Utilizzando eventualmente gli seguenti sviluppi sottoindicati, per $x \to 0$,

$$\sin(x) = x - \frac{x^3}{3!} + o(x^3), \quad \cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + o(x^5), \quad \tan(x) = x + \frac{x^3}{3} + o(x^3)$$

$$\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + o(x^3), \quad \log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$$

$$(1+x)^\alpha = 1 + \alpha x + \alpha(\alpha-1)\frac{x^2}{2!} + \alpha(\alpha-1)(\alpha-2)\frac{x^3}{3!} + o(x^3)$$

calcolare i limiti seguenti:

$$\lim_{x\to 0^+} \frac{\sin(2x) - \tan(2x + x^3)}{x(1-\cos^2(x))} \qquad \lim_{x\to 0^+} \frac{2x\cos(x) - \sin(2x + 5x^3)}{x^3}$$

$$\lim_{x\to 0^+} \frac{e^x - e^{-x} - 2x}{x - \sin(x)} \qquad \lim_{x\to 0^+} \frac{\sin(x - \sin(x))}{\sqrt{x^3+1} - 1}$$

$$\lim_{x\to 0^+} \frac{1 - \sqrt{1+x^2}\cos(x)}{\tan^4(x)} \qquad \lim_{x\to 0^+} \frac{\cos(x) - \exp(-x^2/2)}{\log(1-3x^2)\log(1+2x^2)}$$

$$\lim_{x\to 0^+} \frac{x^x - x^{\sin(x)}}{x(\log(1+x) - x)} \qquad \lim_{x\to 0^+} \frac{\log(1+x^2) - \sin^2(x)}{(1-\cos(x)+x^4)^2}$$

$$\lim_{x\to 0^+} \frac{\exp(\sin(x)) - (1+x)^7}{1 - \cos(x + x^3)} \qquad \lim_{x\to +\infty} \frac{(1+x)^{11} - (1+x+x^2)^{11}}{\exp(x^9) - 1}$$

$$\lim_{x\to 0^+} \frac{x^2 - \sin^2(x)}{x^3(e^x - \cos(x))} \qquad \lim_{x\to 0^+} \frac{(\sin^2(x) - \log(\cos(x)))\log(1+\sin(x))}{\sin(x)(1-\cos(3x))}$$

$$\lim_{x\to +\infty} 2x(x-4) - x^3\log\left(1 + \sin(2/x)\right)$$

$$\lim_{x\to 0^+} \frac{(1+x+x^2/2)^{1/x} - e}{x} \qquad \lim_{x\to 0^+} (e^x + x)^{1/x} \qquad \lim_{x\to 0^+} (1+\sin(x))^{1/x}$$