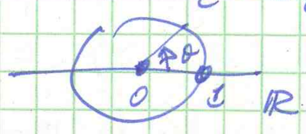
$$\boxed{I = \int_{-\pi}^{\pi} \left| \frac{re^{i\theta} - a}{1 - re^{i\theta}a} \right|^2 d\theta}$$

$z = e^{i\theta}$

$dz = e^{i\theta} \cdot i \cdot d\theta$

$|z| = 1$



$$= \int_{\partial\Delta(0,1)} \left| \frac{rz - a}{1 - rza} \right|^2 \frac{dz}{iz} = \frac{2\pi}{2\pi i} \int_{\partial\Delta(0,1)} \frac{rz - a}{1 - rza} \cdot \frac{r\bar{z} - a}{1 - r\bar{z}a} \frac{dz}{z}$$

$$= 2\pi \cdot \frac{1}{2\pi i} \cdot \int_{\partial\Delta(0,1)} \frac{rz - a}{1 - rza} \frac{r/z - a}{1 - ra/z} \frac{dz}{z}$$

$$= 2\pi \cdot \frac{1}{2\pi i} \int_{\partial\Delta(0,1)} \frac{rz - a}{1 - rza} \cdot \frac{r - az}{z - ra} \frac{dz}{z}$$

$$= 2\pi \cdot \sum_{z_j \in \Delta(0,1)} Res(f; z_j) \quad \text{with} \quad f(z) = \frac{rz - a}{1 - raz} \cdot \frac{r - az}{z - ra} \cdot \frac{1}{z}$$

Singularities of f: (1) $1 - raz = 0$ $\quad z = 1/ra > 1$; $z \notin \overline{\Delta(0,1)}$

(2) $z - ra = 0$ $\quad z = ra \in (0,1)$: $z \in \Delta(0,1)$

(3) $z = 0 \in \Delta(0,1)$

Poles of 1st order.

$$I = 2\pi \cdot [Res(f, 0) + Res(f, ra)]$$

$$= 2\pi \cdot \left[\lim_{z \to 0} z f(z) + \lim_{z \to ra} (z - ra) f(z) \right]$$

$$= 2\pi \cdot \left[\lim_{z \to 0} \frac{(rz - a)(r - az)}{(1 - raz)(z - ra)} + \lim_{z \to ra} \frac{(rz - a)(r - az)}{(1 - raz)\, z} \right]$$

$$= 2\pi \cdot \left[\frac{(-a) \cdot r}{1 \cdot (-ra)} + \frac{(r \cdot ra - a)(r - a\,ra)}{(1 - ra\,ra)\, ra} \right]$$

$$= 2\pi \cdot \left[1 + \frac{a(r^2 - 1)\, r(1 - a^2)}{(1 - r^2a^2)\, ra} \right] = 2\pi \cdot \frac{1 - r^2a^2 + (r^2 - 1)(1 - a^2)}{1 - r^2a^2}$$

$$= \boxed{2\pi \cdot \frac{r^2 + a^2 - 2r^2a^2}{1 - r^2a^2}}$$

Observe that $\lim_{r \to 1} I = 2\pi \frac{1 + a^2 - 2a^2}{1 - a^2} = 2\pi.$