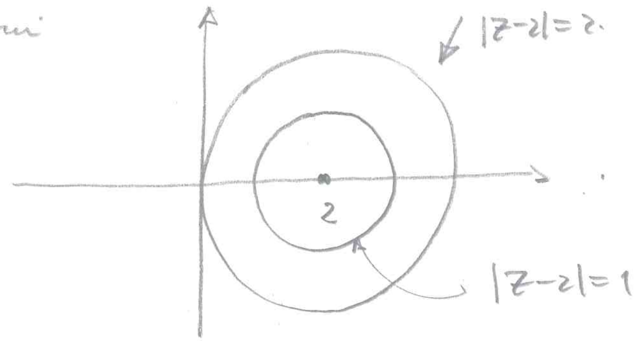
Correzione del 27/10/2014

Es. 1

$|z-2|^2+3|z-2|+2=0$; posto $|z-2|=\rho\geq 0$, otteniamo $\rho^2+3\rho+2=0 \iff (\rho+2)(\rho+1)=0 \iff \rho=-2 \vee \rho=-1$. Quindi $|z-2|=-2 \vee |z-2|=-1$. Le due equazioni ottenute non hanno soluzioni perché $|z-2|\geq 0$. Quindi l'equazione data non ha soluzioni;

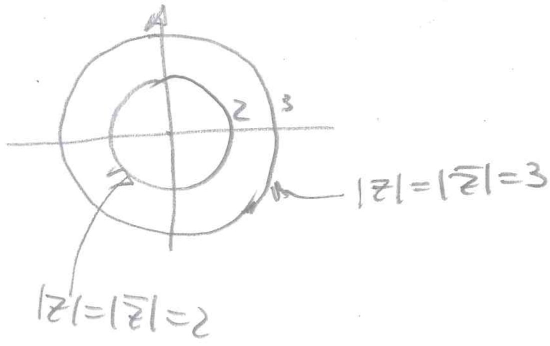
$|z-2|^2-3|z-2|+2=0$; posto $|z-2|=\rho$, otteniamo $\rho^2-3\rho+2=0 \iff (\rho-2)(\rho-1)=0 \iff \rho=2 \vee \rho=1$. Quindi $|z-2|=2 \vee |z-2|=1$ sono le soluzioni



$|\bar{z}|^2-5|\bar{z}|+6=0$; $|\bar{z}|=\rho$, $\rho^2-5\rho+6=0$, $(\rho-3)(\rho-2)=0$

$\rho=3 \vee \rho=2 \to |\bar{z}|=3 \vee |\bar{z}|=2$, ma $|\bar{z}|=|z|$, quindi

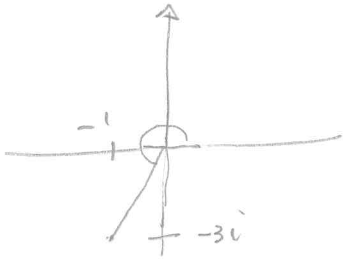
$|\bar{z}|=3 \vee |\bar{z}|=2$



$$\left(z^3+1+3i\right)\left(z^2+(2+i)z+2i\right)=0 \iff z^3=-1-3i \ \vee\ z^2+(2+i)z+2i=0$$

$z^3=-1-3i$, $|-1-3i|=\sqrt{10}$, $\mathrm{Arg}(-1-3i)=\arctan(3)+\pi$

$$z_k=\left(\sqrt{10}\right)^{1/3}e^{i\theta_k}$$



$$\theta_k=\frac{\arctan(3)+\pi+2k\pi}{3}$$

$k=0,1,2$.

Inoltre $z^2+(2+i)z+2i=0 \iff (z+2)(z+i) \iff z=-2 \vee z=-i$

Le soluzioni di $\left(z^3+1+3i\right)\left(z^2+(2+i)z+2i\right)=0$ sono le seguenti:

$$\{z_k \ ; \ k=0,1,2\} \cup \{-2,-i\}.$$

$$\left(z^5+1-2i\right)\left(z^2+3iz-2\right)=0 \iff z^5+1-2i=0 \ \vee\ z^2+3iz-2=0 \iff$$

$$z^5=-1+2i \ \vee\ z^2+3iz-2=0.$$

Iniziamo risolvendo $z^5=-1+2i$. Poiché $|-1+2i|=\sqrt{5}$ e $\mathrm{Arg}(-1+2i)=\arctan(-2)+\pi=-\arctan(2)+\pi$, allora

Poiché è dispari

$-1+2i=\sqrt{5}\,e^{i(\pi-\arctan 2)}$. Quindi $z_k=\left(\sqrt{5}\right)^{1/5}e^{i\theta_k}$, con

$$\theta_k=\frac{\pi-\arctan(2)+2k\pi}{5},\qquad k=0,1,2,3,4.$$

Risolviamo ora $z^2+3iz-2=0 \iff (z+2i)(z+i)=0 \iff$ $z=-2i \vee z=-i$. In alternativa se non ci si accorge della fattorizzazione $\Delta=-9+8=-1$ $h^2=-1 \iff h=\pm i$.

$$z_{1,2}=\frac{-3i\pm i}{2}=\left\langle\begin{array}{l}-\frac{4i}{2}=-2i\\ -\frac{2i}{2}=-i\end{array}\right.$$

. Le soluzioni di

$(z^5+1-2i)(z^2+3iz-2)=0$ sono allora $\{z_k : k=0,1,2,3,4\} \cup \{-2i, -i\}$.

Es. 2.

$f_1(x) = \sin x \qquad f_1 : \mathbb{R} \to \mathbb{R} \qquad df_1(0)(h) = f_1'(0)\, h$ cioè

$df_1(0)(h) = \cos(0) \cdot h$, ovvero $df_1(0) : \mathbb{R} \to \mathbb{R}$, $df_1(0)(h) = h$.

$f_2 : \mathbb{R} \to \mathbb{R}$, $f_2(x) = e^x$, $df_2(0)(h) = f_2'(0)\, h$, cioè $df_2(0) : \mathbb{R} \to \mathbb{R}$

$df_2(0)(h) = h$, perché $f_2'(0) = e^0 = 1$.

$f_3 : \mathbb{R} \to \mathbb{R}$, $f_3(x) = (1+x)^4$, $df_3(0)(h) = 4(1+x)^3_{|x=0} \cdot h$

cioè $df_3(0)(h) = 4 \cdot h$ con $df_3(0) : \mathbb{R} \to \mathbb{R}$.