

ESERCIZI SUI NUMERI COMPLESSI

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Risolvere le seguenti equazioni in \mathbb{C} .

1. $z^4 + 2 = 0$

2. $z^2 + 1 - 2i = 0$

3. $z^{13} + i = 0$

4. $z^{16} - 1 = 0$

5. $(3 + 4i)z^5 = 12 + 5i$

6. $z^2 - z + 1 = 0$

7. $z^2 - 2iz + 1 = 0$

Soluzioni.

1. $z = \sqrt[4]{2} (\pi/4 + k\pi/2) + i \sin(\pi/4 + k\pi/2)$, $k = 0, 1, 2, 3$
2. $z = \pm (\cos([\pi - \arctan(2)]/2) + i \sin([\pi - \arctan(2)]/2))$
3. $z = (\cos(-\pi/26 + k\pi \frac{2}{13}) + i \sin(-\pi/26 + k\pi \frac{2}{13}))$, $k \in \mathbb{Z}$, $0 \leq k \leq 12$
4. $z = 1, -1, i, -i, \frac{1+i}{\sqrt{2}}, \frac{1-i}{\sqrt{2}}, \frac{-1+i}{\sqrt{2}}, \frac{-1-i}{\sqrt{2}}$ e $z = (\cos(\pi/8 + k\pi/4) + i \sin(\pi/8 + k\pi/4))$, $k = 0, 1, 2, 3$
5. $z = \sqrt[5]{\frac{13}{5}} (\cos([-\arctan(33/56) + 2k\pi]/5) + i \sin([-\arctan(33/56) + 2k\pi]/5))$, $k = 0, 1, 2, 3, 4$
6. $z = \frac{1}{2} \pm i \frac{\sqrt{3}}{2}$
7. $z = i(1 \pm \sqrt{2})$