Mathematical Methods - Mathematical Analysis: takehome

Suppose $T : \ell^2(\mathbb{Z}) \to \ell^2(\mathbb{Z})$ is defined as

$$\widehat{Ta}(t) = (\cos^2(t) - \sin^2(t) + 2i\cos(t)\sin(t)) \cdot \hat{a}(t).$$

for all $a \in \ell^2(\mathbb{Z})$.

- Is T linear?
- Is T stable?
- Is T time-invariant?
- Is T realizable?

Motivate your answer.

Find $b \in \ell^2(\mathbb{Z})$ such that

$$Ta(n) = \sum_{k=-\infty}^{+\infty} a(n-k)b(k).$$

Compute $T\delta_b$, where b is the last digit of your student ID.

It is useful to know that:

- $\hat{a}(t) = \sum_{k=-\infty}^{+\infty} a(k)e^{-ikt}$;
- $\delta_k : \mathbb{Z} \to \mathbb{C}$ is a discrete Dirac's delta, defined as

$$\delta_k(n) = \begin{cases} 1 \text{ if } n = k, \\ \hat{a}(t) \text{ if } n \neq k. \end{cases}$$

- for the meaning of linear, stable, time-invariant and realizable see your notes from the class, or the solution to the exercise.