

MIDTERM EXAM

(1) Which of the following functions is not holomorphic in  $\mathbb{C}$ ?

$$f_1(x+iy) = e^x \cos(y) + i \cdot e^x \sin(y)$$

$$f_2(x+iy) = x^4 + 6x^2y^2 + y^4 + i \cdot 4xy(x^2 + y^2)$$

$$f_3(x+iy) = x^4 - 6x^2y^2 + y^4 + i \cdot 4xy(x^2 - y^2)$$

(2) Use the method of residues to compute

$$I_a = \int_{-\pi}^{\pi} \frac{d\theta}{|e^{i\theta} - a|^2} \quad \text{with } a \in \mathbb{R}, -1 < a < 1.$$

Hint: (A) Let  $z = e^{i\theta}$ ,  $|z|=1$ ;

(B) observe that  $z \cdot \bar{z} = 1$  for  $|z|=1$ .