Test I - Soluzioni e svolgimenti

(A) Dominio: $0 < \frac{1-x^2}{x^2} = \frac{(1-x)(1+x)}{x^2} \iff x \in (-1,1)\setminus\{0\}$
$= (-1,0)\cup(0,1)$

f è derivabile in Dominio(f).

$f(x) = \log(1-x^2) - \log(x^2) = \log(1-x) + \log(1+x) - 2\log(x)$

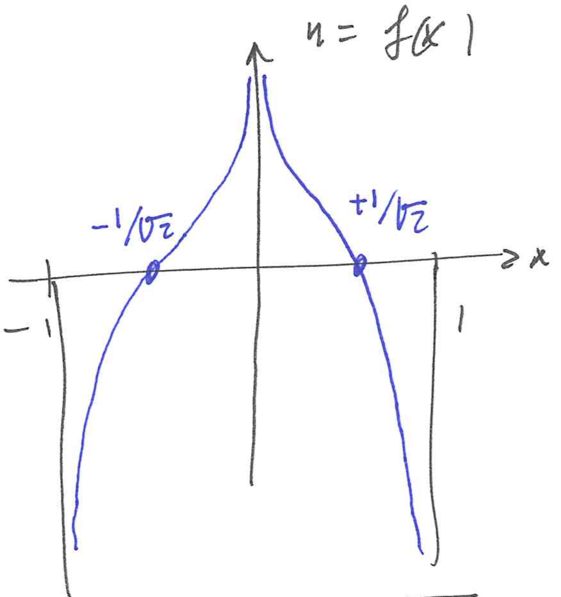
ATTENZIONE: uso la positività dei fattori; nel dubbio è meglio fare qualche conto in più.

$f'(x) = \frac{-1}{1-x} + \frac{1}{1+x} - \frac{2}{x}$

$\lim_{x\to 0} f(x) = \log\frac{1}{0^+} = \log(+\infty) = +\infty$

$\lim_{x\to \pm 1} f(x) = \log 0^+ = -\infty$

$f(x) = 0 \iff \frac{1-x^2}{x^2} = 1$, cioè $\frac{1}{x^2} - 1 = 1 \quad \frac{1}{x^2} = 2 \quad x = \pm\frac{1}{\sqrt{2}}$



← informazioni che ho su f

(B) $\frac{\left(e^{\sqrt{1+x}-1}-1\right)\cdot\left(e^{\sqrt{1+x}+1}+1\right)}{\sin(\log(1+2x))} \underset{x\to 0^+}{\sim}$

è un limite 0/0

$\underset{x\to 0^+}{\sim} \frac{(\sqrt{1+x}-1)\cdot(e^2+1)}{\log(1+2x)} = \frac{[(1+x)^{1/2}-1]\cdot(e^2+1)}{\log(1+2x)}$

$\underset{x\to 0^+}{\sim} \frac{\frac{1}{2}x\cdot(e^2+1)}{2x} \xrightarrow[x\to 0^+]{} \boxed{\frac{e^2+1}{4}}$

(C)
$$\frac{2^{2n+\log n}+3\cdot 4^{n+\log n}}{5\cdot(n+\log n)^4+7\cdot 2^{2n}\cdot n^{2\log 2}} =$$

$$= \frac{2^{2n}\cdot 2^{\log n}+3\cdot 2^{2n}\cdot 2^{2\log n}}{5\cdot n^4\cdot\left(1+\frac{\log n}{n}\right)^4+7\cdot 2^{2n}\cdot n^{2\log 2}}$$

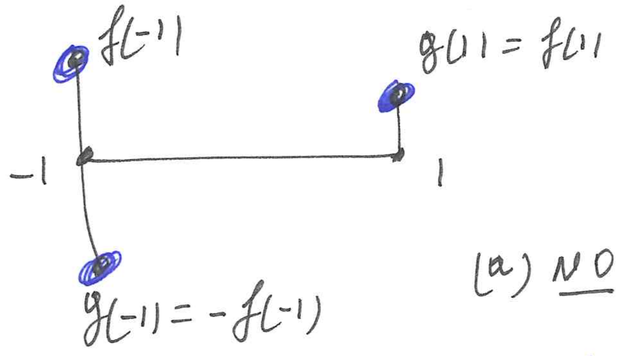
[Osservo che $n^{2\log 2} = (n^{\log 2})^2 = (e^{\log n\cdot\log 2})^2$
$= (2^{\log n})^2 = 2^{2\log n}$

$$\underset{n\to\infty}{\sim}\ \frac{2^{2n}\cdot 2^{\log n}+3\cdot 2^{2n}\cdot 2^{2\log n}}{5\cdot n^4\cdot\left(1+\frac{\log n}{n}\right)^4+7\cdot 2^{2n}\cdot 2^{2\log n}}$$

$$= \cancel{\frac{2^{2n}\cdot 2^{2\log n}}{2^{2n}\cdot 2^{2\log n}}}\cdot\frac{2^{-\log n}+3}{\dfrac{5\cdot n^4\cdot\left(1+\frac{\log n}{n}\right)^4}{2^{2n}\cdot 2^{2\log n}}+7}$$

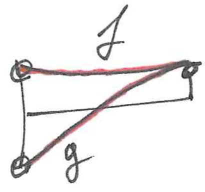
$$\xrightarrow[n\to\infty]{}\ \frac{3}{7}$$ per confronto di infiniti.

(D)

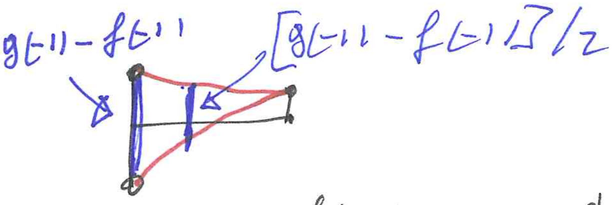


← Dati del problema

(a) NO!



(b) Sì L'idea è:



Dim. $h(x) := g(x) - f(x) - \frac{g(-1)-f(-1)}{2}$; $h\in C([-1,1],\mathbb{R})$

$h(1) = g(1) - f(1) - \frac{g(-1)-f(-1)}{2} = -\ \frac{g(-1)-f(-1)}{2}$

$h(-1) = g(-1) - f(-1) - \frac{g(-1)-f(-1)}{2} = \frac{g(-1)-f(-1)}{2}$

h cambia di segno $\Rightarrow \exists x\in[-1,1]: 0 = h(x)$.

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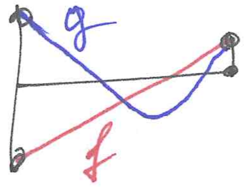
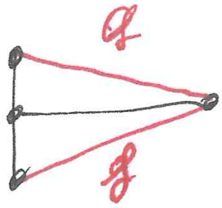
(c) NO:



(d) NO:



$g - f$ è massimo in $x = -1$
e solo lì.