II PROVA PARZIALE SCRITTA DI AN. MAT. I

(1)

$$\frac{e^{2x}-\frac{1}{1-2x}}{x\cdot\sin\left(\frac{2x}{3x+2}\right)\cdot\cos(3x)} \underset{x\to0^+}{\sim} \frac{e^{2x}-\frac{1}{1-2x}}{x\cdot\frac{2x}{2}\cdot 1} =$$

$$\boxed{\text{poiché } \sin\left(\frac{2x}{3x+2}\right)\underset{x\to0^+}{\sim}\frac{2x}{3x+2}\underset{x\to0^+}{\sim}\frac{2x}{2}}$$

$$= \frac{1+(2x)+\frac{(2x)^2}{2}+o(x^2)-\left(1+(2x)+(2x)^2+o(x^2)\right)}{x^2}$$

$$= \frac{2x^2-4x^2+o(x^2)}{x^2} = -2+o(1)\xrightarrow[x\to0]{} -2 \implies \boxed{L=-2}$$

(2)

$$\int_0^{1/3} e^{3x}\cdot\left[\log\left(e^{6x}+1\right)+3x\right]dx =$$

$$= \int_0^{1/3} e^{3x}\cdot\log\left(e^{6x}+1\right)dx + \int_0^{1/3} e^{3x}\cdot 3x\,dx =$$

Pongo $z=e^{3x}$ nel primo integrale: $dz=3\cdot e^{3x}dx$, $x\Big|_0^{1/3} \Longleftrightarrow z\Big|_1^{e}$

$$= \frac{1}{3}\int_1^e \log(z^2+1)\,dz + \left[\frac{e^{3x}}{3}\cdot 3x\right]_0^{1/3} - \int_0^{1/3}\frac{e^{3x}}{3}\cdot 3\,dx$$

$$= \frac{1}{3}\left[z\cdot\log(z^2+1)\right]_1^e - \frac{1}{3}\int_1^e z\cdot\frac{2z}{z^2+1}dz + \frac{e}{3} - \left[\frac{e^{3x}}{3}\right]_0^{1/3}$$

$$= \frac{1}{3}\left[e\log(e^2+1)-\log 2\right] - \frac{2}{3}\int_1^e\left\{\frac{z^2+1-1}{z^2+1}\right\}dz + \frac{e}{3} - \frac{e}{3} + \frac{1}{3}$$

$$= \frac{1}{3}\left[e\log(e^2+1)-\log 2\right] - \frac{2}{3}\int_1^e\left(1-\frac{1}{z^2+1}\right)dz + \frac{1}{3}$$

$$= \frac{1}{3}\left[e\log(e^2+1)-\log 2\right] - \frac{2}{3}\left[(e-1)-\left[\operatorname{arctg}(z)\right]_1^e\right] + \frac{1}{3}$$

(3) $f(x) = |x^2-36| \cdot e^{-|x|/8} + 1$

$$= \frac{|x^2-36|}{e^{|x|/8}} + 1 \xrightarrow[x \to \pm\infty]{} 0 + 1 = 1$$

Se $x^2-36 \neq 0$ e $x \neq 0$ $(x \neq 0, \pm 6)$, allora

$$\exists f'(x) = \operatorname{sgn}(x^2-36) \cdot 2x \cdot e^{-|x|/8} + |x^2-36| \cdot \left(-\frac{\operatorname{sgn}(x)}{8}\right) \cdot e^{-|x|/8}$$

$$= \operatorname{sgn}(x^2-36) \cdot \left\{ 2x + (x^2-36) \cdot \frac{-\operatorname{sgn}(x)}{8} \right\} \cdot e^{-|x|/8}$$

$$= -\frac{\operatorname{sgn}(x)}{8} \cdot \operatorname{sgn}(x^2-36) \cdot e^{-|x|/8} \cdot \left[x^2 - 36 - 16 \cdot \operatorname{sgn}(x) \cdot x \right]$$

$$\left.\begin{aligned} f'(6^+) &= 2 \cdot 6 \cdot e^{-6/8} > 0 \\ f'(6^-) &= -2 \cdot 6 \cdot e^{-6/8} < 0 \end{aligned}\right| \Rightarrow \text{non esiste } f'(6)$$

Allo stesso modo considero $x = -6, 0$:

dominio$(f') = \mathbb{R} \setminus \{6, -6, 0\}$.

Studio il segno di f':

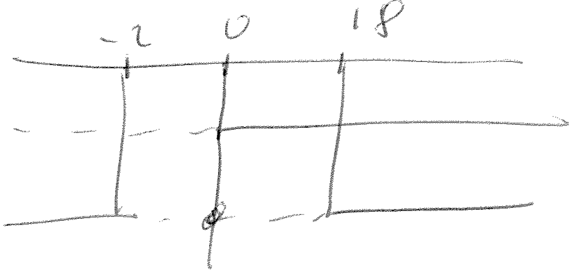
$x^2 - 16 \cdot |x| - 36 \geq 0$.

$\begin{cases} \cancel{x > 0} \\ \cancel{0 \leq x^2 + 16x - 36 = (x+18)(x-2)} \end{cases}$ ⓤ $\begin{cases} \cancel{x < 0} \\ \cancel{0 \leq x^2 + 16x - 36 = (x-18)(x+2)} \end{cases}$

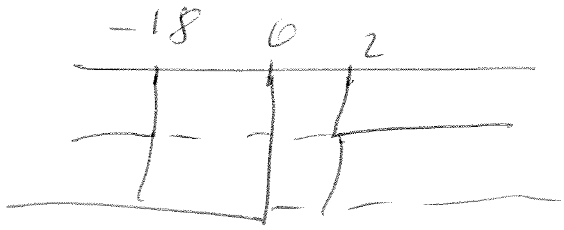
$-18 \quad 0 \quad 2$ $\qquad\qquad$ $\cancel{-2 \quad 0} \quad 18$

$\cancel{x > 2}$ ⓤ $\cancel{x < -2}$

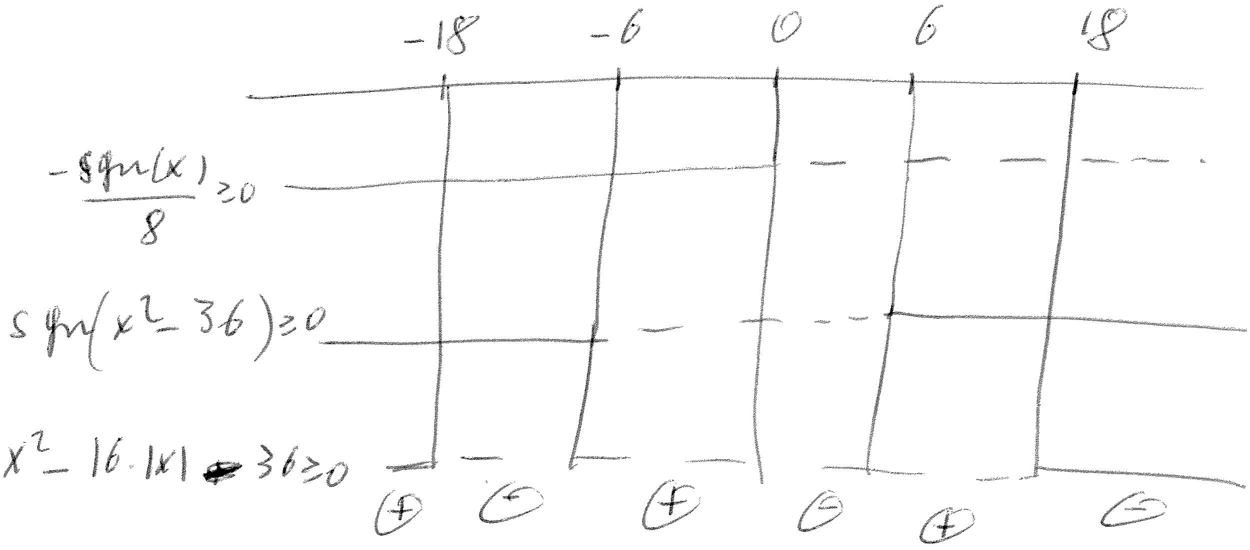
$$\begin{cases} x > 0 \\ x^2 - 16x - 36 \geq 0 \\ \quad \parallel \\ (x-18)(x+2) \end{cases} \quad \text{oppure} \quad \begin{cases} x < 0 \\ x^2 + 16x - 36 \geq 0 \\ \quad \parallel \\ (x+18)(x-2) \end{cases}$$



oppure



$x \geq 18$ oppure $x \leq -18$



f cresce in $]-\infty, -18]$, $[-6, 0]$, $[6, 18]$

e decresce in $[-18, -6]$, $[0, 6]$ e $[18, +\infty[$

PTI MAX rel: $x = -18, 0, 18$.

$f(0) = 37$

$f(\pm 18) = 36 \times 8 \cdot e^{-9/4} + 1$

$\max f = \max\{37,\ 36 \times 8 \cdot e^{-9/4} + 1\}$

grafico di f: