AN. MAT. L-B | 9/9/2010

$\boxed{\text{Es. 1}}$ $z^3 = i = e^{i\pi/2} \iff z = e^{\frac{\pi i}{6} + \frac{2k\pi i}{3}}$

$= \cos\left(\frac{\pi}{6} + \frac{2k\pi}{3}\right) + i \cdot \sin\left(\frac{\pi}{6} + \frac{2k\pi}{3}\right) \qquad k = 0, 1, 2$

o $\quad z^2 - 3(1+i)z + 3i = 0$

$\Delta = 9(1+i)^2 - 12i = 9 \cdot 2i - 12 \cdot i = 6i = 6 \cdot e^{\frac{\pi i}{2}}$

$z = \dfrac{3(1+i) \pm \sqrt{6} \cdot e^{\pi i/4}}{2} = \dfrac{3(1+i) \pm \sqrt{6}\,(1+i)/\sqrt{2}}{2}$

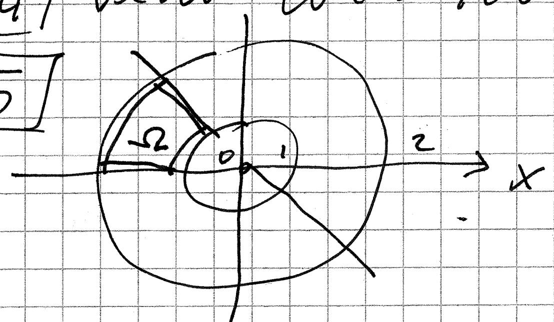
$= \dfrac{3(1+i) \pm \sqrt{3}(1+i)}{2}$

$\boxed{\text{Es. 2}}$ Vedi Correzione A. M. II

$\boxed{\text{Es. 3}}$ Vedi Correzione A. M. II

$\boxed{\text{Es. 4}}$ Vedi Correzione A. M. II

$\boxed{\text{Es. 5}}$



$x = r\cos\theta$

$y = r\sin\theta$

$1 \le r \le 2$

$\frac{3}{4}\pi \le \theta \le \pi$

$I = \int_1^2 r\,dr \int_{\frac{3}{4}\pi}^{\pi} d\theta\,(1 - 2r\cos\theta) = \frac{\pi}{4}\int_1^2 r\,dr - 2\int_1^2 r^2 dr \cdot (\sin\theta)_{\frac{3}{4}\pi}^{\pi}$

$= \frac{\pi}{4}\,\frac{1}{2}(4-1) - 2 \cdot \frac{1}{3} \cdot (8-1) \cdot \left(0 - \frac{1}{\sqrt{2}}\right)$

$= \frac{3}{8}\pi + \sqrt{2} \cdot \frac{7}{3}$

$\boxed{\text{Es. 7}}$ $\dfrac{1+x^\gamma}{x^\gamma + x^{3\gamma}} \underset{x\to\infty}{\sim} \dfrac{1}{x^{2\gamma}}$ e $\displaystyle\int_1^{+\infty} \frac{dx}{x^{2\gamma}}$ conv. $\iff \gamma > \frac{1}{2}$

$\dfrac{1+x^\gamma}{x^\gamma + x^{3\gamma}} \underset{x\to 0+}{\sim} \dfrac{1}{x^\gamma}$ e $\displaystyle\int_0^1 \frac{dx}{x^\gamma}$ conv. $\iff \gamma < 1$

L'integrale converge $\iff \frac{1}{2} < \gamma < 1$

Es. 6 Poiché $-1 \le x \le 1$, $-1 \le -x \le z \le 1-x \le 1-(-1) = 2$

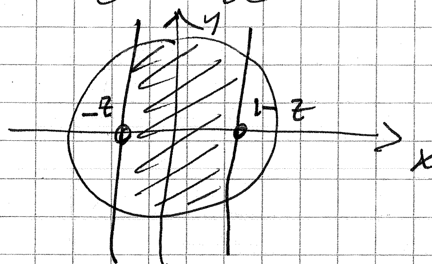
Quindi $a = -1$ e $b = -1$

Per $-1 \le z \le 2$, ho $-z \le x \le 1-z$

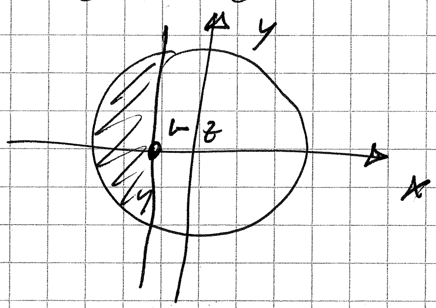
$A(z) = \{(x,y) : x^2+y^2 \le 1$ e $-z \le x \le 1-z\}$

Per essere più precisi, si hanno i casi:

- $0 \le z \le 1 \Rightarrow A(z) = \{(x,y) : x^2+y^2 \le 1$ e $-1 \le -z \le x \le 1-z \le 1\}$



- $1 \le z \le 2 \Rightarrow A(z) = \{(x,y) : x^2+y^2 \le 1$ e $x \le 1-z \le 0\}$



- $-1 \le z \le 0 \Rightarrow A(z) = \{(x,y) : x^2+y^2 \le 1$ e $0 \le -z \le x\}$