Review for the book

Dmitry Panchenko: The Sherrington-Kirkpatrick Model

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The study of the spin-glass problem has been one of the most intriguing research topics in the statistical physics of the last four decades. The spin-glass models, originally motivated by the peculiar thermodynamic behaviour of some magnetic alloys, have become a paradigmatic framework for all disordered systems with random interactions. The methods, techniques and ideas developed within their investigation have proved to be fruitful in fields as diverse as theoretical computer science, biology and also economy and social sciences.

The versatility of the spin-glass theory is strongly based on the elucidation of the mean-field version developed by Giorgio Parisi, within the study of the Sherrington-Kirkpatrick model. The exact (albeit non-rigorous) replica-symmetry-breaking solution provides a set of explicit computational tools of impressive efficacy, together with a picture of the low temperature equilibrium state and all the novel features it comes with.

The book by Panchenko is the first self-contained treatise of the Parisi theory derived in full mathematical rigour. In the last decade, the author has given several important contributions to the development of the field, which finally led him to the much awaited proof of the ultrametricity property.

In this context ultrametricity (a special property of some metric space) appears as a factorisation rule for the overlaps distribution, stating that only isosceles and equilateral triangles are allowed. Three spin configurations whose mutual overlaps are all unequal (scalene triangle), have instead zero probability in the equilibrium state at infinite volume. Physicists, but also mathematicians, have devoted a large effort to understand the origins of such property whose physical significance is still under investigation.

After presenting the basic concepts of the theory, such as free energy and Gibbs measure, together with the necessary technical tools on spin representations, the author introduces the concept of 'Ruelle probability cascades' with his characterization by an invariance property of the equilibrium state. Ultrametricity is shown to follow by contradiction. This because its violation would in turn violate the identified invariance, which is instead proved for a wide class of mean field models. With the factorisation result at hands, the author can obtain the explicit expression of the free energy, shortcutting in a substantial way the Talagrand first derivation.

The logical-deductive style adopted in the presentation, a necessary feature of the rigorous approach, is supported throughout the whole book with a list of well explained references. The ideas and contributions that have accompanied the growth and development of the field in the last two decades are clearly outlined to help the reader build a correct historical perspective.

This book is an ideal reference for the mathematically oriented scholars, especially graduate students and postdocs wanting to enter the field with a rigorous and concise presentation of the mean-field theory. It is also useful to all those scientists, in particular theoretical physicists, who want to understand the roots of disordered systems within a robust probabilistic approach.

Pierluigi Contucci

Department of Mathematics, University of Bologna