AM-2B 10/9/2013

(1) $z^8 - 7i z^4 - 12 = 0$

$\parallel$

$(z^4)^2 - 7i \cdot z^4 - 12$

$\Delta = (-7i)^2 + 4 \cdot 12 = -49 + 48 = -1 = i^2$

$$z^4 = \frac{7i \pm i}{2} = 4i,\ 3i$$

(1.1) $z^4 = 4i = 4e^{i\pi/2}$

$z = \sqrt[4]{4} \cdot e^{i\left(\pi/8 + \frac{k}{2}\pi\right)}$ $\quad k = 0,1,2,3$

$= \sqrt{2} \cdot \left[\cos\left(\frac{\pi}{8} + \frac{k}{2}\pi\right) + i \sin\left(\frac{\pi}{8} + \frac{k}{2}\pi\right)\right]$ $\quad k = 0,1,2,3$

(1.2) $z^4 = 3i \iff z = \sqrt[4]{3} \cdot \left[\cos\left(\frac{\pi}{8} + \frac{k}{2}\pi\right) + i \sin\left(\frac{\pi}{8} + \frac{k}{2}\right)\right]$ $\quad k = 0,1,2,3$

(2) $\arctan(x^\gamma) \underset{x \to 0+}{\sim} x^\gamma$

$\arctan(x^\gamma) \underset{x \to +\infty}{\sim} \pi/2$

se $f(x) = \dfrac{\arctan(x^\gamma)}{x^{2\gamma} + x^{5\gamma}}$ allora

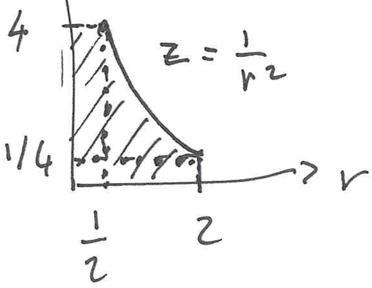
$f(x) \underset{x \to 0+}{\sim} \dfrac{x^\gamma}{x^{2\gamma}} = \dfrac{1}{x^\gamma}$: l'int. converge vicino a 0 $\iff \gamma < 1$

$f(x) \underset{x \to +\infty}{\sim} \dfrac{\pi/2}{x^{5\gamma}}$: l'int. conv. vicino a $+\infty \iff 5\gamma > 1$

l'int. converge $\iff \frac{1}{5} < \gamma < 1$

(3) Posto $\sqrt{\dfrac{x^2}{4} + \dfrac{y^2}{25}} = r \geq 0$:

$$\begin{cases} r^2 z \leq 1 \\ 0 \leq r \\ 0 \leq z \leq 4 \\ r^2 \leq 4 \end{cases}$$



$A = \left\{ (x,y) \in \mathbb{R}^2 : \sqrt{\dfrac{x^2}{4} + \dfrac{y^2}{25}} \leq 2 \right\}$

$$I(x,y) = \begin{cases} [0, 4] \text{ se } \sqrt{x^2/4 + y^2/25} \leq 1/2 \\ \left[0, \dfrac{1}{\frac{x^2}{4} + \frac{y^2}{25}}\right] \text{ se } \frac{1}{2} \leq \sqrt{\dfrac{x^2}{4} + \dfrac{y^2}{25}} \leq 2 \end{cases}$$