Correzione del 22/10

Es 1

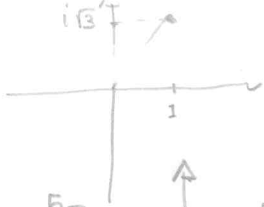
$$\frac{3-i}{5+i}-\frac{6+2i}{5-3i}=\frac{(3-i)(5-i)}{(5+i)(5-i)}-\frac{(6+2i)(5+3i)}{(5-3i)(5+3i)}=\frac{15-1-8i}{25+1}-\frac{30-6+28i}{25+9}$$

$$=\frac{14-8i}{26}-\frac{24+28i}{34}=\frac{17(14-8i)-13(24+28i)}{2\cdot 13\cdot 17}=\frac{238-136i-312-364i}{442}=\frac{-74-500i}{442}$$

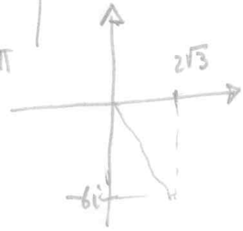
$$=-\frac{74}{442}-\frac{500}{442}i=-\frac{37}{221}-\frac{250}{221}i$$

Es 2

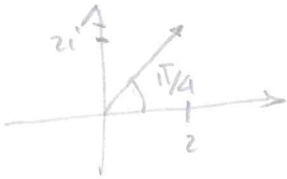
$$|1+\sqrt{3}\,i|=\sqrt{1+3}=\sqrt{4}=2\ ,\quad Ang(1+\sqrt{3}i)=\operatorname{arctg}\sqrt{3}=\frac{\pi}{3}$$



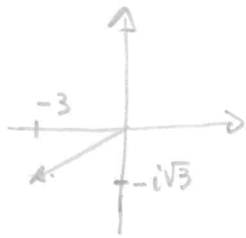
$$|2\sqrt{3}-6i|=\sqrt{(2\sqrt{3})^2+36}=\sqrt{12+36}=\sqrt{48}=4\sqrt{3}\ ;\quad Ang(2\sqrt{3}-6i)=-\operatorname{arctg}\frac{6}{2\sqrt{3}}+2\pi=-\frac{\pi}{3}+2\pi=\frac{5}{3}\pi$$



$$|2+2i|=\sqrt{2^2+2^2}=\sqrt{8}=2\sqrt{2}\ ;\quad Ang(2+2i)=\arctan 1=\frac{\pi}{4}$$

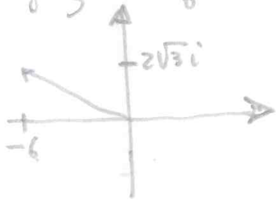


$$|-3-i\sqrt{3}|=\sqrt{9+3}=\sqrt{12}=2\sqrt{3}\ ;\quad Ang(-3-i\sqrt{3})=\operatorname{arctg}\frac{\sqrt{3}}{3}+\pi=\frac{\pi}{6}+\pi=\frac{7}{6}\pi$$



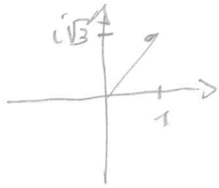
$$|-6+2i\sqrt{3}|=\sqrt{36+12}=\sqrt{48}=4\sqrt{3}\ ;\quad Ang(-6+2i\sqrt{3})=\operatorname{arctg}\left(-\frac{2\sqrt{3}}{6}\right)+\pi=-\operatorname{arctg}\frac{\sqrt{3}}{3}+\pi=-\frac{\pi}{6}+\pi$$

$$=\frac{5}{6}\pi$$



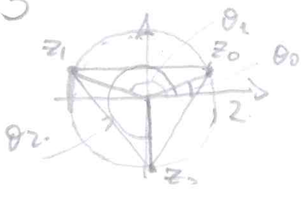
Es 3

$$z^3=1+\sqrt{3}\,i \iff z^3=|1+\sqrt{3}i|\,e^{i\,Ang(1+\sqrt{3}i)}=2\,e^{i\frac{\pi}{3}}$$

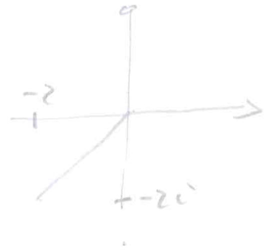


quindi $z_k=2^{1/3}e^{i\theta_k}$, $\quad \theta_k=\dfrac{\frac{\pi}{3}+2k\pi}{3}$ $\quad k=0,1,2$

$\theta_0=\frac{\pi}{9}$, $\theta_1=\frac{7}{9}\pi$, $\theta_2=\frac{13}{9}\pi$

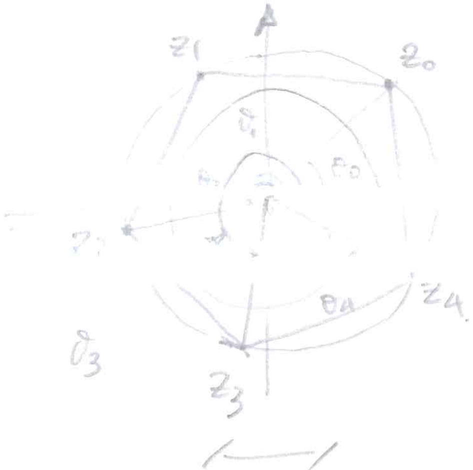


$z^5 = -2-2i = \sqrt{8}\, e^{\frac{5}{4}\pi i}$



$z_k = \left(\sqrt{8}\right)^{1/5} e^{i\theta_k} = 8^{\frac{1}{10}} e^{i\theta_k}$; $\theta_k = \dfrac{\frac{5}{4}\pi + 2k\pi}{5}$, $k=0,1,2,3,4$

$\theta_0 = \dfrac{\pi}{4}$; $\theta_1 = \dfrac{13}{20}\pi$; $\theta_2 = \dfrac{21}{20}\pi$; $\theta_3 = \dfrac{29}{20}\pi$, $\theta_4 = \dfrac{37}{20}\pi$

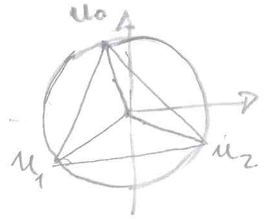


$(z+i)^3 = 1-i$ posto $u = z+i$ risolviamo $u^3 = 1-i$

$u^3 = \sqrt{2}\, e^{\frac{7}{4}\pi i}$

$u_k = \left(\sqrt{2}\right)^{1/3} e^{i\theta_k} = 2^{1/6} e^{i\theta_k}$, $\theta_k = \dfrac{\frac{7}{4}\pi + 2k\pi}{3}$, $k=0,1,2$

$\theta_0 = \dfrac{7}{12}\pi$; $\theta_1 = \dfrac{15}{12}\pi$; $\theta_2 = \dfrac{23}{12}\pi$



D'altra parte $z_k + i = u_k$ $k=0,1,2$, quindi

$z_0 = u_0 - i$; $z_1 = u_1 - i$; $z_2 = u_2 - i$