I PROVA PARZIALE DI ANALISI L-B: SOLUZIONI
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Per le derivate parziali, uso la convenzione $\partial_u f = f_u$. Alle variabili da cui dipende la funzione f ho dato i nomi u, v: $f = f(u, v)$.

Versione 101.

(1) (ii)

(2) Gradiente: $\nabla u(x,y) = (\frac{-x^2-2xy^2+1}{(1+x^2)^2}, \frac{2y}{1+x^2})$;
piano tangente in $(1,1)$: $z - 1 = -1/2(x-1) + (y-1)$;
differenziale in $(1,1)$: $du(1,1)(h,k) = -1/2h + k$.

(3) $u(x,y) = 1 - 1/2(x-1) + (y-1) + 1/2[-2(x-1)(y-1) + (y-1)^2] + o_{(x,y)\to(1,1)}((x-1)^2 + (y-1)^2)$

(4) Punti critici: $(1,0)$, $(-1,0)$. In $(1,0)$ sella, in $(-1,0)$ minimo relativo.

(5) Jacobiana

$$\begin{bmatrix} 0 & f_v(z,y) & f_u(z,y) \\ yf_u(x,z) & f(x,z) & yf_v(x,z) \end{bmatrix}$$

Versione 102.

(1) (i)

(2) Gradiente: $\nabla u(x,y) = (\frac{-x^2+2xy^2+1}{(1+x^2)^2}, \frac{-2y}{1+x^2})$;
piano tangente in $(1,1)$: $z = 1/2(x-1) - (y-1)$;
differenziale in $(1,1)$: $du(1,1)(h,k) = 1/2h - k$.

(3) $u(x,y) = 1/2(x-1) - (y-1) + 1/2[-(x-1)^2 + 2(x-1)(y-1) - (y-1)^2] + o_{(x,y)\to(1,1)}((x-1)^2 + (y-1)^2)$

(4) Punti critici: $(1,0)$, $(-1,0)$. In $(1,0)$ massimo relativo, in $(-1,0)$ sella.

(5) Jacobiana

$$\begin{bmatrix} f_v(z,x) & 0 & f_u(z,x) \\ f(y,z) & xf_u(y,z) & xf_v(y,z) \end{bmatrix}$$

Versione 103.

(1) (iv)

(2) Gradiente: $\nabla u(x,y) = (\frac{x^2-2xy^2-1}{(1+x^2)^2}, \frac{2y}{1+x^2})$;

piano tangente in $(1,1)$: $z = -1/2(x-1) + (y-1)$;

differenziale in $(1,1)$: $du(1,1)(h,k) = -1/2h + k$.

(3) $u(x,y) = -1/2(x-1) + (y-1) + 1/2[((x-1)^2 - 2(x-1)(y-1) + (y-1)^2] + o_{(x,y)\to(1,1)}((x-1)^2 + (y-1)^2)$

(4) Punti critici: $(1,0)$, $(-1,0)$. In $(1,0)$ minimo relativo, in $(-1,0)$ sella.

(5) Jacobiana

$$\begin{bmatrix} 0 & f_u(y,z) & f_v(y,z) \\ yf_v(z,x) & f(z,x) & yf_u(z,x) \end{bmatrix}$$

Versione 104.

(1) (iii)

(2) Gradiente: $\nabla u(x,y) = (\frac{x^2+2xy^2-1}{(1+x^2)^2}, \frac{-2y}{1+x^2})$;

piano tangente in $(1,1)$: $z + 1 = 1/2(x-1) - (y-1)$;

differenziale in $(1,1)$: $du(1,1)(h,k) = 1/2h - k$.

(3) $u(x,y) = -1 + 1/2(x-1) - (y-1) + 1/2[2(x-1)(y-1) - (y-1)^2] + o_{(x,y)\to(1,1)}((x-1)^2 + (y-1)^2)$

(4) Punti critici: $(1,0)$, $(-1,0)$. In $(1,0)$ sella, in $(-1,0)$ massimo relativo.

(5) Jacobiana

$$\begin{bmatrix} f(z,y) & xf_v(z,y) & xf_u(z,y) \\ f_u(x,z) & 0 & f_v(x,z) \end{bmatrix}$$

Versione 105.

(1) (iii)

(2) Gradiente: $\nabla u(x,y) = (\frac{-2x^2-2xy^2+2}{(1+x^2)^2}, \frac{2y}{1+x^2})$;

piano tangente in $(1,1)$: $z - 3/2 = -1/2(x-1) + (y-1)$;

differenziale in $(1,1)$: $du(1,1)(h,k) = -1/2h + k$.

(3) $u(x,y) = 3/2 - 1/2(x-1) + (y-1) + 1/2[-1/2(x-1)^2 - 2(x-1)(y-1) + (y-1)^2] + o_{(x,y)\to(1,1)}((x-1)^2 + (y-1)^2)$

(4) Punti critici: $(1,0)$, $(-1,0)$. In $(1,0)$ sella, in $(-1,0)$ minimo relativo.

(5) Jacobiana

$$\begin{bmatrix} zf_u(x,y) & zf_v(x,y) & f(x,y) \\ f_u(x,z) & 0 & f_v(x,z) \end{bmatrix}$$

Versione 106.

(1) (iii)

(2) Gradiente: $\nabla u(x,y) = (\frac{-2x^2+2xy^2+2}{(1+x^2)^2}, \frac{-2y}{1+x^2})$;
piano tangente in $(1,1)$: $z - 1/2 = 1/2(x-1) - (y-1)$;
differenziale in $(1,1)$: $du(1,1)(h,k) = 1/2h - k$.

(3) $u(x,y) = 1/2 + 1/2(x-1) - (y-1) + 1/2[-3/2(x-1)^2 + 2(x-1)(y-1) - (y-1)^2] + o_{(x,y)\to(1,1)}((x-1)^2 + (y-1)^2)$

(4) Punti critici: $(1,0)$, $(-1,0)$. In $(1,0)$ massimo relativo, in $(-1,0)$ sella.

(5) Jacobiana

$$\begin{bmatrix} yf_v(z,x) & f(z,x) & yf(z,x) \\ 0 & f_u(y,z) & f_v(y,z) \end{bmatrix}$$

Versione 107.

(1) (iii)

(2) Gradiente: $\nabla u(x,y) = (\frac{2x^2-2xy^2-2}{(1+x^2)^2}, \frac{2y}{1+x^2})$;
piano tangente in $(1,1)$: $z + 1/2 = -1/2(x-1) + (y-1)$;
differenziale in $(1,1)$: $du(1,1)(h,k) = -1/2h + k$.

(3) $u(x,y) = -1/2 - 1/2(x-1) + (y-1) + 1/2[3/2(x-1)^2 - 2(x-1)(y-1) + (y-1)^2] + o_{(x,y)\to(1,1)}((x-1)^2 + (y-1)^2)$

(4) Punti critici: $(1,0)$, $(-1,0)$. In $(1,0)$ minimo relativo, in $(-1,0)$ sella.

(5) Jacobiana

$$\begin{bmatrix} 0 & f_u(y,z) & f_v(y,z) \\ zf_u(x,y) & zf_v(x,y) & f(x,y) \end{bmatrix}$$

Versione 108.

(1) (iv)

(2) Gradiente: $\nabla u(x,y) = (\frac{2x^2+2xy^2-2}{(1+x^2)^2}, \frac{-2y}{1+x^2})$;
piano tangente in $(1,1)$: $z + 3/2 = 1/2(x-1) - (y-1)$;
differenziale in $(1,1)$: $du(1,1)(h,k) = 1/2h - k$.

(3) $u(x,y) = -3/2 + 1/2(x-1) - (y-1) + 1/2[1/2(x-1)^2 + 2(x-1)(y-1) - (y-1)^2] + o_{(x,y)\to(1,1)}((x-1)^2 + (y-1)^2)$

(4) Punti critici: $(1,0)$, $(-1,0)$. In $(1,0)$ sella, in $(-1,0)$ massimo relativo.

(5) Jacobiana

$$\begin{bmatrix} f(y,z) & xf_u(y,z) & xf_v(y,z) \\ f_v(z,x) & 0 & f_u(z,x) \end{bmatrix}$$

Versione 109.

(1) (iv)

(2) Gradiente: $\nabla u(x,y) = (\frac{-2x^2-4xy^2+2}{(1+x^2)^2}, \frac{4y}{1+x^2})$;
piano tangente in $(1,1)$: $z-2 = -(x-1)+2(y-1)$;
differenziale in $(1,1)$: $du(1,1)(h,k) = -h+2k$.

(3) $u(x,y) = 2-(x-1)+2(y-1)+1/2[-4(x-1)(y-1)+2(y-1)^2]+o_{(x,y)\to(1,1)}((x-1)^2+(y-1)^2)$

(4) Punti critici: $(1,0)$, $(-1,0)$. In $(1,0)$ sella, in $(-1,0)$ minimo relativo.

(5) Jacobiana

$$\begin{bmatrix} f_u(x,z) & 0 & f_v(x,z) \\ yf_v(z,x) & f(z,x) & f_u(z,x) \end{bmatrix}$$

Versione 110.

(1) (iv)

(2) Gradiente: $\nabla u(x,y) = (\frac{-2x^2+4xy^2+2}{(1+x^2)^2}, \frac{-4y}{1+x^2})$;
piano tangente in $(1,1)$: $z = (x-1)-2(y-1)$;
differenziale in $(1,1)$: $du(1,1)(h,k) = h-2k$.

(3) $u(x,y) = (x-1)-2(y-1)+1/2[-2(x-1)^2+4(x-1)(y-1)-2(y-1)^2]+o_{(x,y)\to(1,1)}((x-1)^2+(y-1)^2)$

(4) Punti critici: $(1,0)$, $(-1,0)$. In $(1,0)$ massimo relativo, in $(-1,0)$ sella.

(5) Jacobiana

$$\begin{bmatrix} 0 & f_u(y,z) & f_v(y,z) \\ yf_v(z,x) & f(z,x) & yf_u(z,x) \end{bmatrix}$$

Versione 100.

(1) $(k,s,n,m) = (a, a+1, a, b)$

(2) Gradiente: $\nabla u(x,y) = (\frac{-Ax^2-2Bxy^2+A}{(1+x^2)^2}, \frac{2By}{1+x^2})$;
piano tangente in $(1,1)$: $z-(A+B)/2 = -B/2(x-1)+B(y-1)$;
differenziale in $(1,1)$: $du(1,1)(h,k) = -B/2h+Bk$.

(3) $u(x,y) = (A+B)/2 - B/2(x-1)+B(y-1)+1/2[(B-A)/2(x-1)^2-2B(x-1)(y-1)+B(y-1)^2]+o_{(x,y)\to(1,1)}((x-1)^2+(y-1)^2)$

(4) Punti critici: $(1,0)$, $(-1,0)$. In $(1,0)$, in $(-1,0)$.

(5) Jacobiana

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$