(1) $z^3 = -i$ o $z^2 - 2iz - 1 = 0$ LB.

- $z^3 = -i = e^{-i\pi/2} \iff z = e^{i\left(-\frac{\pi}{6} + \frac{2k\pi}{3}\right)}$

$= \cos\left(-\frac{\pi}{6} + \frac{2k\pi}{3}\right) + i \cdot \sin\left(-\frac{\pi}{6} + \frac{2k\pi}{3}\right)$

con $k = 0, 1, 2$

- $z^2 - 2iz - 1 = 0$ $\quad \frac{\Delta}{4} = (-i)^2 + 1 = -1 + 1 = 0$

$z = i$ (soluzione doppia).

(2) Sia $f = f(u, v)$. Allora:

$\nabla h(x_0, y_0, z_0) =$

$= \Big(f(y_0, z_0) + y_0 \cdot \partial_v f(z_0, x_0) + z_0 \cdot \partial_u f(x_0, y_0),$

$x_0 \cdot \partial_u f(y_0, z_0) + f(z_0, x_0) + z_0 \cdot \partial_v f(x_0, y_0),$

$x_0 \cdot \partial_v f(y_0, z_0) + y_0 \cdot \partial_u f(z_0, x_0) + f(x_0, y_0) \Big)$

(3) $\begin{cases} f_x(x, y) = 2(x-1)\cdot(x-2) + [(x-1)^2 - y^2] \\ f_y(x, y) = -2y\cdot(x-2) \end{cases}$

$f_y(x, y) = 0 \iff y = 0$ o $x = 2$

$\begin{cases} y = 0 \\ 0 = 2(x-1)(x-2) + (x-1)^2 = (x-1)\cdot(2x - 4 + x - 1) = (x-1)(3x-5 \end{cases}$

$\begin{cases} y = 0 \\ x = 1 \text{ o } x = 5/3 \end{cases}$

o $\begin{cases} x = 2 \\ (x-1)^2 - y^2 = 0 \end{cases}$ $\quad \begin{cases} x = 2 \\ y^2 = 1 \end{cases}$ $\quad \begin{cases} x = 2 \\ y = \pm 1 \end{cases}$

I punti ~~cou~~ critici sono:

$(1,0)$, $(5/3,0)$, $(2,1)$, $(2,-1)$.

$$\mathrm{Hess}\, f(x,y) = \begin{bmatrix} 6x-8 & -2y \\ -2y & -2(x-2) \end{bmatrix}$$

$$\mathrm{Hess}\, f(2,\pm 1) = \begin{bmatrix} 4 & -4 \\ -4 & 0 \end{bmatrix} \to$$ punti di sella perché det Hess < 0

$$\mathrm{Hess}\, f(1,0) = \begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix} \to$$ punto di sella perché det Hess < 0.

$$\mathrm{Hess}\, f\left(\frac{5}{3},0\right) = \begin{bmatrix} 2 & 0 \\ 0 & 2/3 \end{bmatrix} \to$$ punto di min. rel. perché ho autovalori positivi.

$\left(\frac{5}{3},0\right)$ p.to di min. rel.

$(1,0)$, $(2,1)$, $(2,-1)$ p.ti di sella

(4) Eq. Omogenea: $z''-4z=0$

Eq. caratteristica: $\lambda^2-4=0 \iff \lambda=\pm 2$

Int. gen. Omogenea: $z(x) = A\cdot e^{2x} + B\cdot e^{-2x}$

Sol. particolare $y(x) = a\cdot\cos(2x) + b\sin(2x)$

$y'(x) = -2a\cdot\sin(2x) + 2b\cdot\cos(2x)$

$y''(x) = -4a\cdot\cos(2x) - 4b\cdot\sin(2x)$

Sostituisco: $\boxed{LB3}$

$$\sin(2x) = y'' - 4y = -4a\cdot\cos(2x) - 4b\cdot\sin(2x) - 4\big(a\cos(2x) + b\sin(2x)\big)$$

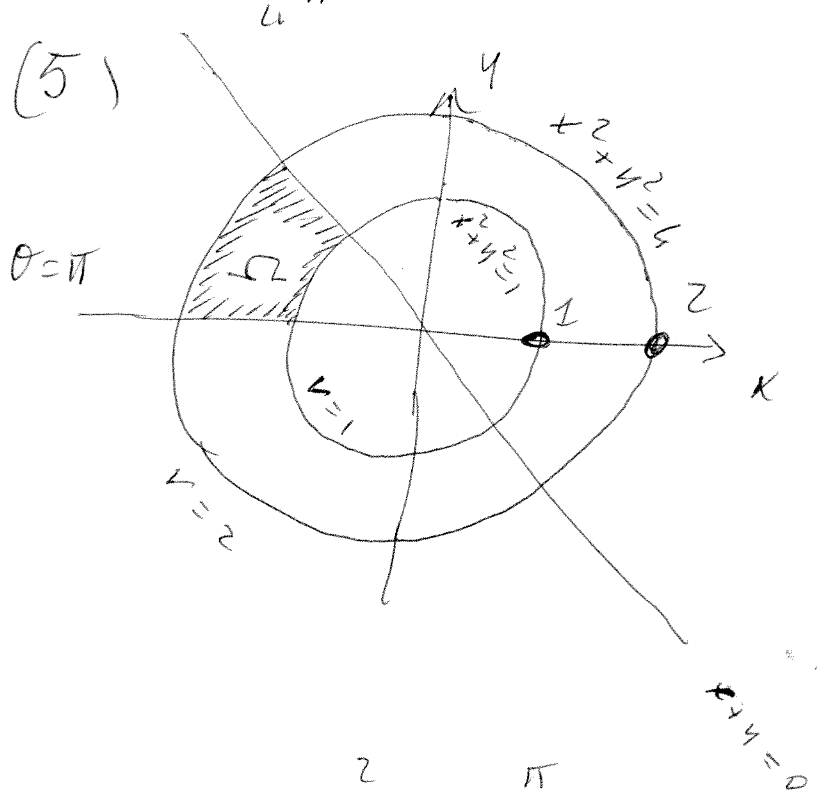
$$= \cos(2x)\cdot[-4a - 4\cdot a] + \sin(2x)\cdot[-4b - 4b]$$

$\Rightarrow a = 0,\ b = -1/8$; l'int. generale è

$$y(x) = A\cdot e^{2x} + B\cdot e^{-2x} - \frac{1}{8}\cdot\sin(2x), \quad \text{con } A, B \in \mathbb{R},\ x \in \mathbb{R}.$$

(5)

$\theta = \frac{3}{4}\pi$



Coordinate polari:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$1 \le r \le 2$

$\frac{3}{4}\pi \le \theta \le \pi$

$dx\,dy = r\,dr\,d\theta$

$$\Rightarrow I = \int_1^2 r\,dr \int_{3/4\pi}^{\pi} d\theta \cdot r\cos\theta =$$

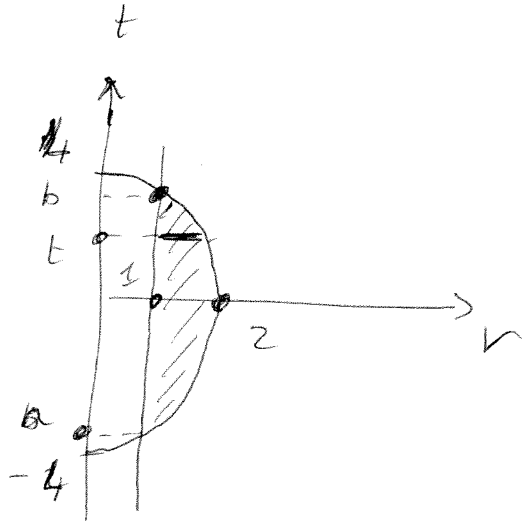
$$= \int_1^2 r^2\,dr \cdot \int_{3/4\pi}^{\pi} \cos\theta\,d\theta = \left(\frac{r^3}{3}\right)\Big|_1^2 \cdot \left(+\sin\theta\right)\Big|_{3/4\pi}^{\pi}$$

$$= \frac{8-1}{3}\cdot\left(-\sin\left(\frac{3}{4}\pi\right)\right) = -\frac{7}{3}\cdot\frac{1}{\sqrt{2}}$$

LB4

(6) Coordinate cilindriche:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ t = z \end{cases} \quad \text{con} \quad \begin{cases} r \geq 0 \\ 0 \leq \theta \leq 2\pi \\ t \in \mathbb{R} \end{cases}$$



$$r^2 + \frac{t^2}{4} \leq 4 \quad ; \quad r^2 \geq 1$$

$t = a$ e b sono soluzioni di:

$$\begin{cases} r^2 + \frac{t^2}{4} = 4 \\ r = 1 \end{cases} \qquad \frac{t^2}{4} + 1 = 4 \qquad t^2 = 12$$

$$t = \pm\sqrt{12} = \pm 2\sqrt{3}$$

$$a = -2\sqrt{3}$$
$$b = 2\sqrt{3}$$

Per $a \leq t = z \leq b$, $\quad 1 \leq r = \sqrt{x^2+y^2} \leq \sqrt{4 - \frac{t^2}{4}}$

$$\boxed{\begin{aligned} a &= -2\sqrt{3} \\ b &= 2\sqrt{3} \\ A(z) &= \left\{ (x,y) : \quad 1 \leq \sqrt{x^2+y^2} \leq \sqrt{4 - \frac{z^2}{4}} \right\} \end{aligned}}$$

(7) $$\sin\left(\frac{1}{n}\right) \underset{n\to\infty}{=} \frac{1}{n} + o\left(\frac{1}{n}\right) \underset{n\to\infty}{\approx} \frac{1}{n} \qquad \text{(Taylor)}$$

$$\left(n + \frac{1}{n}\right)^{\gamma} - 1 = n^{\gamma} \cdot \left[\left(1 + \frac{1}{n^2}\right)^{\gamma} - 1/n^{\gamma}\right] \underset{n\to\infty}{\sim} n^{\gamma} \text{ se } \gamma > $$

$$\Rightarrow \frac{\sin\left(\frac{1}{n}\right)}{\left(n + \frac{1}{n}\right)^{\gamma} - 1} \underset{n\to\infty}{\sim} \frac{\frac{1}{n}}{n^{\gamma}} = \frac{1}{n^{1+\gamma}} \quad e$$

la serie converge (confronto) $\Leftrightarrow \gamma > 0$