Esercitazione del 03/10/2014 (bis)

Calcolare i seguenti limiti

$$\lim_{n\to+\infty} \frac{(\sqrt[3]{n}-\sqrt[3]{n+2})\,n}{\sqrt[3]{4n-5}+\sqrt[3]{n+2}} \quad ; \qquad \lim_{n\to+\infty} \frac{\sqrt{n^3}+5n^{3/2}+\sqrt[5]{n-1}}{\sqrt[5]{n+3}+\sqrt[6]{n^9+1}} \quad ;$$

$$\lim_{n\to+\infty} (\sqrt{n+1}-\sqrt{n-7})\sqrt{n+4} \quad ; \qquad \lim_{n\to+\infty} \frac{(n+3)!+n^5}{n!\sqrt{n^6+3}+n^6} \quad ;$$

$$\lim_{n\to+\infty} \frac{\sqrt{n+1}-\sqrt{n-7}}{\sqrt{3n+3}-\sqrt{3n+4}} \quad ; \qquad \lim_{n\to+\infty} \frac{\log(n^{10}+5)+\sqrt{3n+1}}{\sqrt[3]{n+1}-\sqrt{n}+\log(5n^{10}+6)}$$

$$\lim_{n\to+\infty} \left(1-\frac{1}{n}\right)^{-n} \quad ; \qquad \lim_{n\to+\infty} \frac{2^n+e^{\frac{n}{2}}}{4^{\frac{n}{2}}+e^{\frac{n+1}{2}}}$$

$$\lim_{n\to+\infty} \left(1+\frac{q}{n}\right)^{n} \quad , \ q\in\mathbb{R} \quad ; \qquad \lim_{n\to+\infty} \frac{5^{\frac{n}{2}}+3e^n}{4e^n+\sqrt{5^n}} \quad ;$$

$$\lim_{n\to+\infty} \frac{\log n}{n^\alpha} \quad , \ \alpha>0 \quad ; \qquad \lim_{n\to+\infty} \frac{5^n+n!}{25^{\frac{n}{2}}+3n!}$$

$$\lim_{n\to+\infty} \frac{n!}{n^n} \quad ; \qquad \lim_{n\to+\infty} \frac{\sin n}{n} .$$

Provare che la seguente affermazione è vera:

$\exists\ \bar{n}\in\mathbb{N}$ tale che per ogni $n>\bar{n}$: $n^3+n^5+1>n^2$.

(Fornire le opportune motivazioni)