Problem. *Find a geometric characterization of the multipliers and of the Carleson measures for the infinite dimensional Drury-Arveson space.*

Discussion. The *d-dimensional Drury-Arveson space* is the closure of the complex polynomials on the unit ball $\mathbb{B}_d$ of $\mathbb{C}^d$ with respect to the norm

$$\left\| \sum_{n\in\mathbb{N}^d} a_n z^n \right\|^2_{DA_d} = \sum_{n\in\mathbb{N}^d} |a_n|^2 \frac{n!}{|n|!}.$$

Alternatively, DA_d is the Hilbert function space having reproducing kernel $K(z,w) = (1-\overline{z}\cdot w)^{-1}$. The space DA_d and its multiplier space $M(DA_d)$ were introduced by Drury [3] in connection with the multivariable, commutative version of von Neumann's inequality for contractions. The combinatorial, dimensionless nature of the coefficients and the applications to Nevanlinna-Pick Theory [1] motivate the interest in the infinite dimensional version of DA_d. A function g is a *multiplier* of DA_d if $f \mapsto M_g f = gf$ has finite operator norm $|||M_g|||_d$ on DA_d. A measure μ on $\mathbb{B}_d$ is a *Carleson measure* for DA_d if the imbedding $DA_d \hookrightarrow L^2(\mu)$ has bounded norm $[\mu]^{1/2}_{CM(d)}$. Since DA_d can be viewed as a weighted Dirichlet space on $\mathbb{B}_d$, for fixed integer d one has that $|||M_g|||^2_d \approx \left[|R^{(m)}g(z)|^2(1-|z|^2)^{2m-d}dV\right]_{CM(d)}$ if $m > (d-1)/2$ is fixed. (Here, R is the complex radial derivative in $\mathbb{B}_d$). Unfortunately, this estimate depends on d, hence finding dimension independent Carleson measure and multiplier estimates are, at the current state of knowledge, two distinct problems.

Geometric characterizations of Carleson measures for DA_d where found in [2], then in [4] and [5]. All proofs make use of dyadic decompositions and the behavior of constants with respect to dimension is certainly not the right one. Functional analysis, however, tells us that $[\mu]_{CM(d)}$ is comparable (independently of d) with the best constant $C(\mu)$ in the bilinear estimate

$$\int_{\mathbb{B}_d} d\mu(z) \int_{\mathbb{B}_d} d\mu(w)\varphi(z)\varphi(w)\Re K(z,w) \le C(\mu) \int_{\mathbb{B}_d} \varphi^2 d\mu,$$

restricted to measurable $\varphi \ge 0$ (see [2]).

References

[1] J. Agler, J. McCarthy, Complete Nevanlinna-Pick Kernels, J. Funct. An. Vol. 175 (2000), 111-124

[2] N. Arcozzi, R.Rochberg, E. Sawyer, Carleson Measures for the Drury-Arveson Hardy space and other Besov-Sobolev spaces on Complex Balls, Advances in Mathematics Vol. 218, 4, (2008), 1107-1180.

[3] S. Drury, A generalization of von Neumann's inequality to the complex ball, Proc. Am. Math. Soc. 68, 3 (1978), 300-304.

[4] E. Tchoundja, Carleson measures for the generalized Bergman spaces via a $T(1)$-type theorem, Ark. Mat. 46, 2, 2008 377-406.

[5] A. Volberg, B. Wick, Bergman-type Singular Operators and the Characterization of Carleson Measures for Besov-Sobolev Spaces on the Complex Ball, http://arxiv.org/abs/0910.1142v3.

NICOLA ARCOZZI