x \neq 0$ b) only for $x > 1$ c) only for $x < -1$
   d) only for $x < -1$ and for $x > 1$
   e) none of the above answers is correct
11. The following fractions $\frac{3}{7} + \frac{1}{8}$ and $\frac{1}{\sqrt{3}-1} + \frac{1}{\sqrt{3}+1}$ expressed in the form $m + n\sqrt{3}$ are equal to
   a) $4/5$ and $\frac{1}{3}\sqrt{3}$ respectively b) $31/56$ and $\frac{1}{3}\sqrt{3}$ respectively
   c) $4/5$ and $\sqrt{3}$ respectively d) $31/56$ and $\sqrt{3}$ respectively
   e) none of the above answers is correct

Logic (questions 1 and 8)

1. The product of seven integers is negative. This implies that
   a) all of the numbers are negative
   b) one is negative and the others are positive
   c) three are negative and the others are positive
   d) five are negative and the others are positive
   e) none of the above answers is correct
8. The phrase "it is not true that all students are diligent" is equivalent to the phrase
   a) all students are not diligent b) at least one student is not diligent
   c) no student is diligent d) at least one student is diligent
   e) none of the above answers is correct

Differential calculus (questions 6 and 13)

6. The coordinates and nature of the turning points on $y = 36x - 3x^2 - 2x^3$ are
   a) $(-2, -68)$ is a minimum and $(3, 27)$ is a maximum
   b) $(2, 44)$ is a minimum and $(-3, -81)$ is a maximum
   c) $(2, 44)$ is a maximum and $(-3, -81)$ is a minimum
   d) $(-2, -68)$ is a maximum and $(3, 27)$ is a minimum
   e) none of the above answers is correct
13. The derivative of $(1 - x^2)\ln(1 - x^2)$ with respect to x is
   a) $-2x + 2x\ln(1 - x^2)$ b) $2x - 2x\ln(1 - x^2)$
   c) $-2x + 2x^2\ln(1 - x^2)$ d) $1 - 2x\ln(1 - x^2)$
   e) none of the above answers is correct

Integral calculus (questions 7 and 14)

7. Using integration by parts, the integral $\int_0^\pi x \sin x \, \mathrm{d}x$ is
   a) $\frac{1}{2}\sin(\pi 2)$ b) -2 c) $-\pi$ d) 0
   e) none of the above answers is correct
14. The integral $\int_{-2/3}^{-1/3} (3x + 2)^n \, \mathrm{d}x$ $(n > 1)$ is
   a) $\frac{1}{n+1}$ b) $\frac{3}{n}$ c) $\frac{1}{3(n-1)}$ d) $\frac{1}{3(n+1)}$
   e) none of the above answers is correct

Geometry (questions 5 and 12)

5. A triangle ABC has the angles in B and C of 30° and two sides of 40 cm. Relative to the side BC the height is equal to
   a) $10\sqrt{3}$ cm b) 20 cm c) $20\sqrt{3}/3$ cm d) 80 cm
   e) none of the above answers is correct
12. The two lines in the graph (Fig. 2) meet at

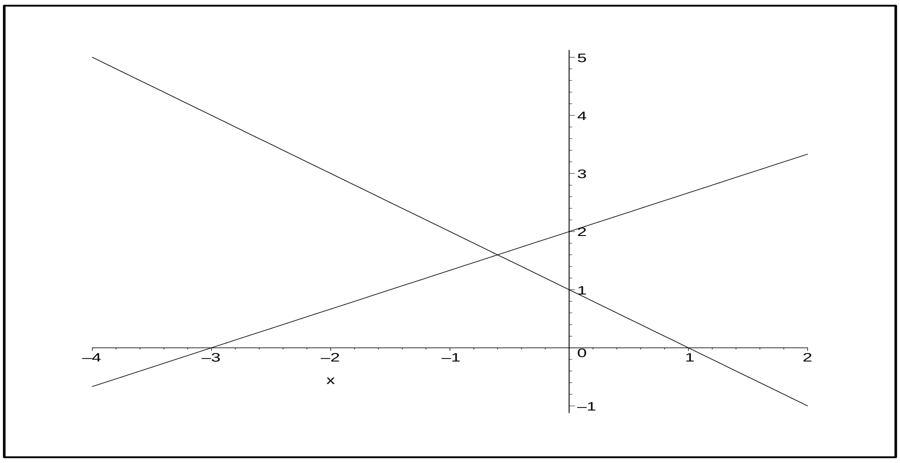

Fig. 2. Graph of Exercise 12

a) $x = -1$ and $y = 2$ b) $x = -2/3$ and $y = 5/3$
c) $x = -3/5$ and $y = 8/5$ d) $x = -11/20$ and $y = 31/20$
e) none of the above answers is correct

Trigonometry (questions 3 and 10)

3. The equation $\sin(2x) = 2\sin x$ holds
   a) for each x b) only for $x = 2k\pi$ with k an arbitrary integer
   c) only for $x = k\pi$ with k an arbitrary integer d) for no value of x
   e) none of the above answers is correct
10. Which of the following graphs (Fig. 3) (a) (b) (c) (d) (e) none of them

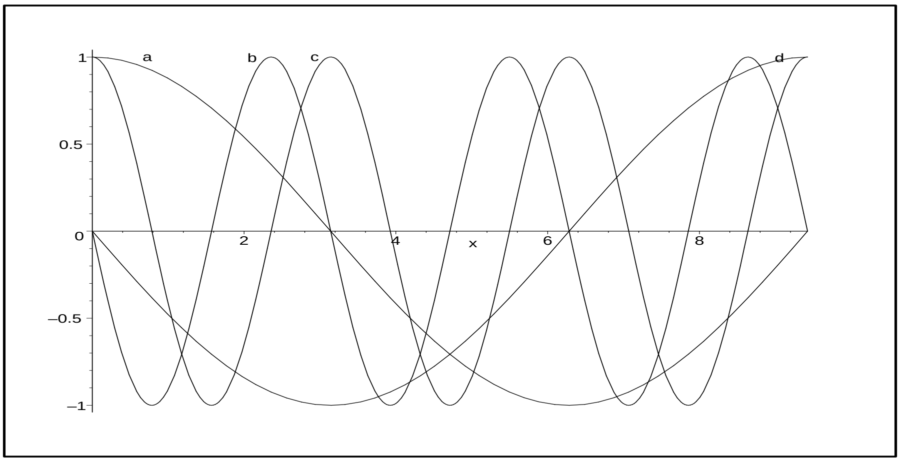

Fig. 3. Graph of Exercise 10

are that of the function $\sin(2x + \pi/2)$?

The questions were designed according to the following criteria:

- to test technical and computational skills and comprehension;
- to reduce the possibility of students guessing by a "none of the above answer is correct" option;
- to trap standard errors;
- to require no calculator;
- to be easy to mark;
- to test the seven different themes in the first seven questions.

The multiple-choice test (exactly one correct answer out of five possibilities) was designed to take fifty minutes. The test questions were given to the students in different orderings so that, in addition to this version A, a second version B was produced by reversing the order.

Test A was presented in Cyprus, England and Germany. In Germany, Test B was also given, but only at the University of Bayreuth were the test results significantly different: for version A (given to 109 students) 45% correct answers over all students and for version B (given to 112 students) 52% correct answers over all students. Other orderings were used in France (1, 3, 5, 7, 9, 11, 13, 2, 4, 6, 8, 10, 12, 14) and in Italy (1, 7, 9, 6, 2, 12, 13, 3, 4, 10, 5, 8, 11, 14) where the questions had been ordered with respect to their (presumed) difficulty, but still matching the above criteria. This choice had been suggested by some faculty members who were concerned that the students could be discouraged by "difficult" questions at the beginning of the test. It turned out that the presumed difficulty did not reflect the real strengths and weaknesses of the students.

In England and Germany it was explicitly stated that there was only one correct answer among the possible choices. In Italy this was relayed to some classes orally.

Unfortunately, we initiated a collaboration with the colleagues S. Mignani and R. Ricci from the Department of Statistics of the University of Bologna only after the questionnaire had already been designed and partially distributed. We have to thank Mignani and Ricci for a deep analysis of the results and for a critical examination of the questionnaire, see their paper [9], published in this volume. Applying techniques of Item Response Theory, they found out that the questionnaire did not grade low abilities of students fine enough.

The participants of the test, collection of data and results

A total number of 3441 students participated in the test. The students were drawn from mathematical, physical and natural sciences, from engineering and from other faculties with service courses in mathematics. The number of students varied considerably from country to country and from university to university: University of Bochum (316), University of Bayreuth (221), University of Freiburg (350), University of Cyprus (196), University of Durham (392), University of Paris 7 (46), University of Bordeaux (60), University of Bologna (1648), University of Catania (275). The number of students of the University of Bologna involved was remarkable (although engineering students could not participate since they had been subjected to another test conducted by a consortium of several engineering faculties), and a detailed statistical study has been carried out on their answers, see Table 3.

The data collected allowed comparisons by question and by discipline. Where available, additional information was provided about the level of mathematics courses attained by the students. In addition, the University of Cyprus, the University of Durham and the German universities listed their results by gender. However, the results did not reflect any significant gender imbalance. Moreover, the German data contained information whether the students attended a basic or an advanced course of mathematics at school. The French and the Italian data distinguished between incorrectly and unanswered questions.

The outcome of the test is given in Table 2.

Table 2. Outcome of the test

Discipline	Correct answers (in %)				
	Italy	Germany	UK	Cyprus	France
Physics	57.2			42.0	
	47.0 (Catania)				
Science		32.4			55.1
		49.0 (Bayreuth)			
		47.5 (Freiburg)			
Computer Sciences	54.3 (Cesena)			49.9	
Mathematics	48.8	45.2	67.0	55.6	
	42.1 (Catania)	47.9 (Freiburg)			
Engineering	45.5 (Catania)	37.5	56.3	46.9	
Industrial Chemistry	48.6				
	22.1 (Faenza)				
Preservation of the Cultural Heritage	44.8				
Biotechnology	43.9				
Statistical Sciences	40.9				
Chemistry and Chem. of Materials	40.4			46.1	
Astronomy	39.8				
Informatics	38.6				
Pharm. Chem. and Technology	36.7				
Architecture	36.6 (Catania)				
Pharmacy	33.0				
Chem. Tech. Envir. Waste Management	32.5 (Rimini)				
Internet Sciences	31.3				
Economics	29.4				
Natural Sciences	26.6		61.3		
Biological Sciences	25.8				
Agriculture	22.4 (Imola)				
Agriculture	10.4				
Other		31.9	54.6		
		51.7 (Freiburg)			
History of Science and Logic					21.4 (Bordeaux)

The results show a striking difference in the mathematical skills between students of universities with entry examinations and universities with free access. Among those with entry examinations, the highest score on the test was achieved by the University of Durham with 61% correct answers over all students, 67% correct answers by Mathematics students, 56% of Engineering students and 55% from other disciplines.

It should be noted that entrance to English universities is competitive and offers to students are normally made subject to sufficiently good A-levels or Scottish Highers - in fact only 11 out of the 392 tested students did not take this route. The maximum achievable tariff score for three A-levels is 360 (three grade A's), the average tariff for Mathematics students of Durham was in the region of 340, the second highest score. Hence, one can say with confidence that the Mathematics students of Durham are a very good group which appears to be homogeneous. Analogously, to be enrolled at the University of Cyprus, students compete through an entrance exam and admission to the School of Engineering requires a minimum grade in the entrance exam.

The other countries' universities scored lower without significant differences, the Mathematics students always being better than those of other disciplines or at least in the group of the strongest disciplines. Hence it can be expected that in general the students' mobility between the test countries is not constrained by their mathematical preparation.

It should be noted that almost two thirds of the students of other disciplines have a completely unsatisfactory mathematical preparation: evidently they will need mathematical tools in their subject.

As for German universities, one can observe that students with an advanced course in mathematics at school clearly outperformed students with a basic course at school (with a margin of more than 10% correct answers).

Table 3 shows the results of the University of Bologna with respect to the various disciplines grouped by cluster analysis. The Mathematics students and the students of Biotechnology are in the first group. It should be noted that Biotechnology is the only discipline (among those listed in Table 3) with competitive entrance and a restricted number of admissions. It should be also noted that the last group has a lower score than the other groups for every single question, except the last one.

An analysis of the results of Mathematics students by question, see Table 4 and Figure 4, shows that in all countries students appeared to be weakest at basic trigonometry (questions 3 and 10), differentiation (in some countries question 6, in other countries question 13) and integration (questions 7 and 14). In general students did best where numeracy was tested (question 11).

Comparison with the studies PISA and TIMSS

Of course, our relatively small project of testing students' mathematics preparation at the very beginning of their university career can hardly be

Table 3. Results of Bologna grouped by cluster analysis

Discipline	Correct answers (in %)
Physics	57.2
Computer Sciences (Cesena)	54.3
Mathematics	48.8
Industrial Chemistry (Bologna)	48.6
Preservation of the Cultural Heritage	44.8
Biotechnology	43.9
Statistical Sciences (Bologna)	40.9
Chemistry and Chemistry of Materials	40.4
Astronomy	39.8
Informatics (M–Z)	38.6
Pharmaceutical Chemistry and Technology	36.7
Pharmacy	33.0
Chem. Techn. Environment Waste Management	32.5
Internet Sciences	31.3
Economics	29.4
Natural Sciences	26.6
Biological Sciences	25.8
Agriculture (Imola)	22.4
Industrial Chemistry (Faenza)	22.1
Agriculture	10.4

Table 4. Results of Mathematics students by question

University	Number of students	Correct answers per question (in %)						
		Qu 1	Qu 2	Qu 3	Qu 4	Qu 5	Qu 6	Qu 7
Bologna	74	51	57	18	55	64	36	18
Bochum	54	63	37	31	7	48	63	24
Freiburg	107	79	27	24	16	44	52	20
Durham	105	79	65	39	22	76	77	56
Cyprus	38	39	81	47	39	78	28	31
Paris 7	46	63	93	61	76	22	72	61
All	424	67	55	34	32	57	57	35
		Qu 8	Qu 9	Qu 10	Qu 11	Qu 12	Qu 13	Qu 14
Bologna	74	95	42	26	86	65	49	23
Bochum	54	87	69	24	65	59	31	24
Freiburg	107	92	77	51	65	58	32	35
Durham	105	94	83	54	92	78	52	70
Cyprus	38	81	39	42	94	78	81	39
Paris 7	46	80	54	43	37	24	57	28
All	424	90	65	42	75	62	47	40

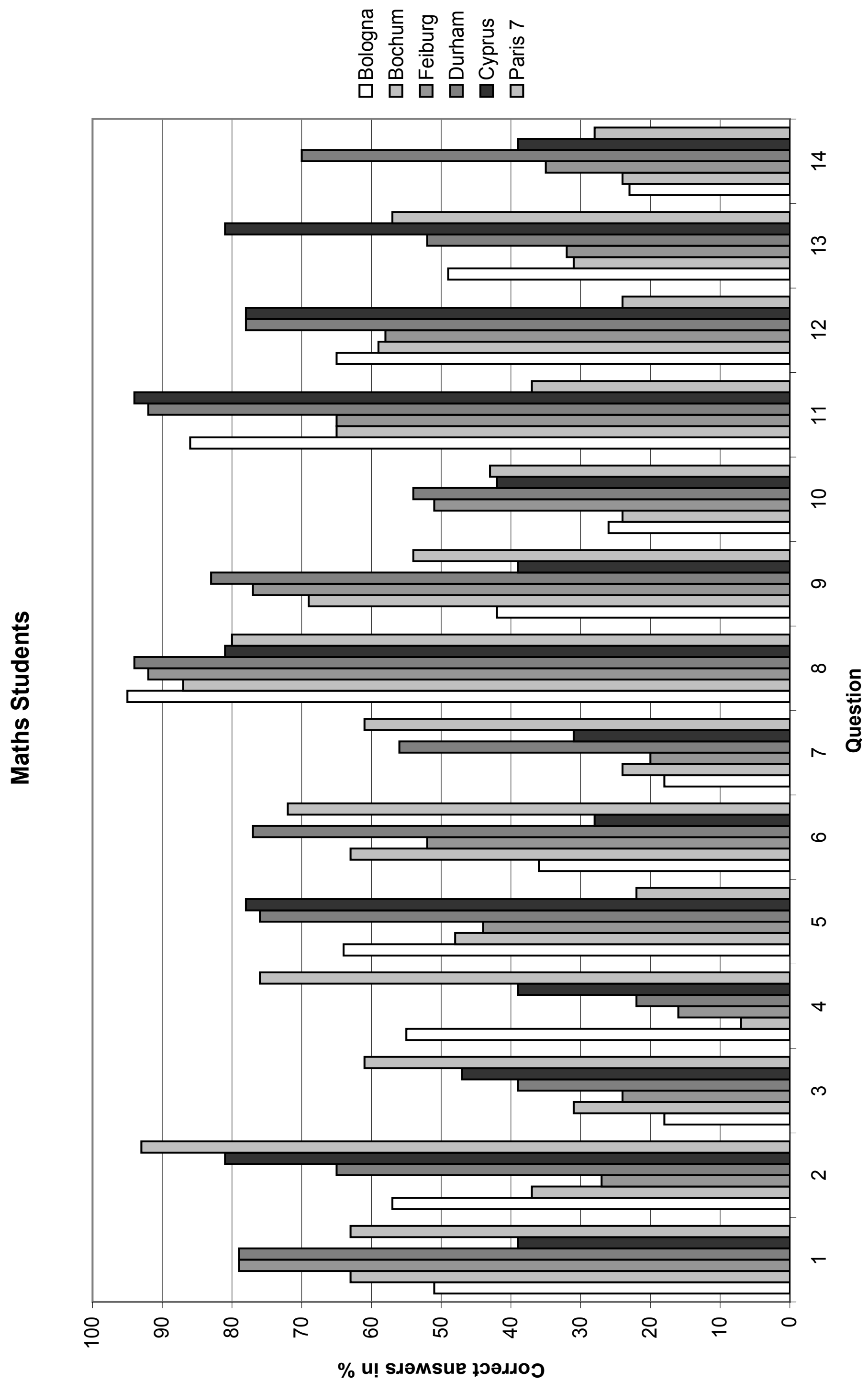

Fig. 4. Results of Mathematics students by question

compared with the huge projects of assessment of students' mathematics and science achievements as PISA and TIMSS, but nevertheless it is interesting to confront them.

Every three years PISA (Programme for International Student Assessment of the Organisation for Economic Co-operation and Development) assesses to what extent students near the end of compulsory schooling have acquired some of the knowledge and skills that are essential for full participation in society. The first PISA survey [8], with a focus on reading, was conducted in 2000, while the second PISA survey [5], with a focus on mathematics and problem solving, was conducted in 2003.

In 2000, about 265.000 students from 32 countries (including all partner countries except Cyprus) took part. Mathematics was only a minor domain in PISA 2000, and the scope of the assessment in this area was more limited, with an emphasis on *change and relationships* and *space and shape*. These concepts were selected to allow a wide range of curriculum strands to be represented, without giving undue weight to number skills.

In 2003, well over a quarter of a million students in 41 countries took part in a two-hour test in their schools, assessing their skills in mathematics, reading, science and problem solving. All 30 OECD member countries participated, as well as 11 partner countries. However, in the United Kingdom the response rate was too low to ensure comparability. Again, Cyprus did not take part.

The PISA 2003 results in mathematics can be found in [5], [10]. They are reported on four content scales: "space and shape", "change and relationships", "quantity", "uncertainty". The rank order positions in mathematics for each partner country (if available) based on sample (with 95% confidence) are given in Table 5.

Table 5. PISA mean scores in mathematics of the partner countries

	Mean score	Standard error	Upper rank	Lower rank
		PISA 2000		
United Kingdom	529	(2.5)	6	10
France	517	(2.7)	10	15
OECD average	**500**			
Germany	490	(2.5)	20	22
Italy	457	(2.9)	26	28
		PISA 2003		
France	511	(2.5)	11	15
OECD average	**500**			
Germany	503	(3.3)	14	18
Italy	466	(3.1)	25	26

While seven OECD countries (e.g. Italy) have very similar results across content areas, 11 (e.g. France and Germany) show especially great differences. Table 5 confirms essentially the results of our test. In fact, in PISA 2000 the United Kingdom is in the first place, followed by France, Germany and Italy. Most countries' relative positions in the PISA 2003 survey remained broadly similar to those in PISA 2000. As for the EU partner countries, one observes that a smaller but still noteworthy improvement in the assessment area of mathematics occurred in Germany.

PISA defines mathematical literacy as the ability to formulate and to resolve mathematical problems in situations encountered in life. Thus in the PISA study much emphasis was placed in applying mathematics to real life problems in different situations, to formulate and communicate the outcomes.

In contrast, our test was designed to assess the mathematical knowledge and skills as they are taught in school (see at the beginning of the section "The scope and the design of the questionnaire"). Another difference is that the students tested in the PISA project were 15 years old whereas our test was given to a self-selected and mathematically more mature group of students entering university.

PISA confirms that, by age 15, gender differences are visible in most countries, with males performing better, particularly at the high end of the performance distribution, but overall the gender gap tends to be small. When interpreting the observed gender differences, it needs to be taken into account that males and females, in many countries at least, make different choices in terms of the schools, tracks and educational programmes they attend. By contrast, the results of our test did not reflect any gender imbalance.

Since its inception in 1959, the International Association for the Evaluation of Educational Achievement (IEA) has conducted a series of international comparative studies designed to provide information to policy-makers, educators, researchers, and practitioners about educational achievement and learning contexts. TIMSS (Trends in International Mathematics and Science Study, formerly known as the Third International Mathematics and Science Study) is the largest and most ambitious of these studies. Every four years TIMSS provides data on the mathematics and science achievement of U.S. students compared to that of students in other countries, see [13]. TIMSS achievement testing in mathematics and science included about 50 countries and 5 grade levels (3rd, 4th, 7th, 8th, and final year of secondary school) and was conducted in 1995, 1999, and 2003. Mathematical content areas of TIMSS include Fractions and number sense, Measurement, Data representation, analysis, and probability, Geometry, and Algebra.

As for the partner countries, the results of the study TIMSS 2003 are reported in Table 6. Note that France and Germany did not take part in the assessment. Again, England achieved one of the highest scores, but it is interesting to remark that Italy is a little above the international averages, while in the PISA studies (15-year-olds) it is much below the international average.

Table 6. TIMSS 2003 scores in mathematics of the partner countries

	Score (standard error)
4th year of secondary school	
England	531 (3,7)
Cyprus	510 (2,4)
Italy	503 (3,7)
International average	**495(0,8)**
8th year of secondary school	
England*	498 (4,7)
Italy	484 (3,2)
International average	**467(0,5)**
Cyprus	459 (1,7)

* Sampling of the classes not approved

For more detailed information on the results of PISA and TIMSS the interested reader is referred to [10] and [13], respectively. The results concerning Italy, also with respect to specific geographic areas and school types, may be found at the website of the Istituto Nazionale per la Valutazione del Sistema dell'Istruzione [6].

Conclusion

Though not giving definitive answers, the results of the test supply significant information on the level of the mathematical knowledge of students on entrance to university. The students of the University of Durham, that is, one of the top universities in the UK, achieved the highest score, but a precise ranking of the other countries cannot be determined by the test. Particular weaknesses were met in trigonometry, differential calculus and integral calculus. However, it should be noted that in some countries, depending on the type of school, the curriculum does not comprise differential and integral calculus (even if it would be desirable). In order to facilitate the mobility of students it is of the utmost importance to harmonise the basic knowledge taught to students in all EU countries. Table 4 may be useful to those drawing up curricula for the first year of study at university in order to realise potential weaknesses of students wishing to switch countries.

The test does not cover all aspects of curricula in the partner countries. For instance, a typical English student would enter university having studied Mechanics in Mathematics which is not the case in any of the other partner countries.

The results of the test indicate that the students who have not taken advanced mathematics at high school are severely disadvantaged if they study mathematics or science in another EU country.

The test was repeated with first year undergraduate students from the University of Cyprus (with a sample of 230 students), the University of Durham (with a sample of 361 students) and the Ruhr-University Bochum in September and October 2004. The results essentially confirmed those of the year before, but Durham's students achieved with 65% (61% in 2003) a better percentage of correct answers over all students. This increase in percentage is probably due to the fact that in 2004 for the first time students were sent material with which they could self-assess their progress prior to their arrival in Durham.

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