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ESERCIZIO 1)

Poiché $(a_{ij}) = A$ è definita positiva $\Rightarrow \sum_{i,j=1}^{n} a_{ij}\,\xi_i\,\xi_j > 0$ $\quad \forall x \in \Omega$
costante $\quad \forall \xi \in \mathbb{R}^n,\ \xi \neq 0$

Dobbiamo dimostrare che:

$$u(x) = \int_{\partial B(x,r)} \langle A \nabla \Gamma(x-y), \nu \rangle_E \, u(y)\, d\sigma(y) - \int_{B(x,r)} \langle A \nabla \Gamma(x-y), \nabla u(y) \rangle_E \, dy$$

Dove $B(x,r) = \{ y \,/\, |\Gamma(x-y)|^{-1} \leq r^{n-2} \}$

Considero $\Omega = B(x,r)$, sia $\varepsilon > 0$ t.c. $B(x,\varepsilon) \subseteq B(x,r)$ $\quad y \in B(x,\varepsilon)$, $y \neq x$

Poiché Γ è soluzione fondamentale di $Lu = 0$, abbiamo che

$$\int_{B(x,r)\setminus B(x,\varepsilon)} L\Gamma(x-y)\, u(y)\, dy = 0$$

no ci manteniamo lontani dalle singolarità.

D'altro canto:

$$\int_{B(x,r)\setminus B(x,\varepsilon)} L\Gamma(x-y)\, u(y)\, dy = \int_{B(x,r)\setminus B(x,\varepsilon)} \sum_{i,j=1}^{n} a_{ij} \frac{\partial^2 \Gamma(x-y)}{\partial x_i \partial x_j}\, u(y)\, dy$$

integriamo per parti

$$= \int_{\partial B(x,r)} \sum_{i,j=1}^{n} a_{ij}\, \partial_j \Gamma(x-y)\, \nu_j(y)\, u(y)\, d\sigma(y) +$$

$$+ \int_{\partial B(x,\varepsilon)} \sum_{i,j=1}^{n} a_{ij}\, \Gamma(x-y)\, \nu_{ext\, B(x,r)\setminus B(x,\varepsilon)}(y)\, u(y)\, d\sigma(y) +$$

$$- \int_{B(x,r)\setminus B(x,\varepsilon)} \sum_{i,j=1}^{n} a_{ij}\, \partial_j \Gamma(x-y)\, \partial_j u(y)\, dy =$$

$$= \int_{\partial B(x,r)} \langle A \nabla \Gamma(x-y), \nu(y) \rangle_E \, u(y)\, d\sigma(y) + \int_{\partial B(x,\varepsilon)} \langle A \nabla \Gamma(x-y), \nu_{ext}(y) \rangle_E \, u(y)\, d\sigma(y) +$$

$$- \int_{B(x,r)\setminus B(x,\varepsilon)} \langle A \nabla \Gamma(x-y), \nabla u(y) \rangle_E \, dy = I_1 + I_2 + I_3$$

Da cui:

$$I_2 = \int_{\partial B(x,\varepsilon)} \langle A \nabla \Gamma(x-y), \nu_{ext}(y) \rangle_E \, u(y)\, d\sigma(y) = - \int_{\partial B(x,\varepsilon)} \langle A \nabla \Gamma(x-y), \nu \rangle_E \, u(y)\, d\sigma(y) \xrightarrow{\varepsilon \to 0} -u(x)$$

per ipotesi.

non giustificato