Prova scritta di Analisi Matematica LB (9/1/2012)

Nome..**Cognome**.. **Matricola**................

(1) [3 pt] Risolvere in $\mathbb{C}$ l'equazione

$$(iz^2 + 2(i-1)z - 4)(iz^3 + 8)$$

(2) [3 pt] Per quali valori di $\gamma \geq 0$ si ha la convergenza di

$$\int_0^{+\infty} \frac{x^\gamma + x^{2\gamma}}{x^{3\gamma} + x^{5\gamma}} dx$$

(3) **[4 pti]** Siano $\Omega \subset \mathbb{R}^3$ l'insieme

$$\Omega = \left\{ (x, y, z) : \sqrt{\frac{x^2}{25} + \frac{y^2}{9}} - 2 \leq z \leq 2 \right\}.$$

e $f \in C(\Omega, \mathbb{R})$ continua.

Trovare $A \subset \mathbb{R}^2$ e, per $(x, y) \in \mathbb{R}^2$, trovare $\alpha(x, y), \beta(x, y) \in \mathbb{R}$, tali che

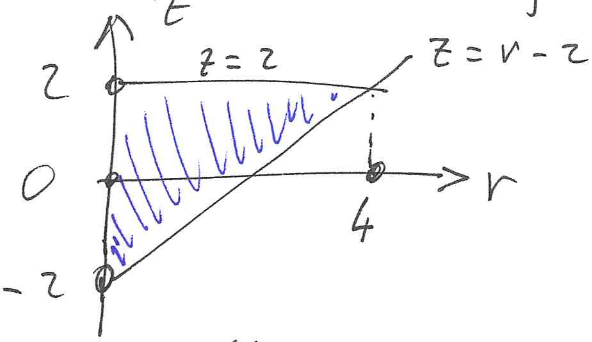
$$\iiint_\Omega f(x, y, z) dxdydz = \iint_A \left[\int_{\alpha(x,y)}^{\beta(x,y)} f(x, y, z) dz \right] dxdy$$

(4) **[5 pti]** Sia $A = \{(x, y) : \; |2x + 3y| \leq \frac{\pi}{2}, \; |3x - 2y| \leq \frac{\pi}{2}|\} \subset \mathbb{R}^2$. Calcolare

$$\iint_A \cos(2x + 3y) dxdy.$$

AM 2-B.

(3) Sia $r = \sqrt{\frac{x^2}{25} + \frac{y^2}{9}} \geq 0$: $\quad r-2 \leq z \leq 2$



Per $r \leq 4$, ho che

$r - 2 \leq z \leq 2$.

Allora,

$$\iiint_{\Omega} f(x,y,z)\,dx\,dy\,dz = \iint_{\left\{(x,y):\ \frac{x^2}{25} + \frac{y^2}{9} \leq 4^2\right\}} \left\{ \int_{\sqrt{x^2/25 + y^2/9} - 2}^{2} f(x,y,z)\,dz \right\} dx\,dy$$

$$\boxed{A = \left\{(x,y):\ \frac{x^2}{25} + \frac{y^2}{9} \leq 4^2\right\} \text{ e } \alpha(x,y) = \sqrt{\frac{x^2}{25} + \frac{y^2}{9}} - 2 \text{ e } \beta(x,y) = 2}$$

(6) Sia $f = f(u,v)$:

$$\partial_x h(x_0,y_0) = \partial_u f(\alpha(x_0,y_0), \beta(x_0,y_0)) \cdot \partial_x \alpha(x_0,y_0) + \partial_v f(\alpha(x_0,y_0), \beta(x_0,y_0)) \cdot \partial_x \beta(x_0,y_0)$$

$$\partial_y h(x_0,y_0) = \partial_u f(\alpha(x_0,y_0), \beta(x_0,y_0)) \cdot \partial_y \alpha(x_0,y_0) + \partial_v f(\alpha(x_0,y_0), \beta(x_0,y_0)) \cdot \partial_y \beta(x_0,y_0).$$