PAG 2

$$I_1 = \int_{\partial B_{A^{-1}}(x,r)} u(y) \langle A\nabla\Gamma_L(x-y), \nu\rangle \, d\sigma(y) - \sum \int_{\partial B_{A^{-1}}(x,\varepsilon)} u(y) \langle A\nabla\Gamma(x-y), \nu(y)\rangle \, d\sigma(y)$$

Invece, $I_2 = -\int_{\Omega_\varepsilon} \langle A\nabla\Gamma(x-y), \nabla u\rangle \, dy$.

Ora vedo che:

$$\left| \int_{\partial B_{A^{-1}}(x,\varepsilon)} u(y) \langle A\nabla\Gamma(x-y), \nu(y)\rangle d\sigma(y) - \int_{\partial B_{A^{-1}}(x,\varepsilon)} u(x) \langle A\nabla\Gamma(x-y), \nu\rangle d\sigma(y) \right| \leq$$

$$\int_{\partial B_{A^{-1}}(x,\varepsilon)} |u(y) - u(x)| \, |\langle A\nabla\Gamma(x-y), \nu\rangle| \, d\sigma(y) \leq \sup_{y\in\partial B_{A^{-1}}(x,\varepsilon)} |u(y) - u(x)|$$

$$\int_{\partial B_{A^{-1}}(x,\varepsilon)} \langle A\nabla\Gamma(x-y), \nu\rangle d\sigma(y) \xrightarrow{\varepsilon\to 0} 0.$$

(per continuità della u e perché $\int_{\partial B_{A^{-1}}(x,\varepsilon)} \langle A\nabla\Gamma(x-y), \nu\rangle d\sigma(y) \xrightarrow{\varepsilon\to 0} 1$)

Da questo concludo che esiste

$$\lim_{\varepsilon\to 0} \int_{\partial B_{A^{-1}}(x,\varepsilon)} u(y) \langle A\nabla\Gamma(x-y), \nu(y)\rangle d\sigma(y) =$$

$$\lim_{\varepsilon\to 0} u(x) \int_{\partial B_{A^{-1}}(x,\varepsilon)} \langle A\nabla\Gamma(x-y), \nu(y)\rangle d\sigma(y) = u(x).$$

Devi spiegare perché quando metti $|\cdot|$ non continui a convergere

Ora considero I_2. Vedo che:

$$|\langle A\nabla\Gamma(x-y), \nabla u\rangle| \leq |A\nabla\Gamma(x-y)| \, |\nabla u| \leq \|A\| \, |\nabla\Gamma(x-y)| \, |\nabla u|$$

dove $\|A\|$ indica la norma matriciale di A.