

ESERCIZI SUI LIMITI DI SUCCESSIONI

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Analisi Matematica L-A

Calcolare i seguenti limiti di successioni

$$\lim_{n \rightarrow \infty} (1 - (-1)^n) \sin(1/n) \quad (1)$$

$$\lim_{n \rightarrow \infty} (\sqrt{2n+1} - \sqrt{n-2}) \quad (2)$$

$$\lim_{n \rightarrow \infty} (\sqrt{n^2 + 2n} - \sqrt{n^2 - n + 4}) \quad (3)$$

$$\lim_{n \rightarrow \infty} (2 - 3e^{-n}) \sqrt{n} (\sqrt{n+2} - \sqrt{n-2}) \quad (4)$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{2n+11} - \sqrt{2n+4}}{\sqrt{n^2+2} - \sqrt{n^2-1}} \quad (5)$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{2n+11} - \sqrt{2n+4}}{\sqrt{n^2+2n} - \sqrt{n^2-n}} \quad (6)$$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{2n+11} - \sqrt{2n+4}}{\sqrt{n^2+2\sqrt{n}} - \sqrt{n^2-\sqrt{n}}} \quad (7)$$

$$\lim_{n \rightarrow \infty} (1 + \cos(n)/n)(1 - \cos(1)) \quad (8)$$

$$\lim_{n \rightarrow \infty} \left(\frac{n+1}{n-1} \right)^{n+3} \quad (9)$$

$$\lim_{n \rightarrow \infty} \frac{n^5 - 3n^3 + 2}{1 - 3n^4 + 2n^8} \quad (10)$$

$$\lim_{n \rightarrow \infty} \cos\left(\frac{\pi n}{3n+1}\right) \frac{n^2 - 3n + 2}{1 - 3n + 2n^2} \quad (11)$$

$$\lim_{n \rightarrow \infty} \frac{2^n + n^2 - \log^{12}(n)}{3^n - n\sqrt[n]{n} + 5} \quad (12)$$

$$\lim_{n \rightarrow \infty} \left(\frac{2^n + n^2 - \log^{12}(n)}{3^n - n\sqrt[n]{n} + 5} \right)^{1/n} \quad (13)$$

$$\lim_{n \rightarrow \infty} \left(\frac{2^n + n^2 - \log^{12}(n)}{3^n - n\sqrt{n} + 5} \right)^{1/n^2} \quad (14)$$

Soluzioni. (1) 0; (2) $+\infty$; (3) $3/2$; (4) 4; (5) $+\infty$; (6) 0; (7) $\frac{7}{6}\sqrt{2}$; (8) $1 - \cos(1)$; (9) e^2 ; (10) 0; (11) $1/4$; (12) 0; (13) $2/3$; (14) 1.

Calcolare

$$\lim_{n \rightarrow \infty} a_n$$

dove $a_n =$

| | | | |
|---|--------------------------------------|--|---|
| (1) $\frac{2^n}{3^n}$ | (2) $\frac{(\log n)^2}{\sqrt{n}}$ | (3) $\frac{\log n}{2^n}$ | (4) $\frac{n \log n}{2^n}$ |
| (5) $\frac{n2^n}{3^n}$ | (6) $\frac{n^{1/10}}{(\log n)^{10}}$ | (7) $\frac{2^{2n}}{3^n}$ | (8) $\frac{n^{100}}{2^{\frac{n}{10}}}$ |
| (9) $\frac{3n+n^2}{2n^2+1}$ | (10) $\frac{n}{n^2+1}$ | (11) $\frac{(\log n)^2-n}{\sqrt{n}+1}$ | (12) $\frac{n^3-2n+n^2}{n-3n^2-2n^3}$ |
| (13) $\frac{\log(n^2)+1}{\log(n)}$ | (14) $\frac{\log(3^n+n^2)}{n}$ | (15) $\frac{n-\log(n)}{\sqrt{n}-2n}$ | (16) $\frac{2^n-4n^4}{3 \cdot 2^n+n^5}$ |
| (17) $\frac{n^2-3n \log(n)}{\log(n)-n^2}$ | (18) $\frac{\sqrt[3]{1+1/n}-1}{1/n}$ | (19) $\frac{e^{1/n}-1}{1/n}$ | (20) $n \log(1/n)$ |

Soluzioni.

| | | | |
|-----------|---------------|----------------|----------------|
| (1) 0 | (2) 0 | (3) 0 | (4) 0 |
| (5) 0 | (6) ∞ | (7) ∞ | (8) 0 |
| (9) $1/2$ | (10) 0 | (11) $-\infty$ | (12) $-1/2$ |
| (13) 2 | (14) $\log 3$ | (15) $-1/2$ | (16) $1/3$ |
| (17) -1 | (18) $1/3$ | (19) 1 | (20) $-\infty$ |