

## Mathematical Methods - Mathematical Analysis: takehome

Suppose  $T : \ell^2(\mathbb{Z}) \rightarrow \ell^2(\mathbb{Z})$  is defined as

$$\widehat{Ta}(t) = \frac{\hat{a}(t)}{1 - 2e^{it}}.$$

for all  $a \in \ell^2(\mathbb{Z})$ .

- Is  $T$  linear?
- Is  $T$  stable?
- Is  $T$  time-invariant?
- Is  $T$  realizable?

Motivate your answer.

Find  $b \in \ell^2(\mathbb{Z})$  such that

$$Ta(n) = \sum_{k=-\infty}^{+\infty} a(n-k)b(k).$$

Compute  $T\delta_b$ , where  $b$  is the last digit of your student ID.

**It is useful to know that:**

- $\hat{a}(t) = \sum_{k=-\infty}^{+\infty} a(k)e^{-ikt}$ ;
- $\delta_k : \mathbb{Z} \rightarrow \mathbb{C}$  is a discrete Dirac's delta, defined as

$$\delta_k(n) = \begin{cases} 1 & \text{if } n = k, \\ \hat{a}(t) & \text{if } n \neq k. \end{cases}$$

- for the meaning of linear, stable, time-invariant and realizable see your notes from the class, or the solution to the exercise.