Problemi percettivi

C'è una differenza significativa fra quello che guardiamo e quello che percepiamo

- L'immagine è individuata a meno di contrasto
- Problemi di segmentazione e di completamento

Leggi della Gesthalt

vicinanza,

somiglianza,

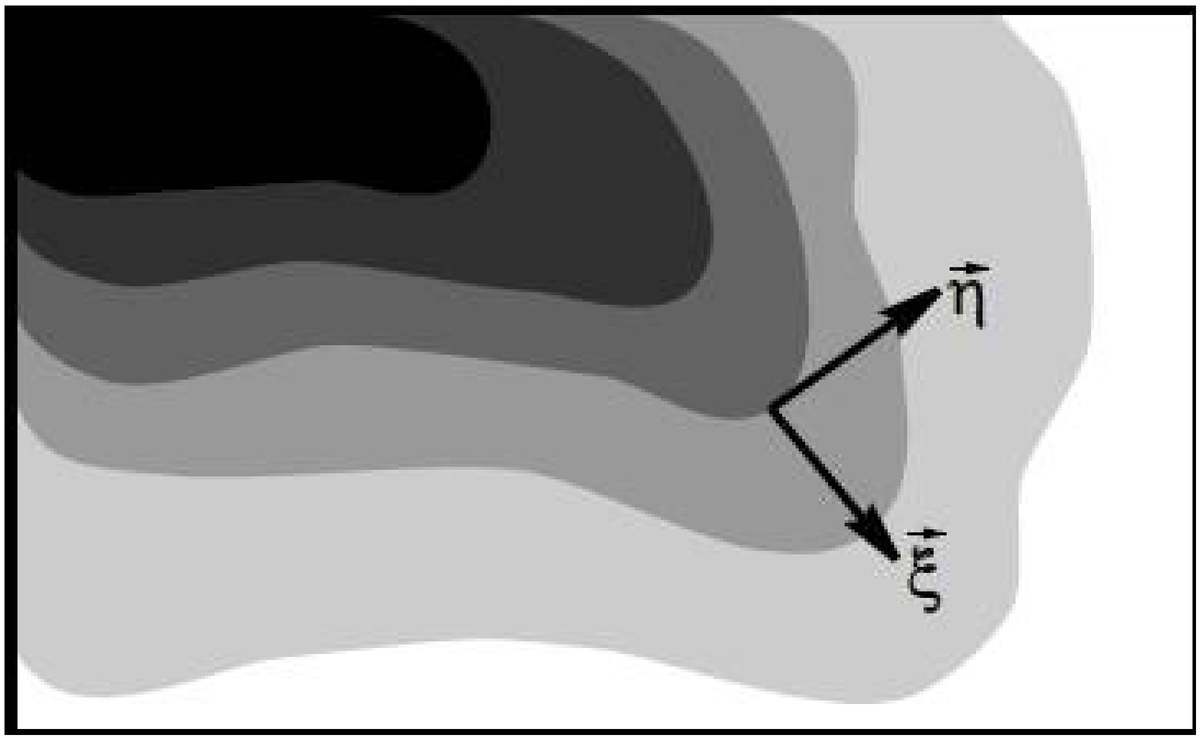
continuità di direzione – continuazione di bordi

chiusura,

convessità,

Completamento amodale – terminazione di bordi

Geometric diffusion



C'è l'esigenza di mantenere I bordi e I level set dell'immagine ,
che hanno un forte significato secondo la teoria della Gesthalt

Modelli variazionali

Modelli che dipendono dal gradiente definiti

- da un funzionale, di cui ci interessano i minimi o punti critici

- da un'equazione differenziale che potrebbe essere
la steepest descent del funzionale

Mumford Shah

$$MS(u,\gamma) = \int_{\Omega-\gamma} (u-g)^2 + \int_{\Omega-\gamma} |\nabla u|^2 + length(\gamma)$$

Il funzionale somma di tre termini:

- il primo termine forza l'immagine ad essere vicinal al dato iniziale
- il secondo termine regolarizza l'immagine, lontano da eventuali singolarit di tipo salto
- il terzo termine minimizza la lunghezza dei bordi

Ambrosio Tortorelli approximation

$$\mathcal{AT}_\epsilon(u,v) = \int_\Omega (u-g)^2 dx + \lambda \int_\Omega v^2 |\nabla u|^2 dx + \alpha \mathcal{M}_\epsilon(v),$$

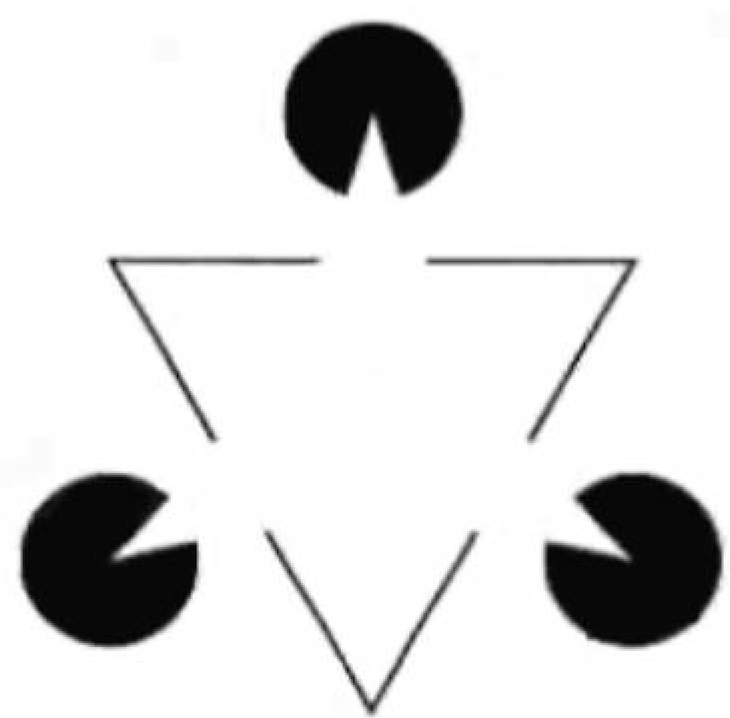
$$\mathcal{M}_\epsilon(v) = \int_\Omega \left(\epsilon |\nabla v|^2 + \frac{(1-v)^2}{\epsilon} \right) dx.$$

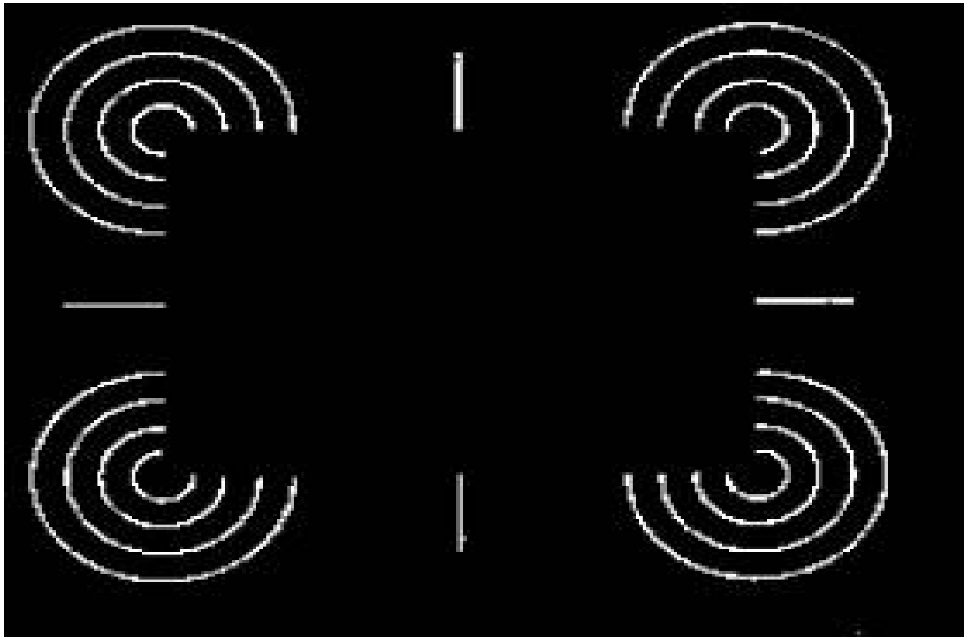
Curved boundary completion

The Munford Shah functional produces
piecewise constant minima, with linear interfaces

Munford introduce an elastica term:

$$\int_\gamma (1 + \phi(k^2))\, ds.$$





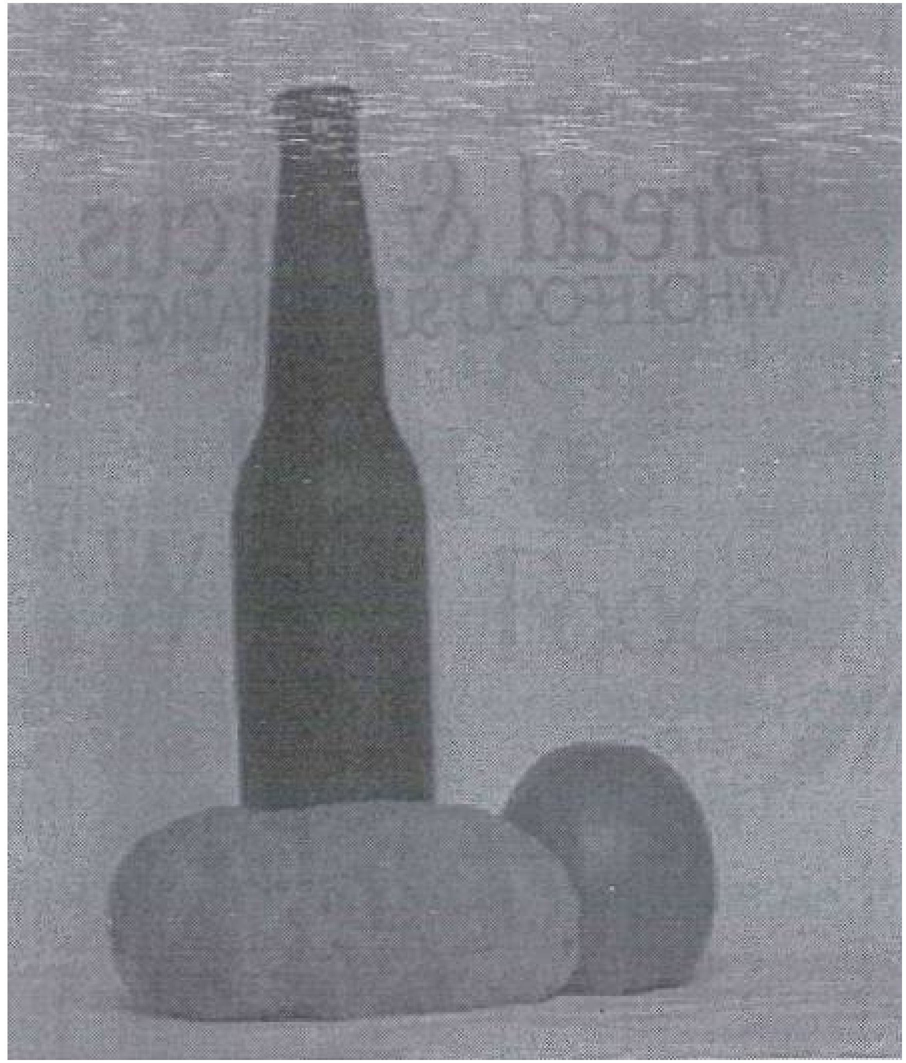
Se   è lineare all'infinito permette la ricostruzione
Di contorni con spigoli

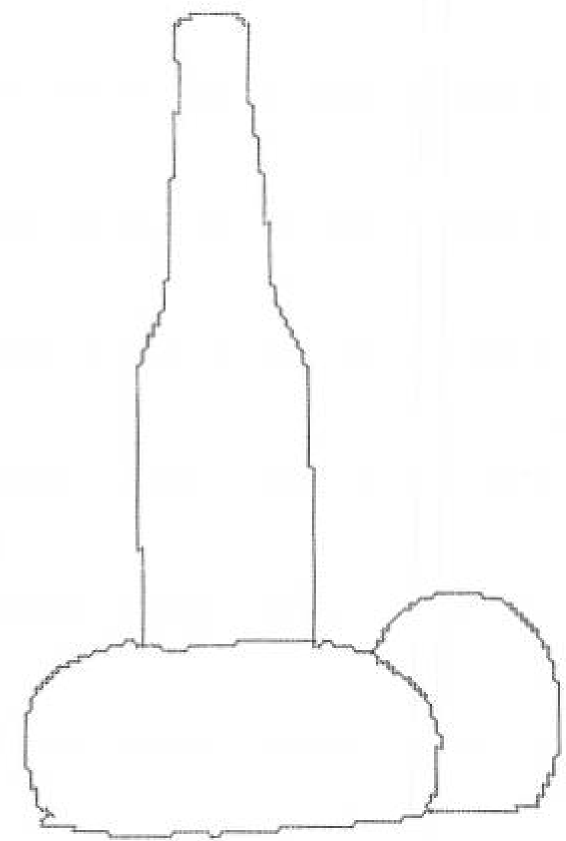
Mumford Nitzberg Shiota Segmentation with depth

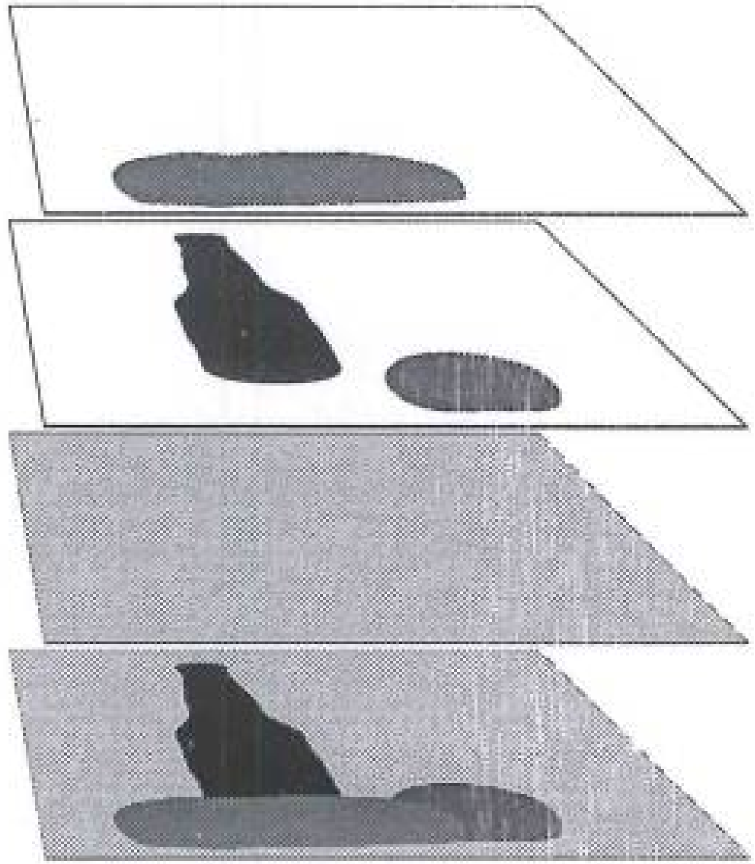
$$\mathcal{MS}^{\mathrm{j}}(u,B)=\int_{\Omega\setminus B}(u-g)^2dx+\lambda\int_{\Omega\setminus B}|\nabla u|^2dx+\int_B\left(\alpha+\beta k^2\right)ds,$$

Questo operatore permette il completamento a-modale

Vastissima letteratura per lo studio di questi problemi







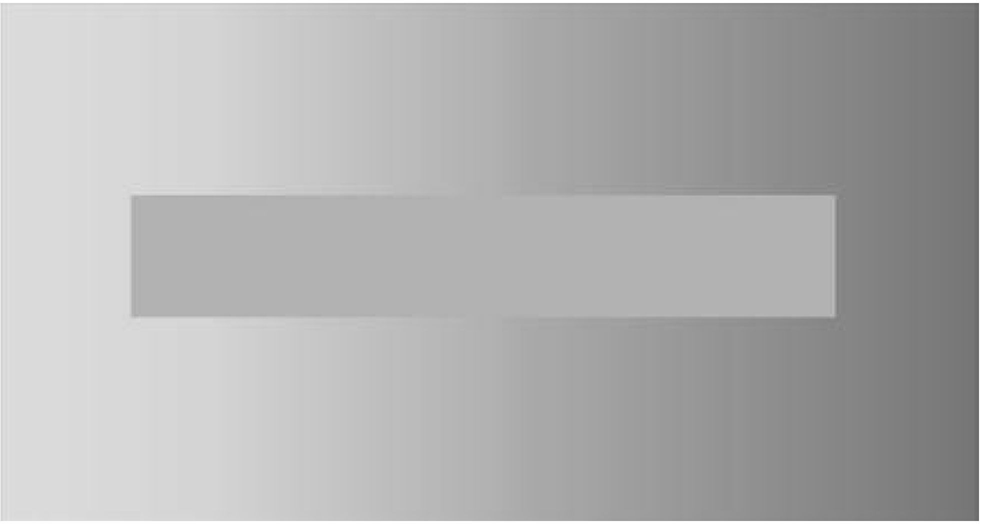
Second order functionals

$$\int_{\gamma} (1 + \phi(k^2))\, ds.$$

$$\int_{\Omega} |\nabla u| \left(1 + \left| div\left(\frac{\nabla u}{|\nabla u|}\right)\right|\right)$$

Curvature operator of each level set: Masnou Morel

contrasto



Il retinex

$I = LR \qquad h = log(I) = log(L) + Log(R) = \phi + r$

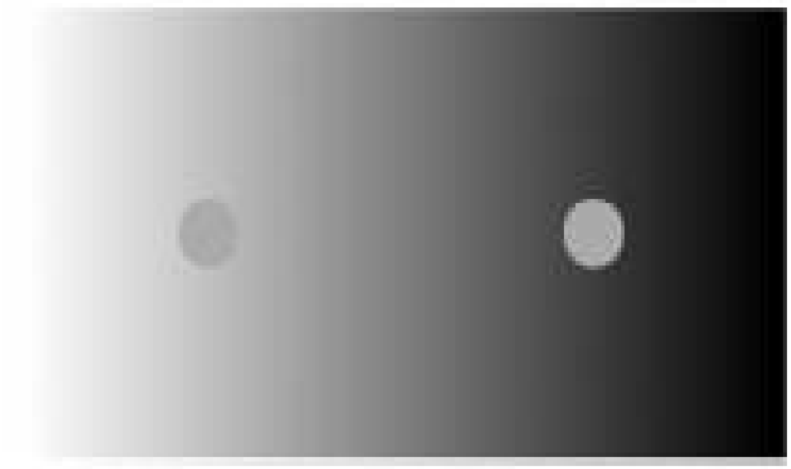
$$J(\phi) = \int_\Omega |\nabla(\phi - h)|^2 + \alpha |\nabla \phi|^2$$

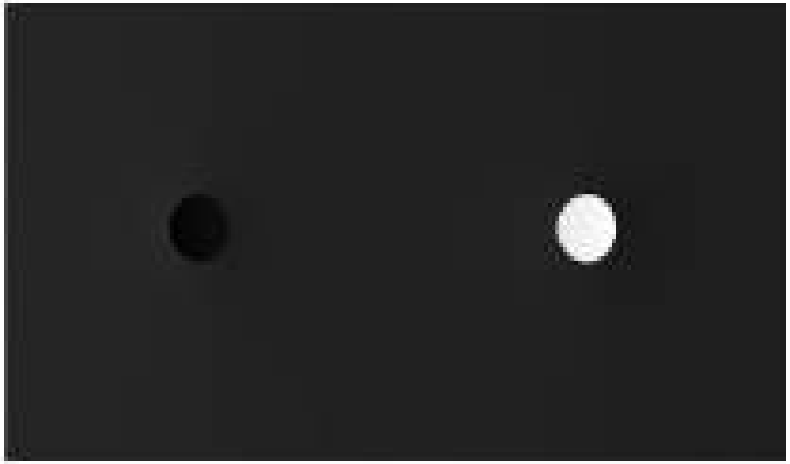
$$\phi \geq h \quad \partial_\nu \phi = 0 \quad \text{on } \partial\Omega$$

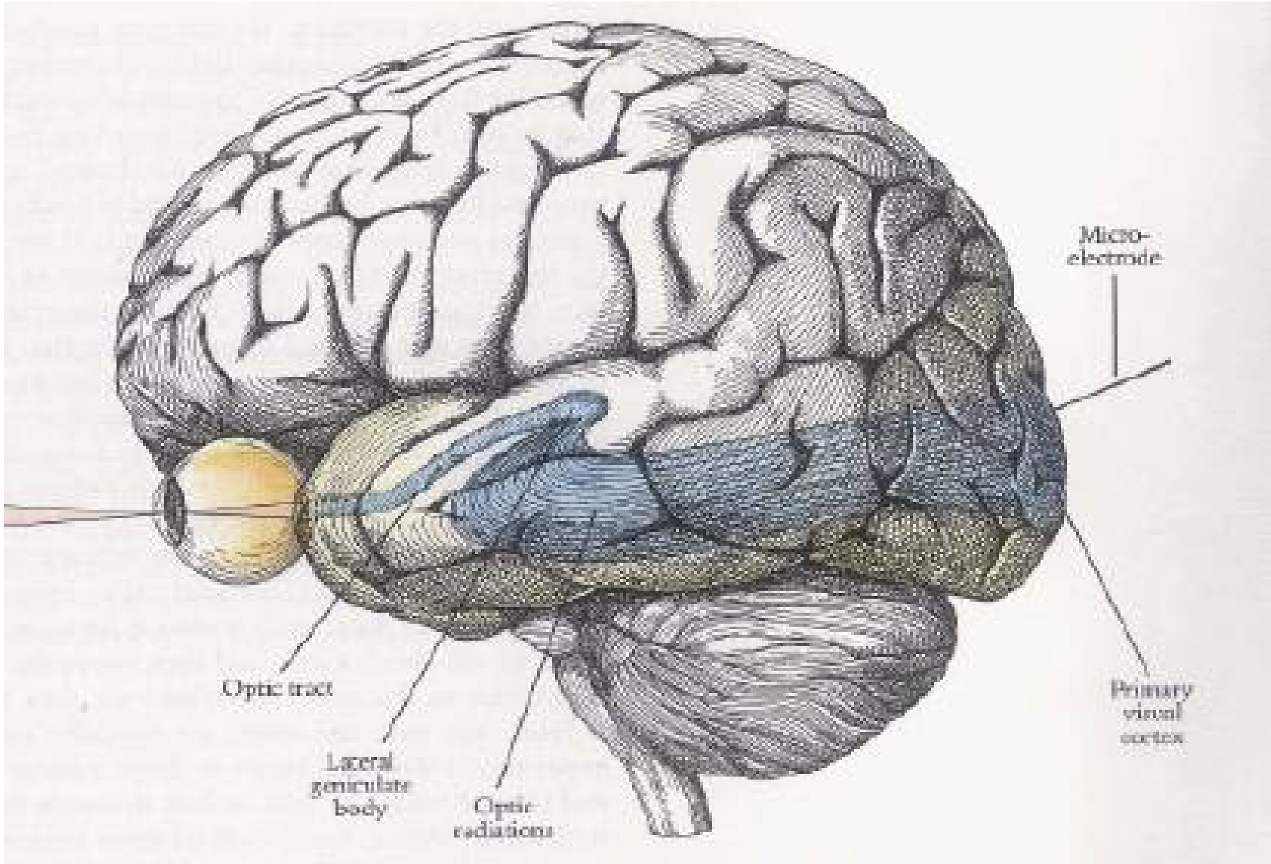
Land, McCann: '71, Horn '74
R. Kimmel, M. Elad, D. Shaked, R. Keshet, I. Sobel 2003

Variational models: Morel, Pedro, Sbert 2010

$$J(\phi) = \int_\Omega |\nabla(\phi - h)|^2 \cdot \qquad \Delta\phi = \Delta h$$





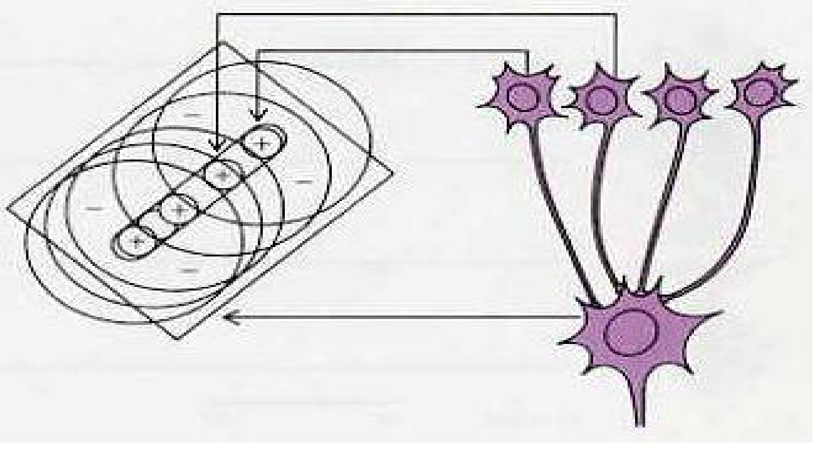


[Hubel Wiesel] [W.C. Hoffmann '89]
[Petitot and Tondut '99], [Petitot '03]
[S. Zucker ' 05]

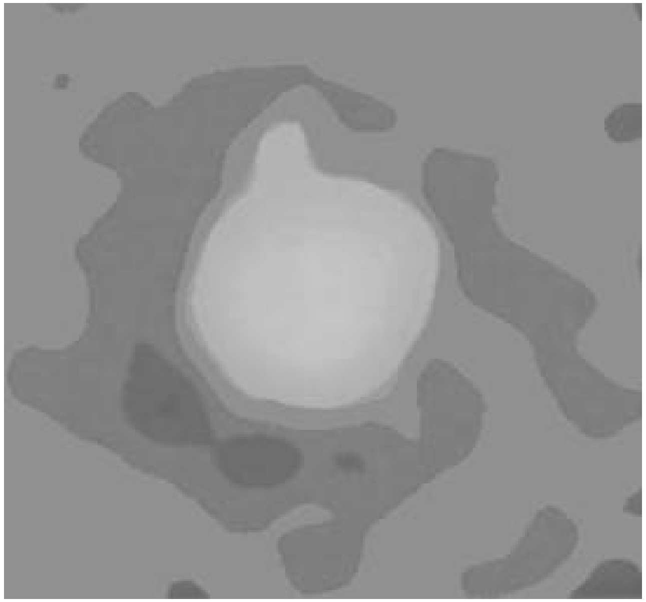
[C.- Sarti] [Sarti, C- Petitot '08]
[Hladky and Pauls '08].
[Duits, van Almsick, Franken, ter Haar Romeny '05, '08]

Campo recettore

- L'insieme dei recettori retinici che portano informazioni ad una specifica cellula cella corteccia visiva

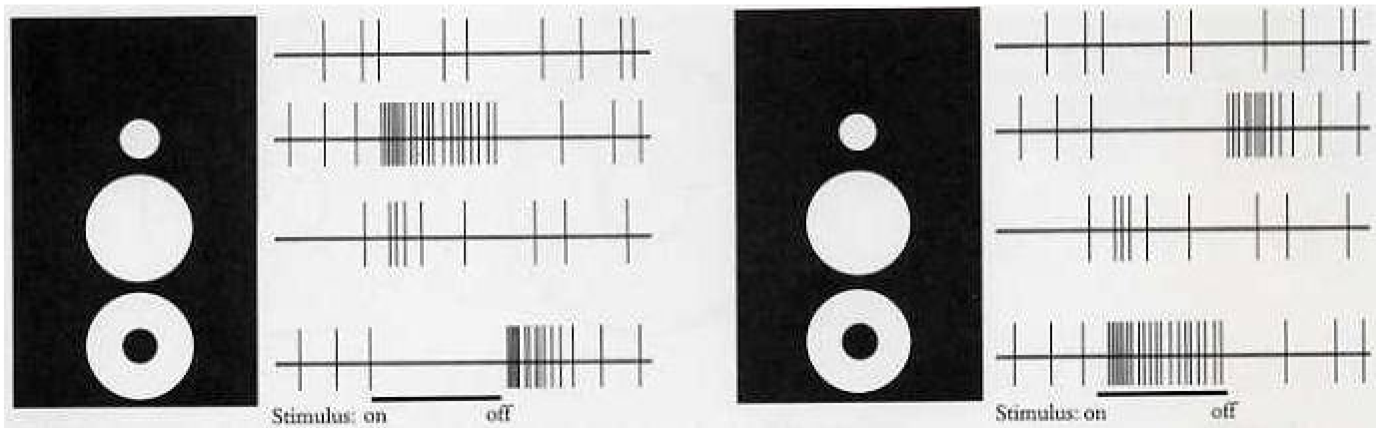


Profilo recettore



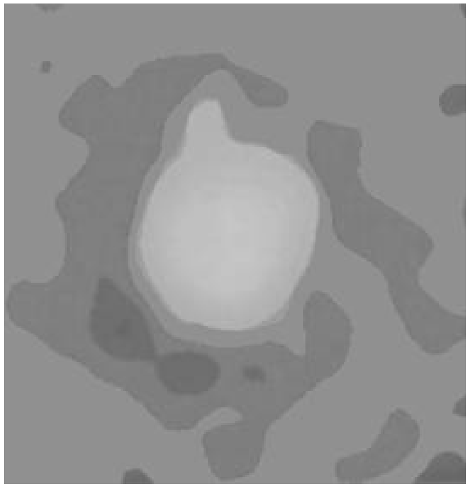
Funzione definita sul campo recettore e che descrive l'intensità della risposta ad uno stimolo visivo

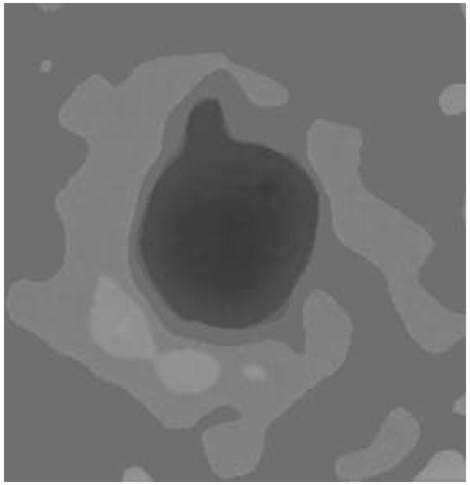
Retinal and Lgn cells



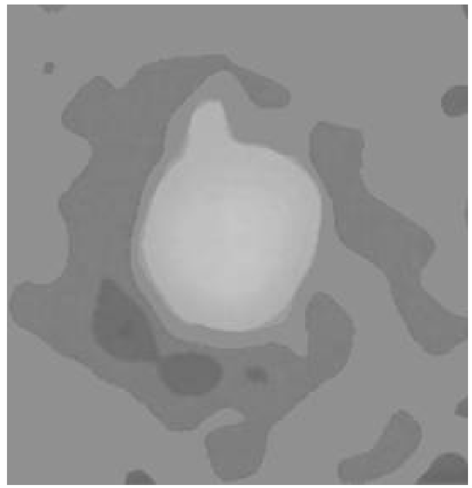
Four recordings from a typical oncenter cell.
In the resting state at the top, there is no stimulus
responses to a small (optimum size) spot,
a large spot covering the receptive-field center and surround
a ring covering the surround only.

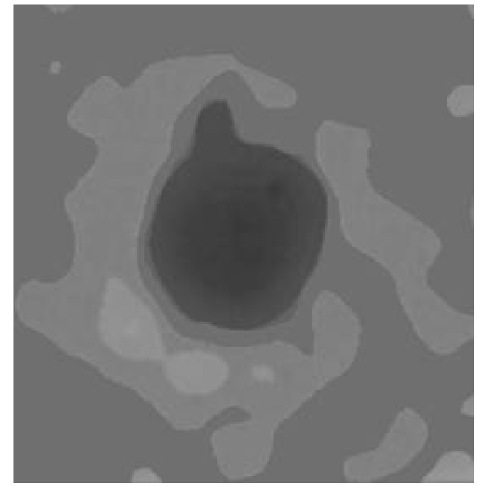
Determinare una funzione che descriva il profilo



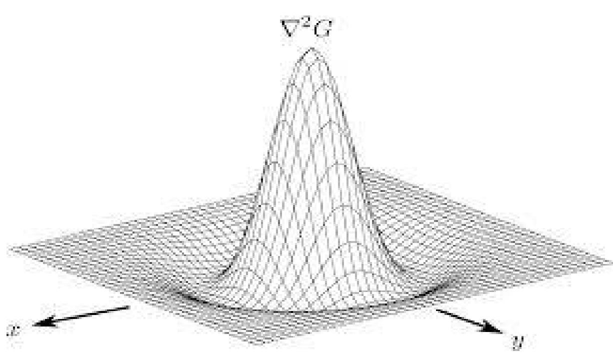


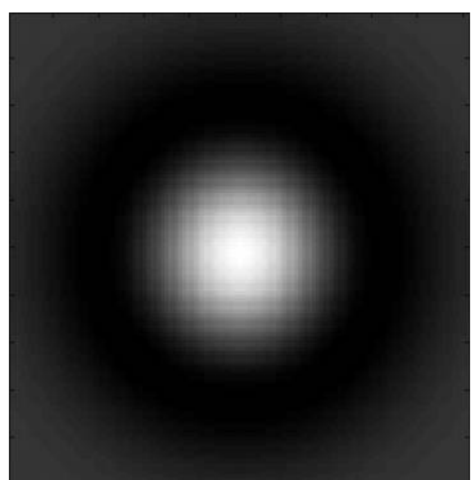
LGN cells

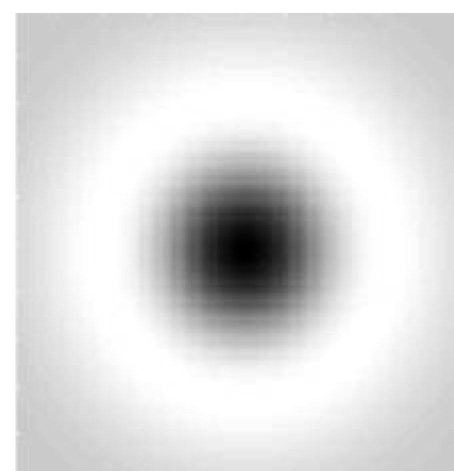




Il profilo recettore è modellato attraverso un Laplaciano di gaussiana:







Azione delle cellule

- In presenza di un'immagine I

$$O = \int \Delta G(x, y) log I(x, y) dx dy$$

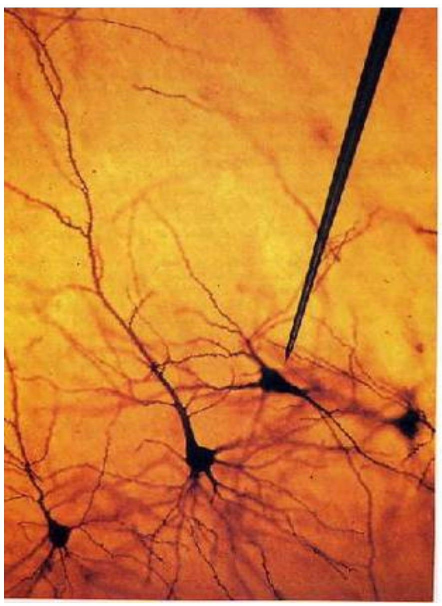
La stessa struttura è ripetuta in ogni punto

$$O(x_0, y_0) = \int \Delta G(x_0 - x, y_0 - y) log I(x, y) dx dy =$$

$$\Delta G * log I(x_0, y_0) = \Delta h_G(x_0, y_0)$$

diffusion in LGN

- The visual signal propagates along the axon of the cells: Due to the isotropy of the structure we can model the connectivity kernel as fundamental solution of an isotropic Laplacian

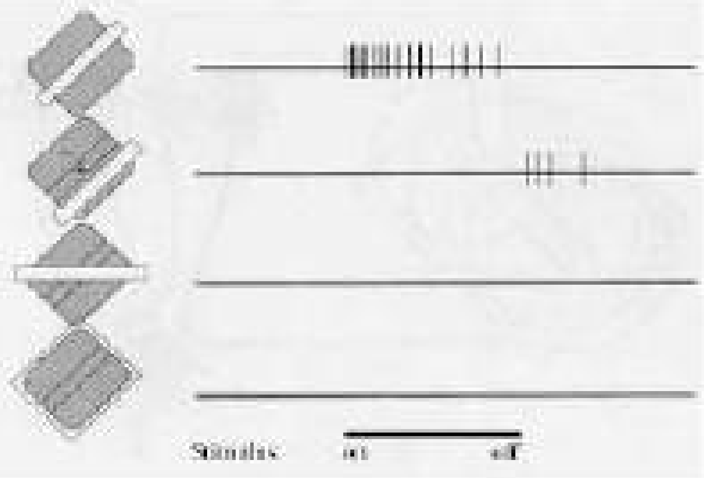


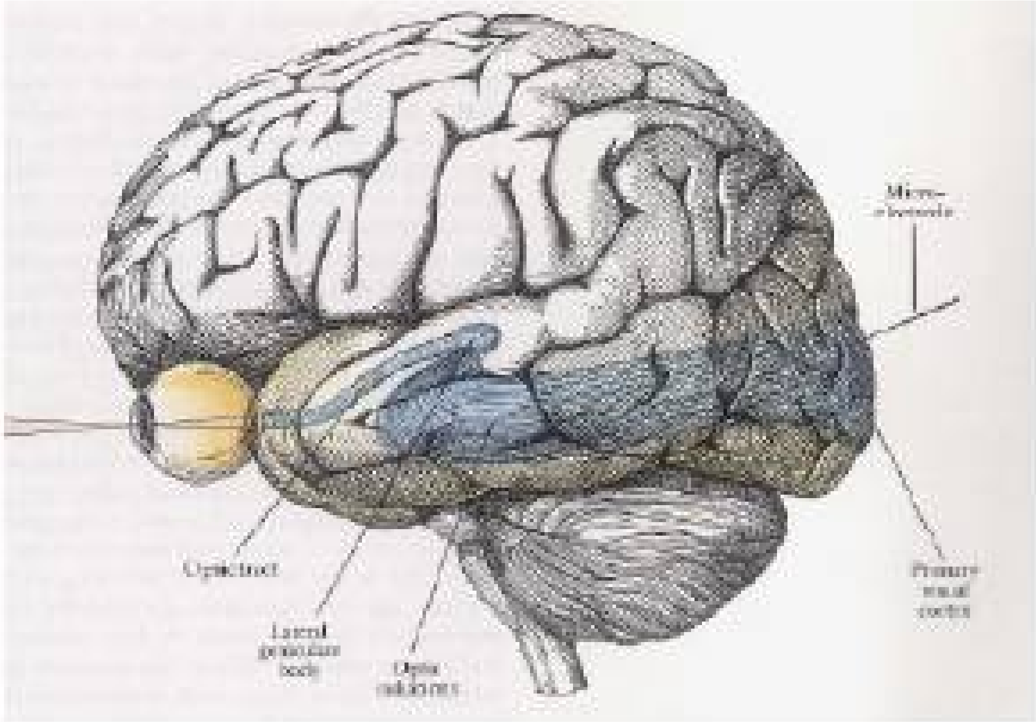
$$\Gamma(x,y) = -\log\sqrt{x^2+y^2}.$$

$$\phi = \Gamma * \Delta h_G$$

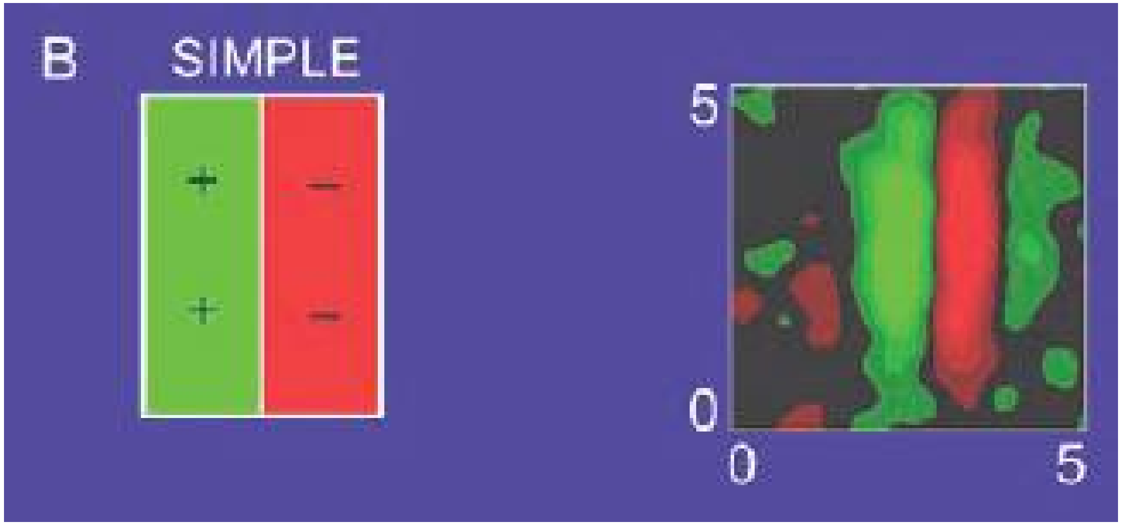
$$\Delta\phi = \Delta h_G$$

The diffusion in LGN produces a contrast invariant perception

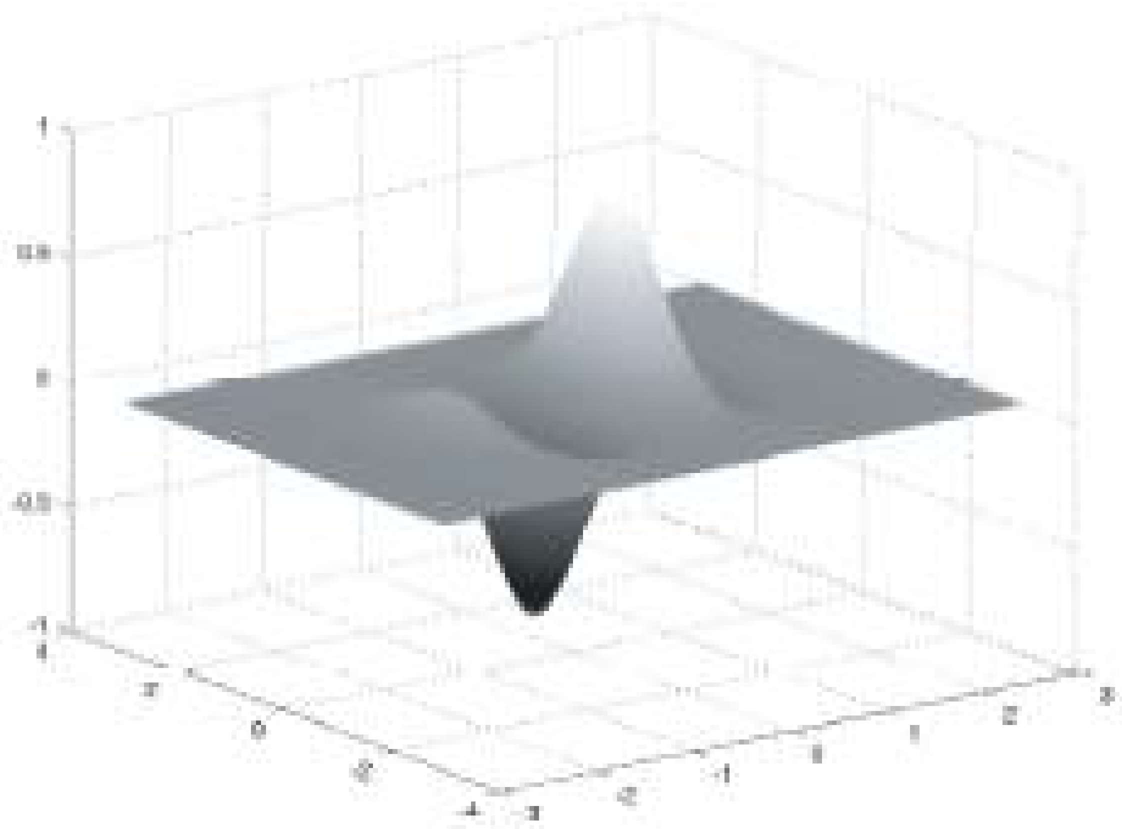

Stimulus
on
off

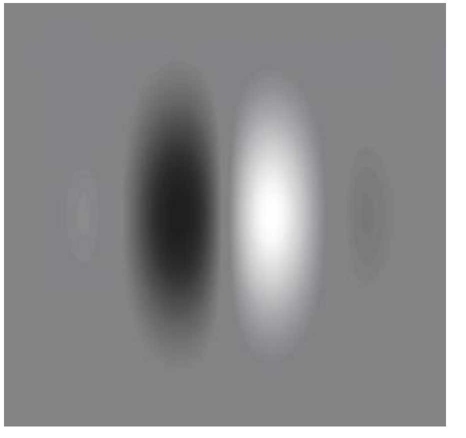


Receptive profile of a simple cell



Receptive profile of a simple cell

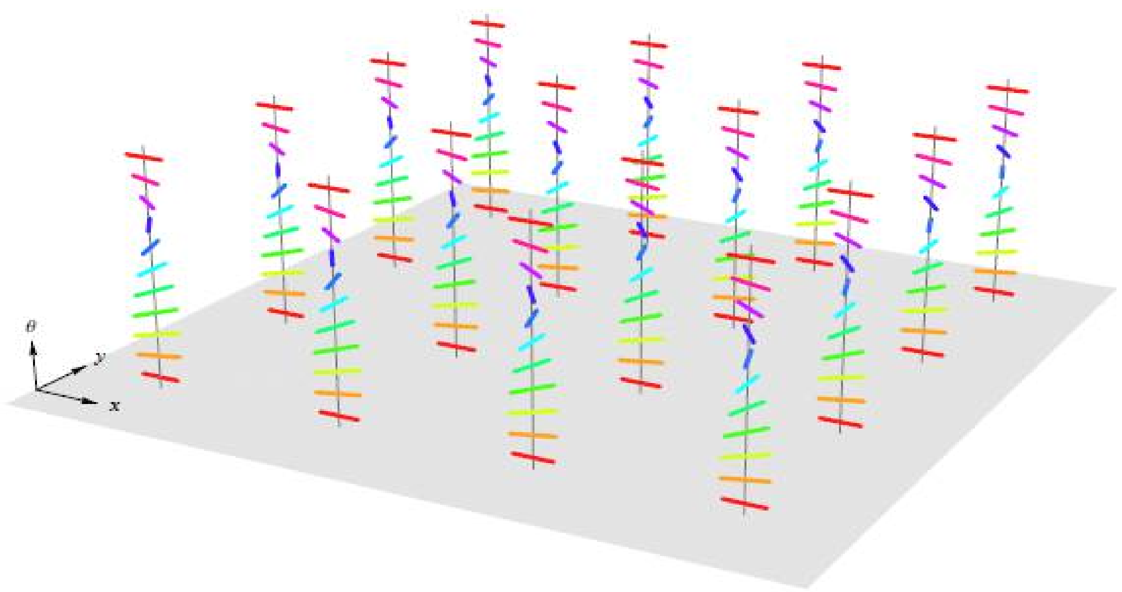




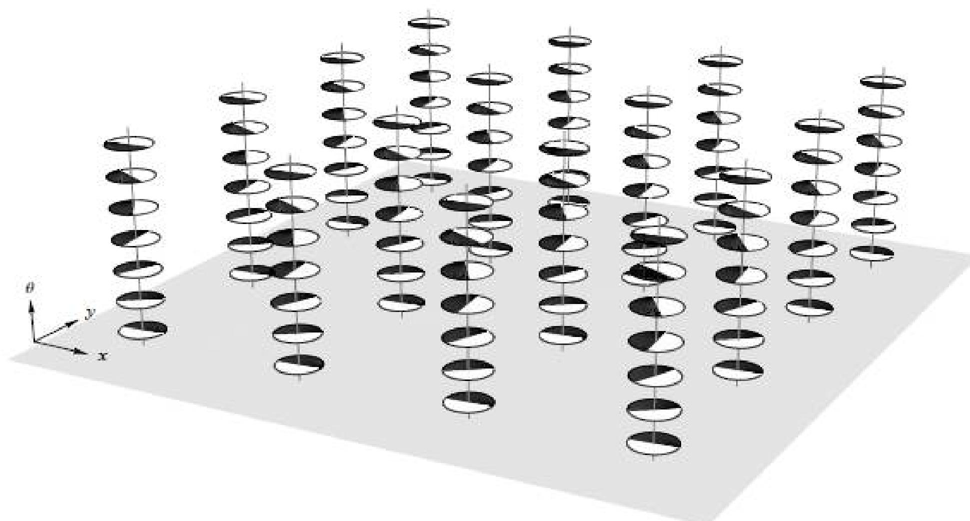
Simple cells can be modelled as derivative of a Gaussian

The cortical structure

- Retinotopic structure
- Ipercolumnar structure



The simple cells as a Lie group



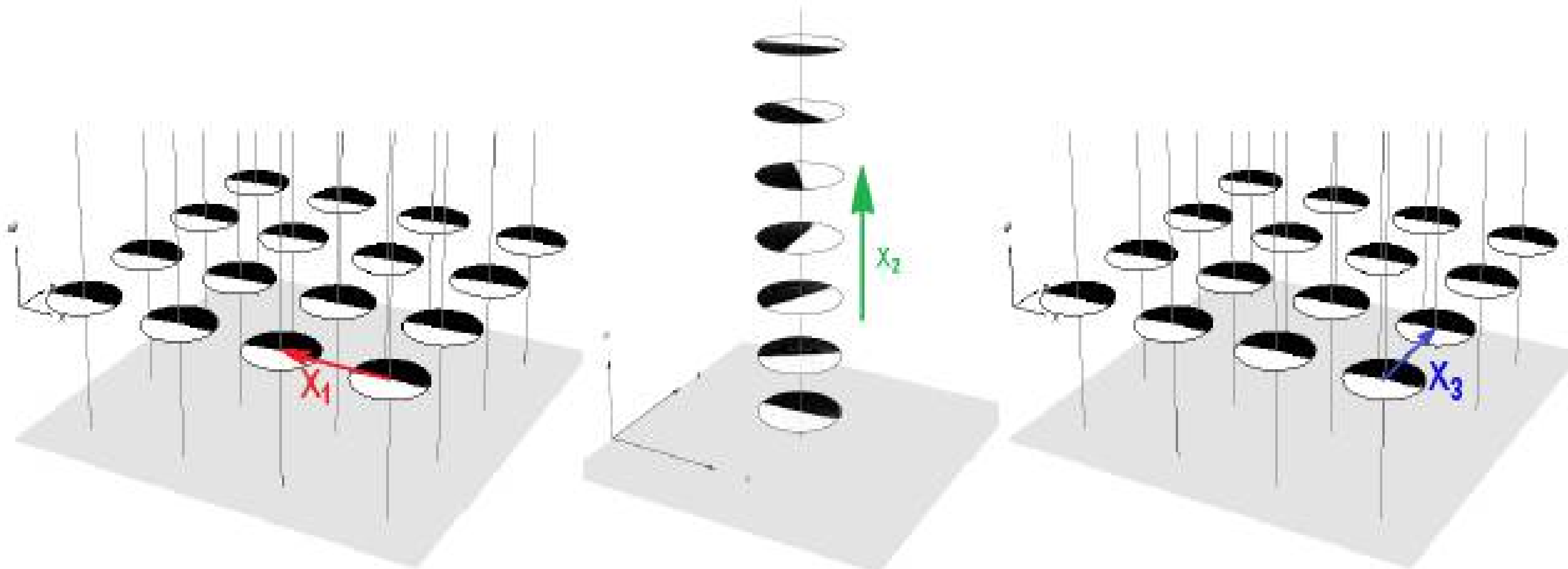
T_{x_1,y_1} the translation of the vector (x_1, y_1)

R_θ a rotation matrix $\qquad \mathrm{R}_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$

Legge di gruppo: composizione di traslazione e rotazione

The associated Lie algebra is generated by the vector fields

$$X_1 = \cos(\theta)\partial_1 + \sin(\theta)\partial_2, \quad X_2 = \partial_\theta, \quad X_3 = -\sin(\theta)\partial_1 + \cos(\theta)\partial_2$$



Campi invarianti a sinistra

$$\phi_0(x, y) = \partial_y \exp(-(x^2 + y^2))$$

$$\phi_\theta(x, y) = \phi_0 \circ R_\theta(x, y)$$

$$= X_3 \exp(-(x^2 + y^2))$$

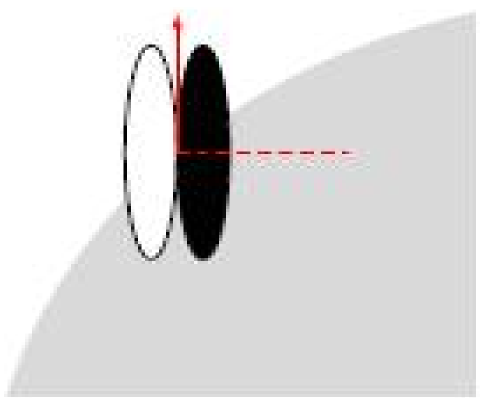
$$X_3 = -\sin\theta\partial_x + \cos\theta\partial_y.$$

General simple cell:

$$)\phi_e(x - x_0, y - y_0)$$

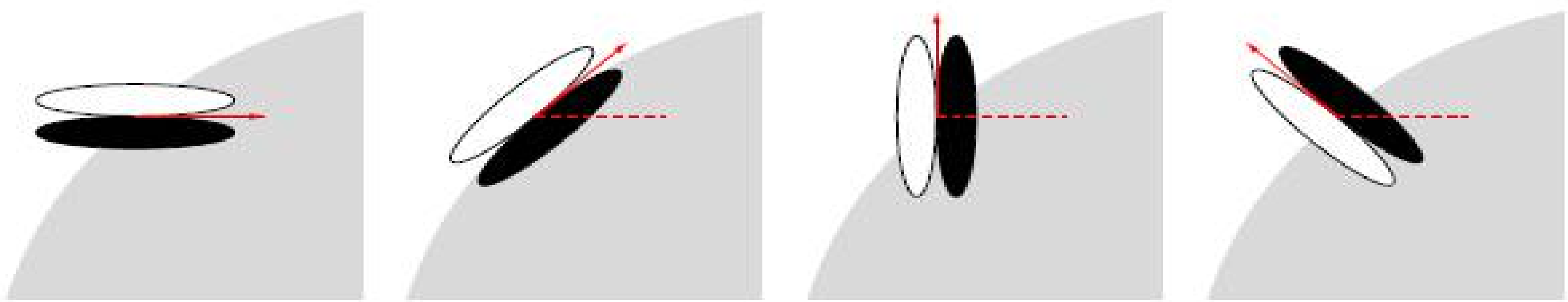
Output of a cell

$$O_e = \int I(x,y)\phi_e(x,y)\,dx\,dy$$



Azione delle cellule semplici

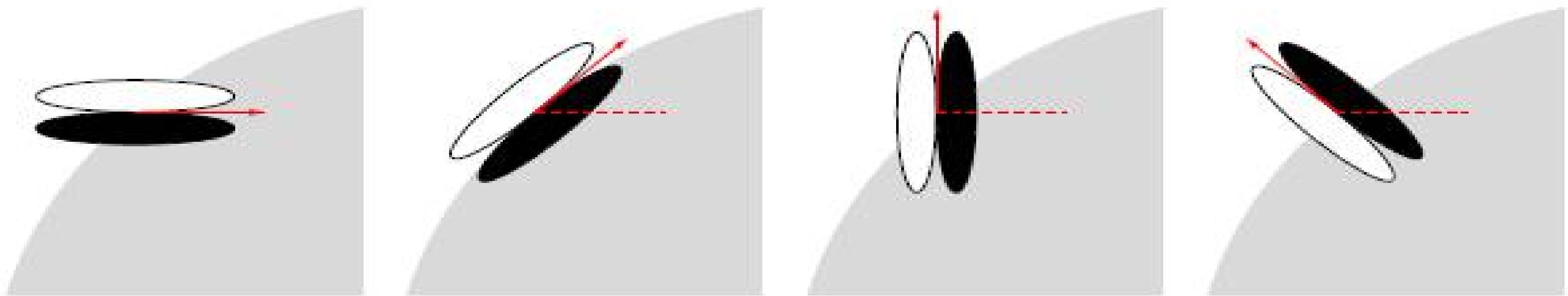
$$O(x, y, \theta) = \int \phi_\theta(x - x', y - y') I(x', y') \, dx' \, dy'$$



Derivazione direzionale

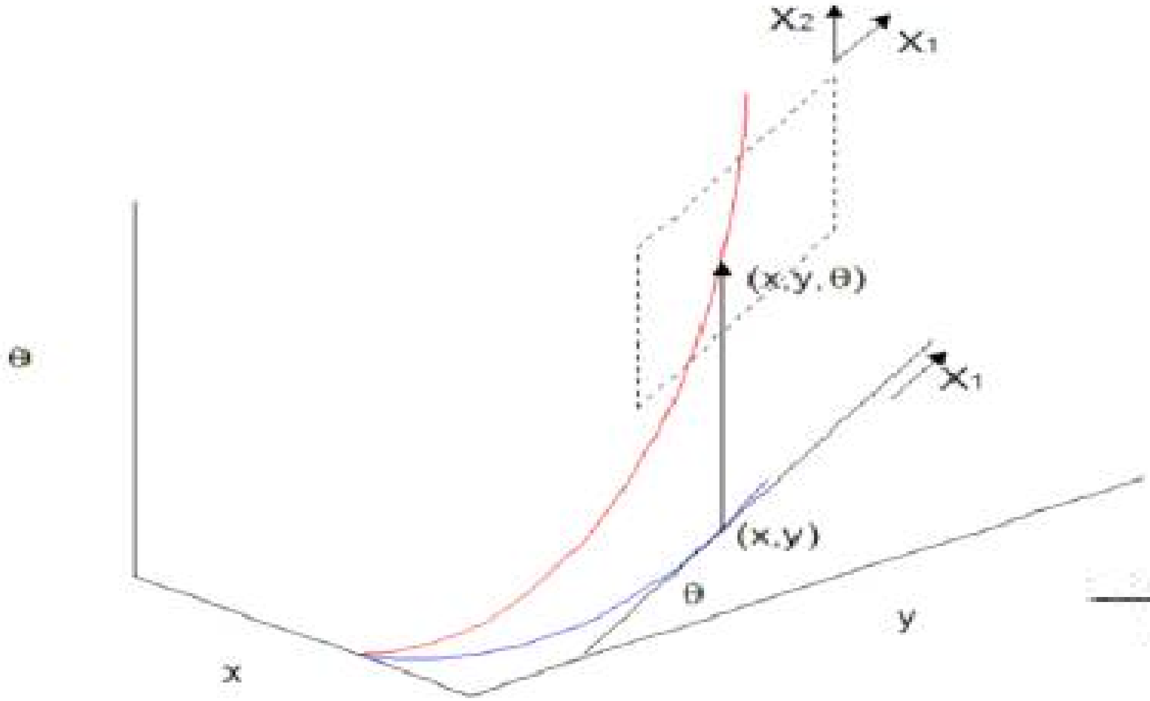
Suppressione dei non massimali

$$O(x, y, \theta^*) = \max_{\theta} O(x, y, \theta).$$



θ^* Indica l'orientazione del bordo

Lifted curved



$$\gamma_{2D}(t) = (x(t), y(t)),$$

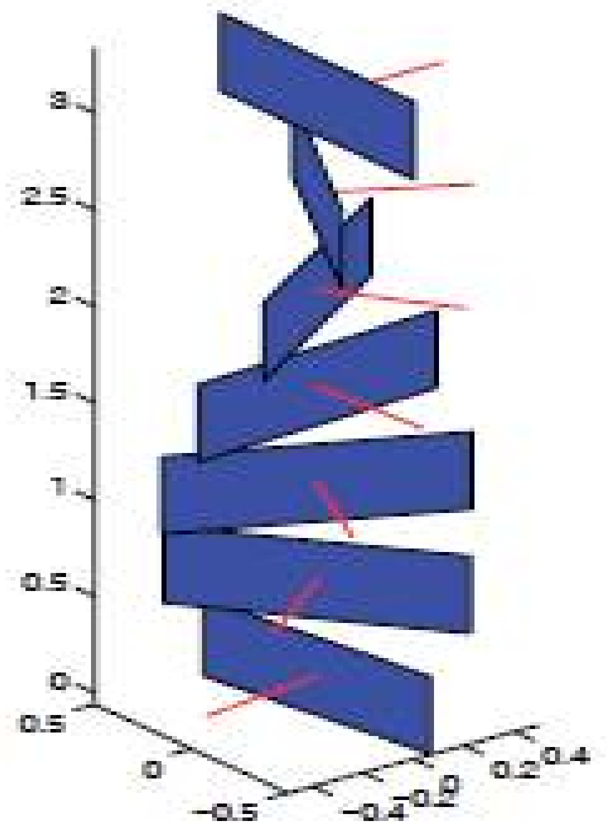
$$\rightarrow (x(t), y(t), \theta(t)).$$

$$X_2 = \partial_\theta$$

$$X_1 = \cos\theta\,\partial_x + \sin\theta\,\partial_y$$

Horizontal tangent plane

- We have selected two vector fields at every point such that

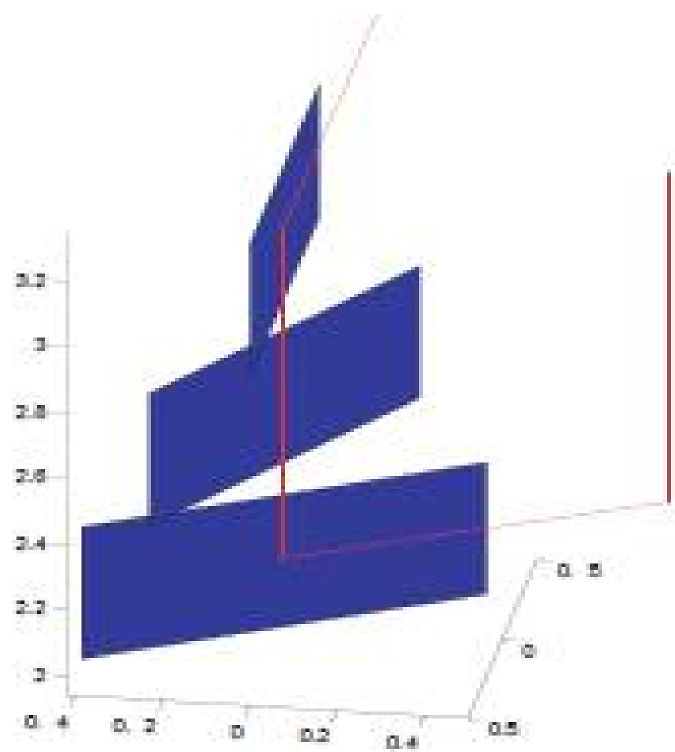


$$X_2 = \partial_\theta$$

$$X_1 = \cos\theta\,\partial_x + \sin\theta\,\partial_y$$

Campi di Hormander e connettività

$$X_3 = [X_1, X_2] = -\sin(\theta)\partial_x + \cos(\theta)\partial_y.$$



Norm on the horizontal plane

$$\|\alpha_1 X_1 + \alpha_2 X_2\|_g = \sqrt{\alpha_1^2 + \alpha_2^2}$$

Riemannian approximating norm

Control distance

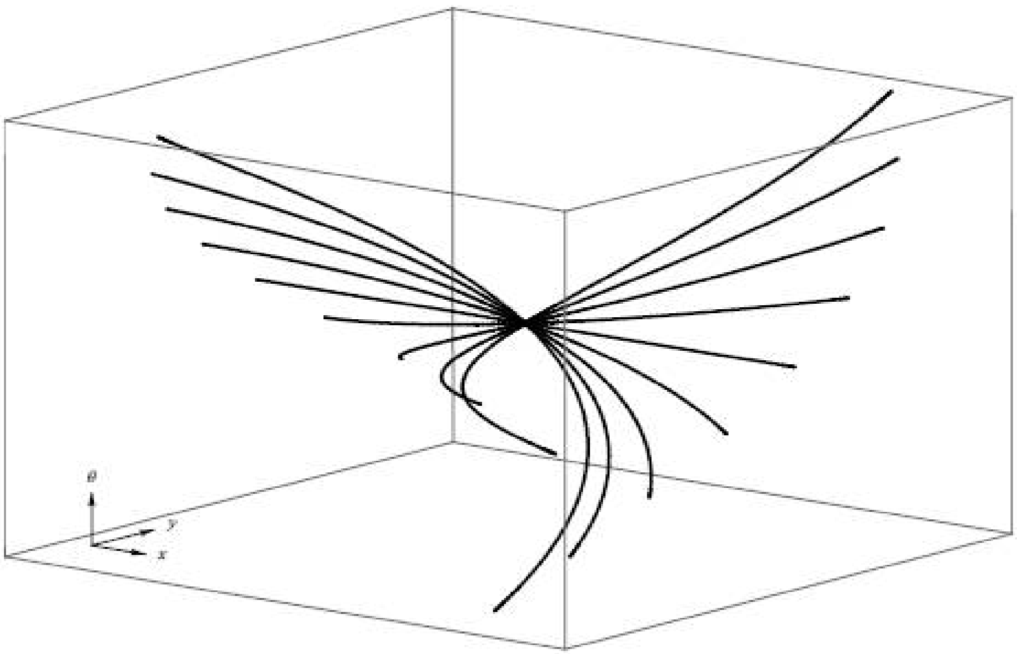
$$\lambda(\gamma) = \int_0^1 |\gamma'(t)|\, dt.$$

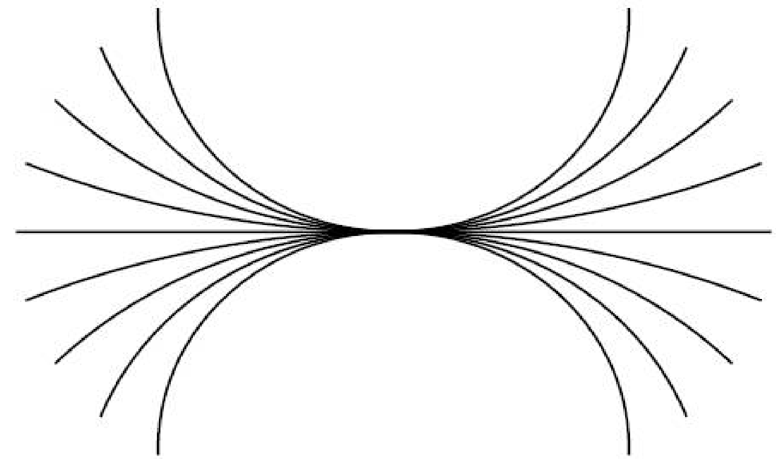
$$d(\xi, \xi_0) = \inf\{\lambda(\gamma) \ : \ \gamma \text{ is a horizontal curve connecting } \xi \text{ and } \xi_0\}.$$

Attenzione l'insieme delle curve ammissibili non è quello euclideo

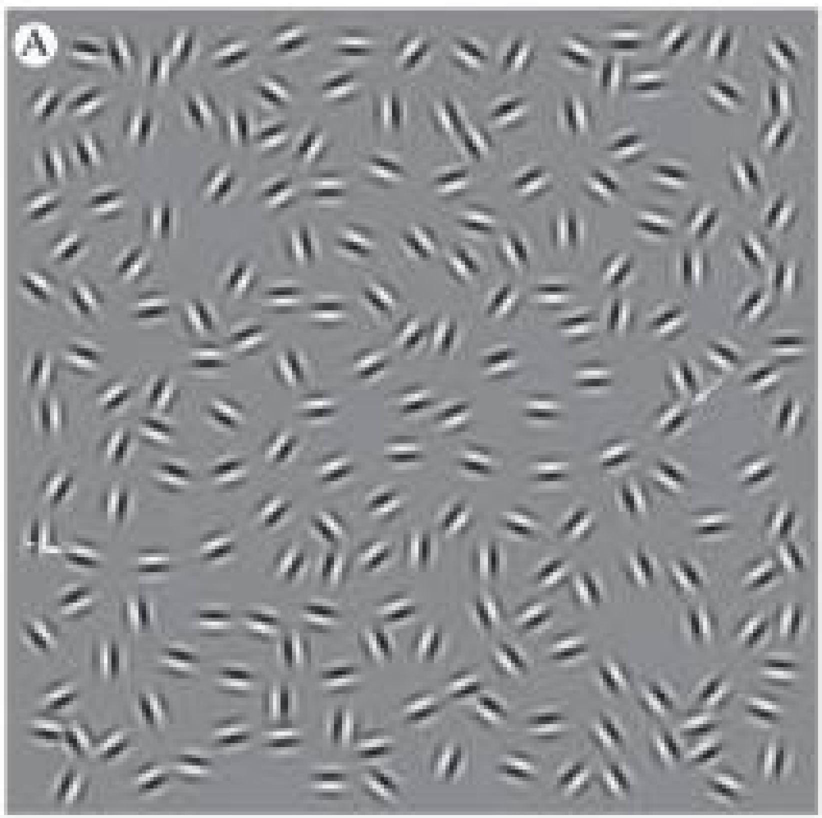
Norma omogenea e gauge

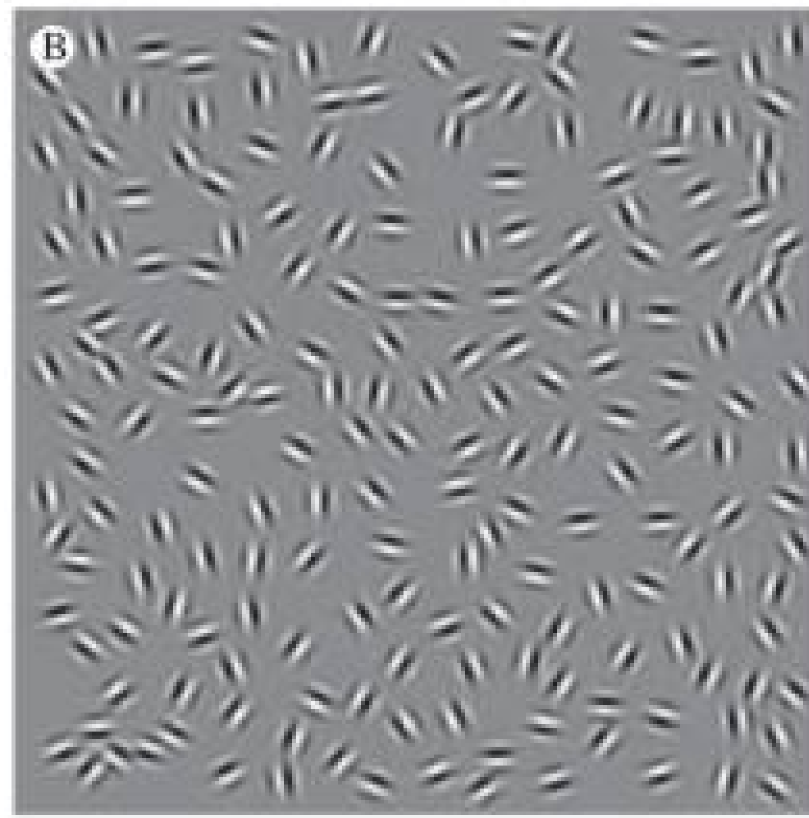
Integral curves of the vector fields

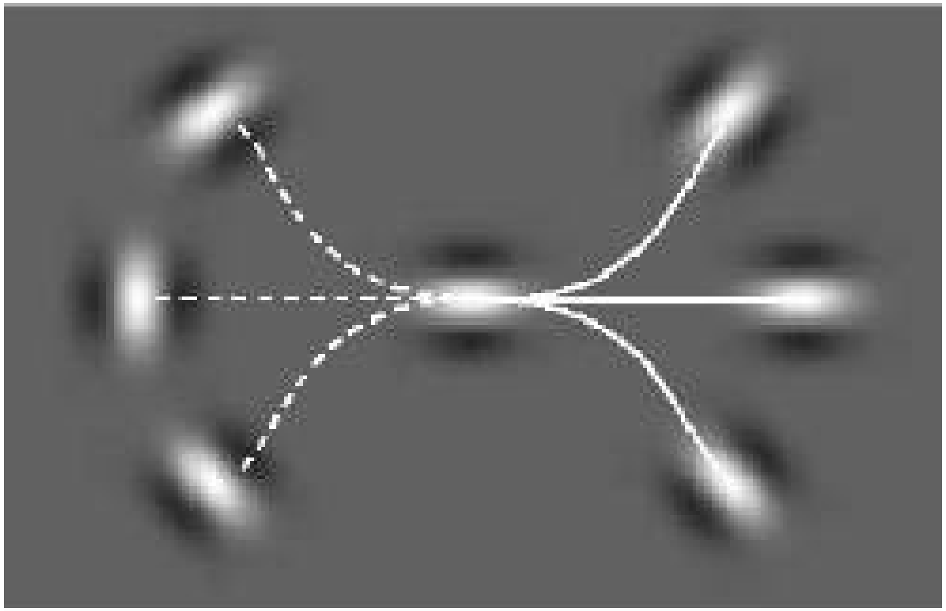




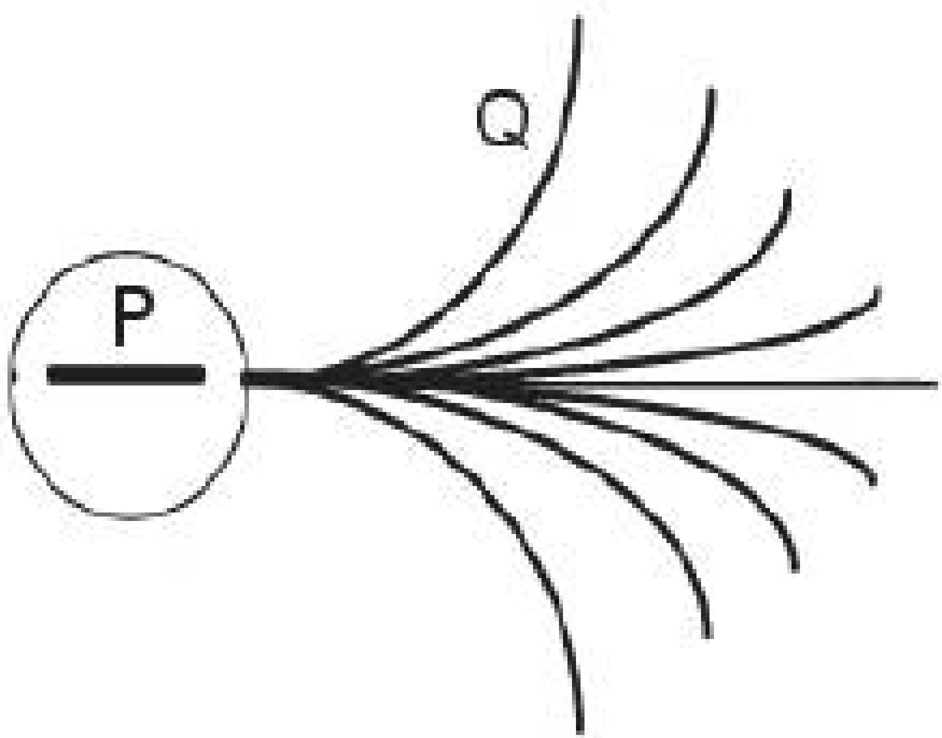
Experimento di Field, Hayes, Hess

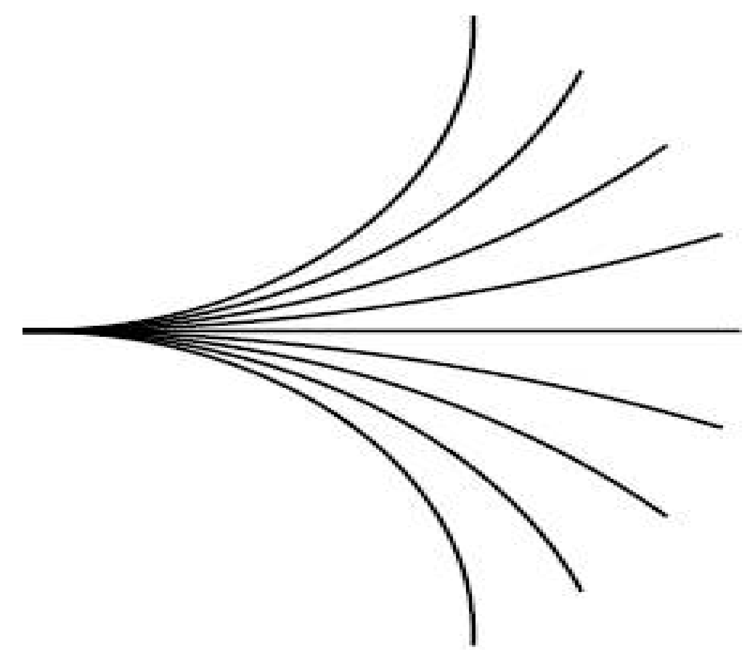






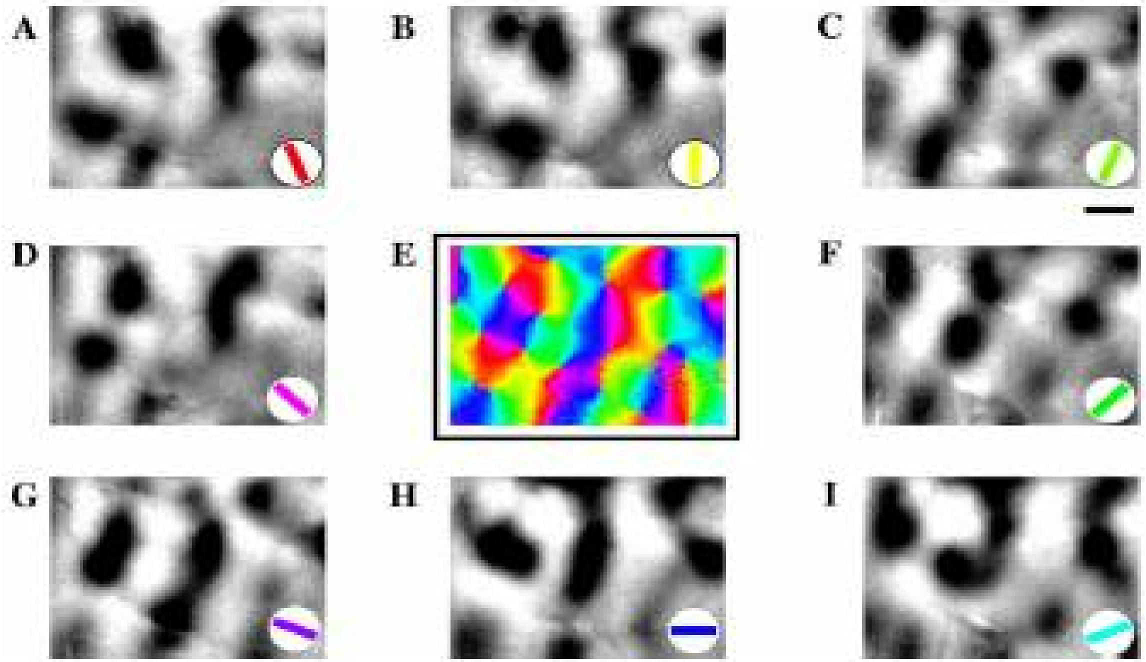
Campi di associazione

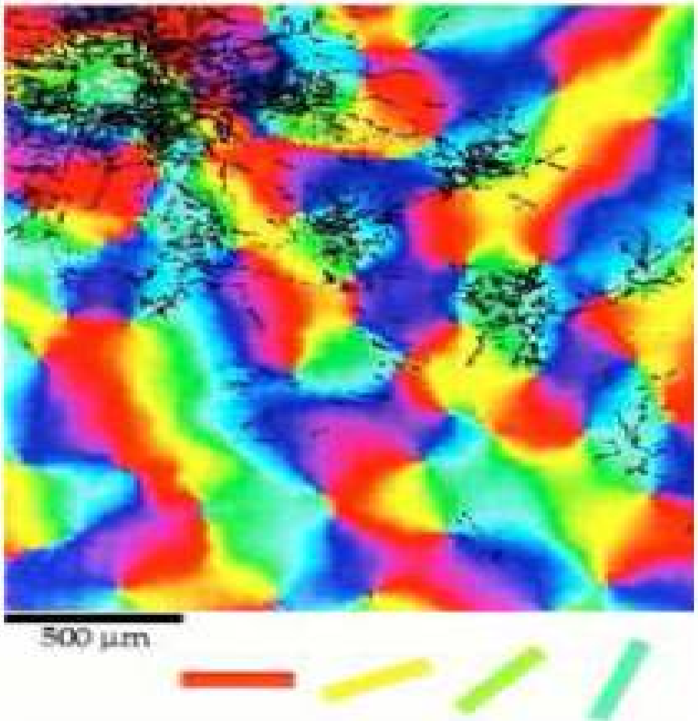




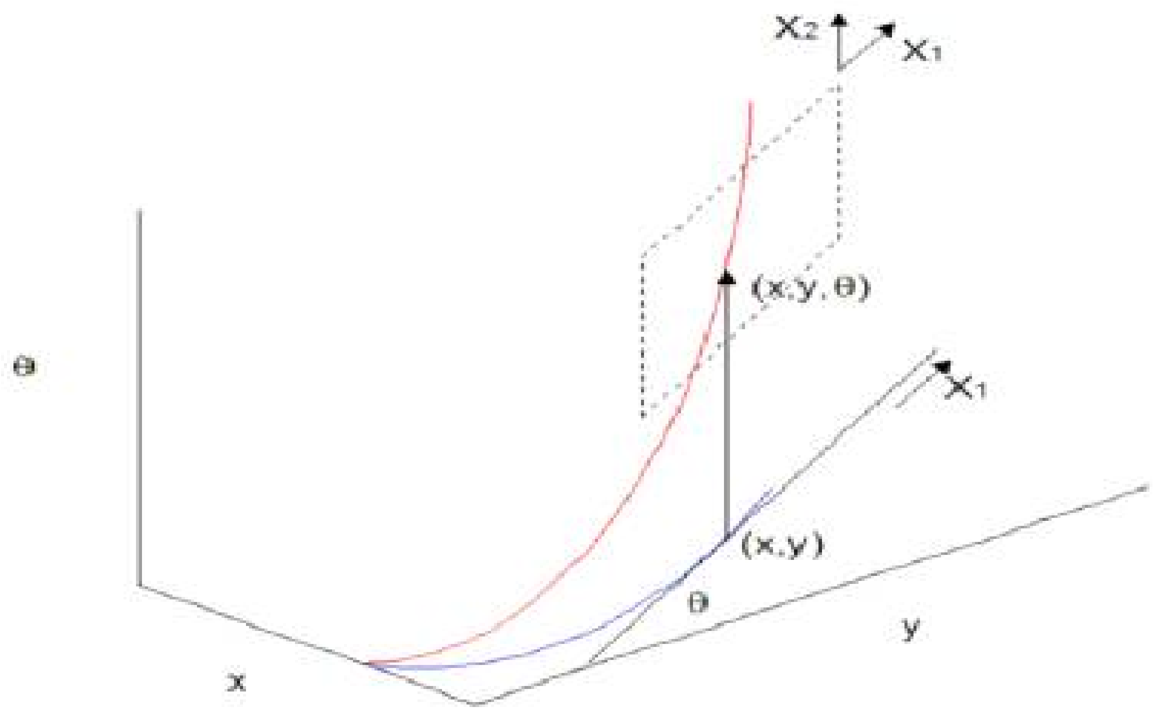
La distribuzione spaziale dei profili

$$\mathscr{P}(x,y) = \frac{1}{2} arg \int_0^{\pi} e^{i2\theta} u_{\theta}(x,y) d\theta. \tag{4.23}$$



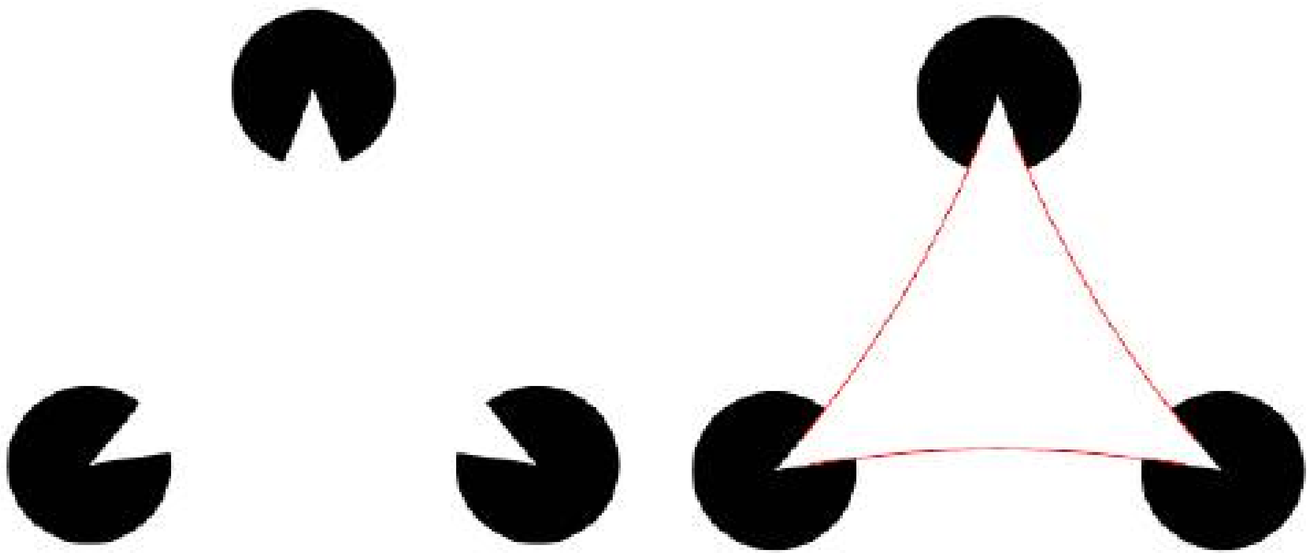

500 µm

Geodesics and elastica

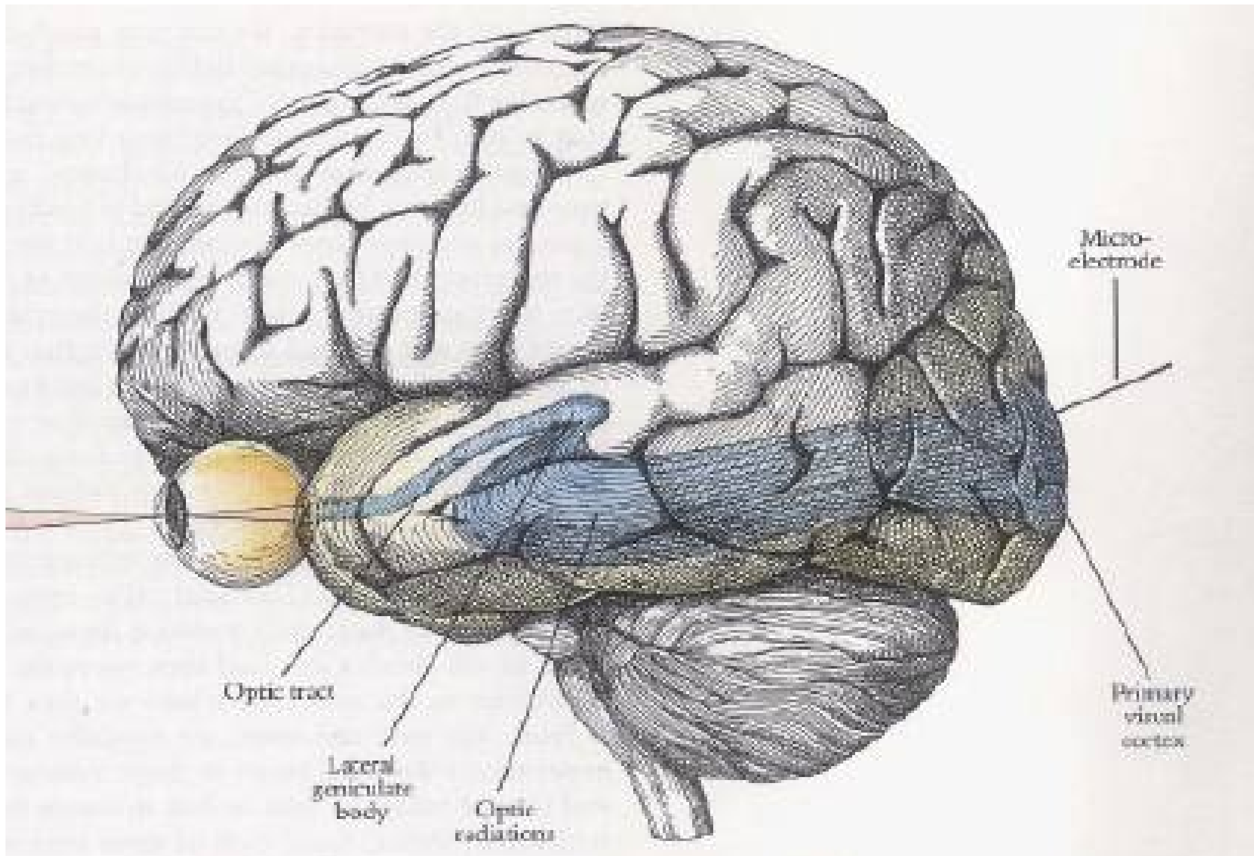


Il completamento è quello operato dal modello di Mumford

Completion of boundaries



La struttura della corteccia è modulare



Cellule selezionano: movimento, curvatura, colore