Spectral theory on spheres and Gauss space

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Abstract

Mehler observed that the Gaussian measure on $\mathbb{R}$ can be approximated by the normalized Hausdorff measure on the spheres $\mathbb{S}^{n-1}(\sqrt{n})$ of dimension $n-1$ and radius n. In 1973, Mk Kean showed how this observation can be extended to the extent that the geometry of the spheres tends to the differential calculus in Gauss space and gave an interpretation of this in terms of probability theory and Itô calculus. This bridge between geometry and probability is popular among symmetrizers (Baernstein II, Beckner, Carlen and Loss). In fact, in order to be transferable to Gauss' space, estimates on $\mathbb{S}^{n-1}$ must be best possible or, at least, exhibit the right behavior with respect to the dimension n.

In this note I give a partial overview of results with this flavor and show how the spectral theory of the Laplacian on $\mathbb{S}^{n-1}$ tends to the spectral theory of the Hermite operator.

Notation. ∂_j and ∂_{jj} denote the first and the second partial derivative with respect to x_j in Euclidean space. If $(X, \mathcal{F}, \mu)$ is a measure space, $f: X \to \mathbb{R}^n$ is a measurable function and $p \in [1, \infty)$, the L^p norm of f is defined by $\|f\|_p = \|f\|_{L^p(X,\mathbb{R}^n)} = (\int_X |f|^p dx)^{\frac{1}{p}}$. If S is a linear operator which maps $\mathbb{R}^n$ valued functions of L^p into $(X, \mathcal{F}, \mu)$ to $\mathbb{R}^m$ valued L^p functions on $(X_1, \mathcal{F}_\infty, \mu_1)$, $\|S\|_p = \sup\{\|Sf\|_p \colon \|f\|_p = 1\}$ is the operator norm of S.

0 Geometry of Gauss' space

The m dimensional Gauss space is the measure space $(\mathbb{R}^m, \gamma)$, where $\gamma(dx) = (2\pi)^{-\frac{m}{2}} e^{-\frac{|x|^2}{2}} dx$, $x \in \mathbb{R}^m$, is the m - dimensional Gaussian measure. Let $\nabla_{\mathbb{R}^m} D = (\partial_1, \ldots, \partial_m)$ be the gradient in $\mathbb{R}^m$ and D^* be its formal adjoint with respect to the measure γ. If $X = (X_1, \ldots, X_m)$ is a smooth vector field on $\mathbb{R}^m$, then

$$D^* X = -\sum_{j=1}^{m} (\partial_j X_j - x_j X_j)$$

and

$$A = -D^*D = \sum_{j=1}^{m} \partial_{jj} - x_j\partial_j$$

is a negative operator, the m - *dimensional Hermite operator.* A plays the role of the Laplacian in Gauss space. The *carré du champ* associated with A is the quadratic form $\Gamma(f,f) = \frac{1}{2}(Af^2 - 2fAf) = |Df|^2$, the square modulus of the Euclidean gradient.

The *m-dimensional Ornstein-Uhlenbeck process* is the process generated by A. Its densities are given by the *Ornstein-Uhlenbeck semigroup* $P_t = \exp(tA)$.

The analysis of Brownian motion naturally leads to the study of an infinite dimensional Ornstein - Uhlenbeck process, in which the Gaussian measure γ is supported on an infinite dimensional Fréchet space instead of $\mathbb{R}^m$. The connection is the following. Let B be a 1 - dimensional Brownian motion and $e_1, \ldots, e_m, \ldots$ be an orthonormal basis of $L^2([0,\infty))$. Consider the stochastic integral with "sure" integrand $\mathcal{B}_j = \int_0^\infty e_j(t)dB_t$. $\mathcal{B}_1, \ldots, \mathcal{B}_m, \ldots$ are independent Gaussian random variables. Hence, for each m, the density of the distribution of $\mathcal{B}_1, \ldots, \mathcal{B}_m$ is the Gaussian measure γ on $\mathbb{R}^m$. A complete exposition of the theory can be found, for instance, in [BH]. This explains the importance of having estimates on the finite dimensional Gauss spaces that are optimal with respect to the dimension. Such estimates, in fact, tell something about Brownian motion. On the other hand, estimates that are not optimal are lost in the limit $n \to \infty$.

Gauss' space has two features that make it more difficult to develop a harmonic analysis on it. First of all, it is not a homogeneous space in the sense of Coifman-Weiss, since it does not satisfy a doubling condition. Secondly, though it has total measure 1 and a large group of measure preserving transformations, hence it resembles the homogeneous space of a Lie group, the operator A is not a Casimir operator on $\mathbb{R}^m$. In fact

$$A\partial_j - \partial_j A = \partial_j$$

A is nonetheless some sort of Casimir operator. Let

$$\mathcal{T}_{ij} = x_i\partial_j - x_j\partial_i \tag{1}$$

Then,

$$\mathcal{T}_{ij}A = A\mathcal{T}_{ij}$$

In fact, $\mathcal{T}_{ij}$ is derivative with respect to the angular coordinate in the (x_i, x_j) plane and the operator A commutes with the orthogonal group $SO(m)$. Unfortunately, A does not lie in the enveloping algebra engendered by the $\mathcal{T}_{ij}$'s. We will see below that it fails to do so by a small error, in a suitable sense.

In the analysis of the Ornstein - Uhlenbeck process a crucial role is played by the eigenfunctions of A. A *generalized Hermite polynomial of degree* k is a solution P of the equation

$$AP + kP = 0 \tag{2}$$

in $\mathbb{R}^m$, where $k \in \mathbb{N}$. For $m = 1$, the unique solutions of (2) are the constant multiples of the classical Hermite polynomial of degree k. $\mathfrak{H}_k^{\infty,m}$ denotes the space of the generalized Hermite polynomials of degree k on $\mathbb{R}^m$ and $\mathfrak{I}_k^{\infty,m} = \bigoplus_{j=0}^{k} \mathfrak{H}_j^{\infty,m}$ is the space of their finite linear combinations up to degree k. It is classical that

$$L^2(\mathbb{R}^m, \gamma) = \bigoplus_{j=0}^{\infty} \mathfrak{H}_j^{\infty,m} \tag{3}$$

Actually, (3) holds for $m = \infty$ as well, but in this note we only deal with the finite dimensional case. For the arguments leading to the infinite dimensional case see, for instance, [BH], [Me3].

1 Mehler's principle

From now on, $\mathbb{S}_n = \mathbb{S}^{n-1}(\sqrt{n})$ will be the $(n-1)$ - dimensional sphere of radius $\sqrt{n}$. We endow $\mathbb{S}_n$ with its natural Riemannian metric and with the $SO(n)$ invariant measure μ_n normalized so that $\mu_n(\mathbb{S}_n) = 1$. The L^p norms on $\mathbb{S}_n$ are taken with respect to this measure. Many geometric objects on $\mathbb{S}_n$ pass to the limit to corresponding objects on the infinite dimensional Gauss space. This observation is attributed to Poincaré in [McK], but it seems to be due to Mehler, see [Be1]. We shall call this heuristic observation *Mehler's principle*. It has been used in several works dealing with sharp estimates on the n - dimensional sphere in order to obtain corresponding results for the Ornstein - Uhlenbeck process, see [Be1], [Be2], [CL]. In the same spirit, see [A].

With $\mathcal{T}_{lk}$ as in (1), if $F\colon \mathbb{S}_n \to \mathbb{R}$ is smooth enough, we have

$$\Delta_{\mathbb{S}_n} F = \frac{1}{n} \sum_{1 \le l < k \le n} \mathcal{T}_{lk} \mathcal{T}_{lk} F \text{ and} \tag{4}$$

$$\left|\nabla_{\mathbb{S}_n} F\right|^2 = \frac{1}{n} \sum_{1 \le l < k \le n} |\mathcal{T}_{lk} F|^2 \tag{5}$$

If F is a spherical harmonic of degree k, then,

$$\Delta_{\mathbb{S}_n} F + \frac{k(n-2+k)}{n} F = 0 \tag{6}$$

Let m be a fixed positive integer. Let $\Pi_n\colon \mathbb{S}_n \to \mathbb{R}^m$ be the projection $\Pi_n(x,y) = x$, if $x \in \mathbb{R}^m$, $y \in \mathbb{R}^{n-m}$ and $|x|^2 + |y|^2 = n$. If $f\colon \mathbb{R}^m \to \mathbb{R}$, $f_n = f \circ \Pi_n$.

Then, if f is smooth,

$$\Delta_{\mathbb{S}_n} f_n(x) = \Delta_{\mathbb{R}^m} f(x) - \frac{n-1}{n}\sum_{i=1}^{m} x_i \partial_i f(x) - \frac{1}{n}\sum_{i,j=1}^{n} x_i x_j \partial_{ij} f(x) \tag{7}$$

where ∂_{ij} is the cross derivative, and

$$|\nabla_{\mathbb{S}_n} f_n|^2 = \sum_{j=1}^{m} (\partial_j f)^2 - \frac{1}{n}\left(\sum_{j=1}^{m} x_j \partial_j f\right)^2 \tag{8}$$

In (7) and (8) we made use of the fact that that the left hand side only depends on the $x \in \mathbb{R}^m$ component of a point $(x,y) \in \mathbb{S}_n$ and we omitted the $y \in \mathbb{R}^{n-m}$ component.

It is clear that, as $n \to \infty$,

$$\Delta_{\mathbb{S}_n} f_n(x) \to Af(x) \tag{9}$$

and

$$|\nabla_{\mathbb{S}_n} f_n(x)|^2 \to |\nabla_{\mathbb{R}^m} f(x)|^2 \tag{10}$$

The last equality can be read as a limit involving *carrés des champs* on different spaces. See [Me1], [Me2]. (9) and (10) might be called the *differential geometric Mehler's principle.*

Given the commutativity relation $\Delta_{\mathbb{S}^{n-1}} \mathcal{T}_{ij} = \mathcal{T}_{ij} \Delta_{\mathbb{S}^{n-1}}$, which ultimately comes from the fact that $SO(n)$ is a compact Lie group with a biinvariant metric that projects on that of $\mathbb{S}^{n-1}$, one might expect some similar relation to hold for A. In fact, we saw above that $\mathcal{T}_{ij}$ does commute with A. Still, (4) and (7) , and probabilistic intuition, show that A can not directly be written in terms of such vector fields.

The original observation by Mehler, the *measure theoretic Mehler's principle*, is that, if $E \subseteq \mathbb{R}^m$ is measurable, then

$$\begin{aligned}\int_{\mathbb{S}_n} \chi_E \circ \Pi_n \, d\mu_n &= \frac{\int_{\substack{\mathbb{R}^m \\ |x|^2 \le n}} \chi_E(x)\left(1 - \frac{|x|^2}{n}\right)^{\frac{n-m-2}{2}} dx}{\int_{\substack{\mathbb{R}^m \\ |x|^2 \le n}} \left(1 - \frac{|x|^2}{n}\right)^{\frac{n-m-2}{2}} dx} \\ &\to \int_{\mathbb{R}^m} \chi_E \, d\gamma\end{aligned}$$

as $n \to \infty$. It is not difficult to check that, in fact, if $f: \mathbb{R}^m \to \mathbb{R}$ has polynomial growth, then

$$\int_{\mathbb{S}_n} f_n \, d\mu_n \to \int_{\mathbb{R}^m} f \, d\gamma \tag{11}$$

Hence, if f is a polynomial in $x \in \mathbb{R}^m$ and $1 \leq p < \infty$, then

$$\lim_{n\to\infty} \|\nabla_{\mathbb{S}_n} f_n\|_{L^p(\mathbb{S}_n)} = \|\nabla_{\mathbb{R}_m} f\|_{L^p(\gamma)} \tag{12}$$

$$\lim_{n\to\infty} \|\Delta_{\mathbb{S}_n} f_n\|_{L^p(\mathbb{S}_n)} = \|Af\|_{L^p(\gamma)} \tag{13}$$

In fact, by (8) and (7), in the integrals above we have error terms with respect to n, with polynomial growth with respect to $x \in \mathbb{R}^m$.

2 Spectral theoretic Mehler's principle

The main goal of this section is Theorem 2.3.

Let $\mathcal{A}_k^m$ be the space of polynomials of degree not greater than k in $x = (x_1, \ldots, x_m)$. $\mathcal{H}_k(\mathbb{R}^n)$ is the space of homogeneous harmonic polynomials of degree k in $\mathbb{R}^n$ and $\mathfrak{H}_k^{n,m}$ is the space of those $Y \in \mathcal{H}_k(\mathbb{R}^n)$ that are invariant under $SO(n-m)$, the subgroup of $SO(n)$ that pointwise fixes the first factor of $\mathbb{R}^n = \mathbb{R}^m \times \mathbb{R}^{n-m}$, $n > m$. Then $Y \in \mathfrak{H}_k^{n,m}$ if and only if it is a spherical harmonic of degree k on $\mathbb{R}^n$ that can be written as $Y(x_1, \ldots, x_n) = \phi(x_1, \ldots, x_m, x_{m+1}^2 + \ldots + x_n^2)$, where ϕ is a polynomial in $m+1$ variables.

Mimicking the reasoning in [SW], chpt. IV, it is easy to verify that $\dim(\mathfrak{H}_k^{n,m}) = \dim(\mathfrak{H}_k^{\infty,m}) = d_k^m$ is independent of n. In fact, $d_k^m = \sharp\{\alpha \in \mathbb{N}^k : |\alpha| = \alpha_1 + \ldots + \alpha_m = k\}$.

Let now $P \in \mathfrak{H}_k^{\infty,m}$, $n > m$ and let P_n be its restriction to $\mathbb{S}_n$. Then

$$P_n = \sum_{j \leq k} Q_j^{n,m}(P) \tag{14}$$

where $Q_j^{n,m}(P)$ is the $L^2(\mathbb{S}_n)$ - orthogonal projection of P onto $\mathcal{H}_j(\mathbb{R}^n)$, a spherical harmonic of degree k, that we extend to a homogeneous polynomial on $\mathbb{R}^n$. Then, by $SO(n-m)$ invariance, $Q_j^{n,m}(P) \in \mathfrak{H}_k^{n,m}$. The following lemma shows how the spectral decomposition of P_n semplifies as $n \to \infty$. In [Ma], C. Martini has a result with the same flavor.

Lemma 2.1 *Let $P \in \mathfrak{H}_k^{\infty,m}$ and consider its decomposition as in (14). Then $Q_k^{n,m}(P)$ is the leading term of P_n in the L^2 sense.*

(i) $\lim_{n\to\infty} \|Q_k^{n,m}(P)\|_{L^2(\mathbb{S}_n)} = \|P\|_{L^2(\gamma)}$ *and*

(ii) $\lim_{n\to\infty} \left\|Q_j^{n,m}(P)\right\|_{L^2(\mathbb{S}_n)} = 0$, *if* $j < k$.

PROOF . If Q is a spherical harmonic of degree j, then $(-\Delta_{\mathbb{S}_n})^{\frac{1}{2}} Q = \sqrt{\frac{j(n-2+j)}{n}}$, then

$$\begin{aligned}\lim_{n\to\infty}\sum_{j=0}^{k}\frac{j(n-2+j)}{n}\left\|Q_j^{n,m}(P)\right\|^2_{L^2(\mathbb{S}_n)} &= \lim_{n\to\infty}\left\|(-\Delta_{\mathbb{S}_n})^{\frac{1}{2}}P_n\right\|^2_{L^2(\mathbb{S}_n)}\\ &= \lim_{n\to\infty}\|\nabla_{\mathbb{S}_n}P_n\|^2_{L^2(\mathbb{S}_n)}\\ &= \|\nabla_{\mathbb{R}^m}P\|^2_{L^2(\gamma)}\\ &= \|(-A)^{\frac{1}{2}}P\|^2_{L^2(\gamma)} = k\|P\|^2_{L^2(\gamma)}\\ &= \lim_{n\to\infty}\sum_{j=0}^{k}k\left\|Q_j^{n,m}(P)\right\|^2_{L^2(\mathbb{S}_n)}\end{aligned}$$

Comparing the first and the last term in the chain of equalities and taking into account that $\left\|Q_j^{n,m}(P)\right\|^2_{L^2(\mathbb{S}_n)} \leq \|P\|^2_{L^2(\mathbb{S}_n)}$ is bounded in n, since $\|P_n\|^2_{L^2(\mathbb{S}_n)} \to \|P\|^2_{L^2(\gamma)}$, we obtain (ii) for $0 < j < k$. The case $j = 0$ is obtained through (11), and (i) follows.□

Let now $\mathfrak{I}_k^{n,m} = \bigoplus_{j=0}^{k}\mathfrak{H}_j^{n,m}$, $m < m \leq \infty$. A consequence of Lemma 2.1 is that

$$\left\|(-\Delta_{\mathbb{S}_n})^{\frac{1}{2}}P_n\right\|_{L^2(\mathbb{S}_n)} \to \left\|(-A)^{\frac{1}{2}}P\right\|_{L^2(\gamma)} \tag{15}$$

as $n \to \infty$, if $P \in \mathfrak{I}_k^{\infty,m}$. The lemma below is the key to extend (15) to $1 \leq p < \infty$. The real problem is $p > 2$, the case $p < 2$ being easily reduced to that of $p = 2$.

LEMMA 2.2 *Let* $1 \leq p < \infty$. *There exist* $K_p = K(p,m,k)$ *and* $N = N(m,k)$ *such that, if* $F \in \mathfrak{I}_k^{n,m}$,

$$\|F\|_{L^p(\mathbb{S}_n)} \leq K_p\|F\|_{L^2(\mathbb{S}_n)} \tag{16}$$

PROOF . If $p \leq 2$, (16) follows from Jensen's inequality, with $K_p = 1$.

Let $p > 2$. If $F \in \mathfrak{I}_k^{n,m}$, then F_n, the restriction of F to $\mathbb{S}_n$, is the restriction to $\mathbb{S}_n$ of a polynomial $\phi_n \in \mathcal{A}_k^m$ that only depends on $x = (x_1,\dots,x_m)$. By Schwarz's inequality we have

$$\|F\|^p_{L^p(\mathbb{S}_n)} \leq \frac{(2\pi)^{\frac{m}{2}}}{\int_{|x|^2\leq n}\left(1-\frac{|x|^2}{n}\right)^{\frac{n-2-m}{2}}dx}C^0(n,m)$$

$$
\begin{aligned}
& \cdot \left(\int_{|x|^2 \le n} |\phi_n(x)|^{2p} d\gamma(x) \right)^{\frac{1}{2}} \\
\le\ & C^1(m) \|\phi_n\|^p_{L^{2p}(\gamma)} \\
\le\ & C^2(m,k,p) \|\phi_n\|^{\frac{p}{2}}_{L^2(\gamma)}
\end{aligned}
$$

where $C^j(\cdot)$ represent various positive constants dependent on the arguments in parenthesis and, in particular,

$$
C^0(n,m) = \left(\int_{|x|^2 \le n} \left(1 - \frac{|x|^2}{n}\right)^{n-2-m} e^{|x|^2} d\gamma(x) \right)^{\frac{1}{2}}
$$

is bounded in n, for fixed m. The last inequality follows from the fact that $\mathcal{A}_k^m$ is a finite dimensional Banach space with any of its L^p norms.

Consider now on $\mathcal{A}_k^m$ the norms $[\cdot]_n$, $m < n \le \infty$, $[f]_n = \|f_n\|_{L^2(\mathbb{S}_n)}$, $[f]_\infty = \|f\|_{L^2(\gamma)}$. By (11) and simple considerations about finite dimensional Hilbert spaces, we have that

$$
C^3(m,k)[f]_\infty \le [f]_n \le C^4(m,k)[f]_\infty \tag{17}
$$

for $n \ge N(m,k)$. Together with the chain of inequalities above, (17) implies (16).□

THEOREM 2.3 *Let $P = P^{(1)} + \ldots + P^{(k)} \in \mathfrak{I}_k^{\infty,m}$, $P^{(l)} \in \mathfrak{H}_l^{\infty,m}$. Then $\sum_{l=1}^k Q_l^{n,m}(P^{(l)})$ is the leading term of P_n in the L^p sense, for $1 \le p < \infty$.*

(i) $\lim_{n\to\infty} \left\| \sum_{l=0}^k Q_l^{n,m}(P^{(l)}) \right\|_{L^p(\mathbb{S}_n)} = \|P\|_{L^p(\gamma)}$ *and*

(ii) $\lim_{n\to\infty} \left\| Q_j^{n,m}(P^{(l)}) \right\|_{L^p(\mathbb{S}_n)} = 0$, *if $j < l$.*

PROOF . Part (ii) of the theorem is just Lemma 2.2. For $0 \le l \le k$, consider the decomposition of $P_n^{(l)}$ in spherical harmonics,

$$
P_n^{(l)} = \sum_{j=0}^{l} Q_j^{n,m}(P^{(l)})
$$

Then, by (i),

$$
\sum_{l=0}^{k} \sum_{j=0}^{l-1} \|Q_j^{n,m}(P^{(l)})\|_{L^p(\mathbb{S}^{n-1})} \to 0 \tag{18}
$$

as $n \to \infty$, hence, by (11), (18)and a triangulation

$$\|P\|_{L^p(\gamma)} = \lim_{n\to\infty} \|\sum_{l=0}^{k} Q_l^{n,m}(P^{(l)})\|_{L^p(\mathbb{S}^{n-1})}$$

which is (i).□

The following application of Theorem 2.3 is in [A], and it is instrumental in transfering L^p estimates for the Riesz transform on $\mathbb{S}^{n-1}$ to an estimate for the Riesz transform for the Ornstein - Uhlenbeck process.

COROLLARY 2.4 *Let $1 \leq p < \infty$. If P is a finite linear combination of generalized Hermite polynomials, then*

$$\left\|(-\Delta_{\mathbb{S}_n})^{\frac{1}{2}} P_n\right\|_{L^p(\mathbb{S}_n)} \to \left\|(-A)^{\frac{1}{2}} P\right\|_{L^p(\gamma)} \tag{19}$$

as $n \to \infty$.

PROOF . Suppose $P = P^{(1)} + \ldots + P^{(k)}$, $P^{(l)} \in \mathfrak{H}_l^{\infty,m}$. By (6), the multiplier of $(-\Delta_{\mathbb{S}_n})^{\frac{1}{2}}$ is the sequence $m^n(k) = \left(\frac{(n-2+k)k}{n}\right)^{\frac{1}{2}} \to k^{\frac{1}{2}} = m^\infty(k)$, which is the multiplier of $(-A)^{\frac{1}{2}}$. Hence, by Theorem 2.3 the leading term, in the L^p sense, of the decomposition of $(-\Delta_{\mathbb{S}_n})^{\frac{1}{2}} P_n$ in spherical harmonics is

$$\left(\frac{(n-1)}{n}\right)^{\frac{1}{2}} Q_1^{n,m}(P^{(1)}) + \cdots + \left(\frac{(n-2+k)k}{n}\right)^{\frac{1}{2}} Q_k^{n,m}(P^{(k)})$$

(19) immediately follows.□

Corollary 2.4 is useful because we have good estimates for the L^p norms of the Riesz transform on $\mathbb{S}^{n-1}$. It is likely that the same method can be successfully applied, for instance, to find conditions under which a multiplier operator is L^p bounded in $L^p(\gamma)$, provided we have a similar result on $\mathbb{S}^{n-1}$, for all n, with nearly sharp estimates of the L^p norms. For instance, we have the following

PROPOSITION 2.5 *Let ϕ be a continuous, bounded function on $\mathbb{N}$ such that, for all n, $\phi(-\Delta_{\mathbb{S}_n})$ is a bounded multiplier on $L^p(\mathbb{S}^{n-1})$. Suppose, more, that*

$$\|\phi(-\Delta_{\mathbb{S}_n})f\|_{L^p(\mathbb{S}_n)} \leq C\|f\|_{L^p(\mathbb{S}_n)}$$

with C independent of n. Then, $\phi(-A)$ is a bounded multiplier on $L^p(\gamma)$.

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