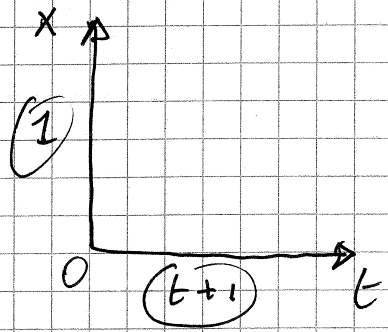
ES. 2



Cerco le curve caratteristiche $x = x(t)$:

$$\frac{d}{dt} v(x(t), t) = v_x \cdot \dot{x} + v_t$$

$$:= v_t + e^x \cdot v_x \iff \boxed{\dot{x} = e^x}$$

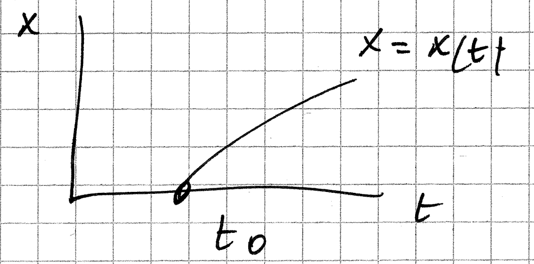
Per curve caratteristiche ~~che~~ aventi inizio sull'asse t:

$$\begin{cases} \frac{dx}{dt} = \dot{x}(t) = e^x \\ x(t_0) = 0 \end{cases} \qquad \begin{cases} e^{-x}\, dx = dt \\ x(t_0) = 0 \end{cases}$$

$$\int_{0 = x(t_0)}^{x(t)} e^{-y}\, dy = \int_{t_0}^{t} ds = t - t_0$$

$$\| \quad \left(-e^{-y}\right)_0^{x(t)} = 1 - e^{-x(t)}$$

$$\iff x(t) = \cancel{1 + t_0 - t} \log(1 + t_0 - t)^{-1}$$

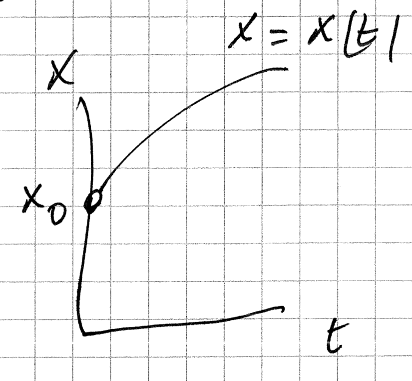


Per curve caratteristiche aventi inizio ~~su~~ sull'asse x:

$$\begin{cases} \frac{dx}{dt} = \dot{x} = e^x \\ x(0) = x_0 \end{cases} \qquad \int_{x_0}^{x(t)} e^{-y}\, dy = \int_0^t ds = t$$

$$e^{-x_0} - e^{-x(t)}$$

$$\iff x(t) = \log\left(e^{-x_0} - t\right)^{-1}$$



Pongo $\varphi(t) = v(x(t), t)$:

$$\begin{cases} \dot{\varphi}(t) = \frac{d}{dt} v(x(t),t) = A \cdot v(x(t),t) = A \cdot \varphi(t) \\ \varphi(t_0) = v(x(t_0),t_0) = v(0,t_0) = t_0+1 \quad \text{se } t_0 \geq 0 \end{cases}$$

$$\varphi(t) = K \cdot e^{At}$$

$$\varphi(t_0) = K \cdot e^{At_0} = t_0+1 \ : \quad K = e^{-At_0}(t_0+1)$$

$$v(x(t),t) = e^{-At_0}(t_0+1)$$

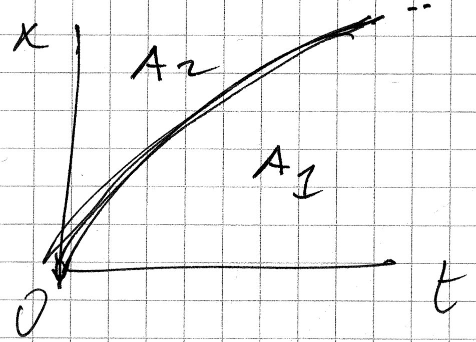
Poiché $t - t_0 = 1 - e^{-x}$,

$$t_0 = t - 1 + e^{-x} \quad \text{e}$$

$$\boxed{v(x,t) = e^{-A(t-1+e^{-x})} \cdot (t+e^{-x})}$$

sulla regione A_1 individuata dalle curve caratteristiche che finiscono sull'asse t:

$$x = \log(1-t)^{-1}$$



sull'altra regione, A_2, $\begin{cases} \dot{\varphi} = A\varphi \\ \varphi(0) = v(x_0,0) = 1 \end{cases}$

$\varphi(t) = K \cdot e^{At}$ con $\varphi(0) = K = 1$:

$$\varphi(t) = e^{At}.$$

Cioè, $\boxed{v(x,t) = e^{At} \quad \text{su } A_2}$.