Il risultato del limite (10) è errato (c'è un refuso nella funzione).

(13) $$f(x) = \frac{(2+x)^{2/3} - e^{\frac{1}{3}x} \cdot 4^{1/3}}{x \cdot \sin(x)}$$

$\lim_{x \to 0} f(x) = \frac{0}{0}$ è una forma d'indeterminazione.

$$\sin(x) \underset{x \to 0}{\sim} x \Rightarrow f(x) \underset{x \to 0}{\sim} \frac{(2+x)^{2/3} - e^{\frac{1}{3}x} \cdot 4^{1/3}}{x^2}$$

Sviluppo il numeratore al II ordine.

Uso $(1+y)^{\alpha} = 1 + \alpha y + \frac{\alpha \cdot (\alpha - 1)}{2} y^2 + o(y^2)$, $y \to 0$

$$(2+x)^{2/3} = \left[2 \cdot \left(1 + \frac{x}{2}\right)\right]^{2/3} = 2^{2/3} \cdot \left(1 + \frac{x}{2}\right)^{2/3} =$$

$$= 2^{2/3} \left[1 + \frac{2}{3} \frac{x}{2} + \frac{2/3 \cdot (2/3 - 1)}{2} \cdot \left(\frac{x}{2}\right)^2 + o(x^2)\right]$$

$$= 2^{2/3} \cdot \left[1 + \frac{x}{3} + \frac{1}{2} \cdot \frac{2}{3}\left(-\frac{1}{3}\right) \cdot \frac{x^2}{4} + o(x^2)\right]$$

$$= 2^{2/3} \cdot \left[1 + \frac{x}{3} - \frac{x^2}{3^2 \cdot 4} + o(x^2)\right] = 4^{1/3} \cdot \left[1 + \frac{x}{3} - \frac{x^2}{3^2 \cdot 4} + o(x^2)\right]$$

$$\Rightarrow f(x) \underset{x \to 0}{\sim} \frac{4^{1/3} \left\{\left[1 + \frac{x}{3} - \frac{x^2}{3^2 \cdot 4} + o(x^2)\right] - e^{x/3}\right\}}{x^2}$$

$$= \frac{4^{1/3}}{x^2} \cdot \left\{1 + \frac{x}{3} - \frac{x^2}{3^2 \cdot 4} + o(x^2) - \left[1 + \frac{x}{3} + \frac{1}{2} \cdot \left(\frac{x}{3}\right)^2 + o(x^2)\right]\right\}$$

$$= \frac{4^{1/3}}{x^2} \cdot \left\{-x^2\left(\frac{1}{3^2 \cdot 4} + \frac{1}{2 \cdot 3^2}\right) + o(x^2)\right\} \xrightarrow[x \to 0]{} -\frac{4^{1/3}}{3^2 \cdot 2}\left(1 + \frac{1}{2}\right)$$

$$= -\frac{4^{1/3}}{3^2 \cdot 2} \cdot \frac{3}{2} = -\frac{4^{1/3}}{12}$$

$$\boxed{\lim_{x \to 0} f(x) = -\frac{4^{1/3}}{12}}$$