Calcolare

①

$$\int_{\sqrt{5\pi}-4}^{2\sqrt{\pi}-4} (t+4)^3 \sin\left((t+4)^2\right) dt$$

$(t+4)^2 = s \qquad ds = 2(t+4)dt$

$$= \frac{1}{2}\int_{5\pi}^{4\pi} s \sin s \, ds = \frac{1}{2}\Big[-s\cos s\Big]_{s=5\pi}^{s=4\pi} + \frac{1}{2}\int_{5\pi}^{4\pi} \cos s \, ds$$

$$= \frac{1}{2}\Big[-s\cos s + \sin s\Big]_{s=5\pi}^{s=4\pi} = \frac{1}{2}\left(-4\pi\cos 4\pi + \sin 4\pi + 5\pi\cos 5\pi - \sin 5\pi\right)$$

$$= \frac{1}{2}\left(-4\pi - 5\pi\right) = -\frac{9}{2}\pi$$

Determinare l'integrale generale dell'eq. diff.

$$y'' + 12y' + 36y = e^{7t} + 6t^2$$

L'eq. omogenea associata è $y'' + 12y' + 36y = 0$.
L'eq. caratt. è: $\lambda^2 + 12\lambda + 36 = 0$. Gli autovalori sono $\lambda_1 = -6 = \lambda_2 = -6$
Quindi $V_2 = \text{span}\{e^{-6t}, te^{-6t}\}$.

Cerco una sol di $y'' + 12y' + 36y = e^{7t}$ nella forma
$Y_1 = Ke^{7t}$, $Y_1' = 7Ke^{7t}$, $Y_1'' = 49Ke^{7t}$. Pertanto
$49Ke^{7t} + 84Ke^{7t} + 36Ke^{7t} = e^{7t}$, da cui $169Ke^{7t} = e^{7t}$ cioè
$K = \frac{1}{169}$
Analogamente per $y'' + 12y' + 36y = 6t^2$, cerco una sol. nella forma $Y_2 = At^2 + Bt + C$. Quindi $Y_2' = 2At + B$, $Y_2'' = 2A$
da cui $2A + (2At + B)12 + 36(At^2 + Bt + C) = 6t^2$

cioè

$$36At^2 + (36B+24A)t + 36C + 12B + 2A = 6t^2$$

Pertanto

$$\begin{cases} 36A = 6 \\ 36B + 24A = 0 \\ 36C + 12B + 2A = 0 \end{cases} \quad \begin{cases} A = \frac{1}{6} \\ 36B + 4 = 0 \\ 36C + 12B + \frac{1}{3} = 0 \end{cases} \quad \begin{cases} A = \frac{1}{6} \\ B = -\frac{1}{9} \\ 36C = +\frac{4}{3} - \frac{1}{3} \end{cases} \quad \begin{cases} A = \frac{1}{6} \\ B = -\frac{1}{9} \\ C = \frac{1}{36} \end{cases}$$

Finalmente l'integrale generale è:

$$LV_2 = V_2 + \frac{e^{7t}}{169} + \frac{1}{6}t^2 - \frac{1}{9}t + \frac{1}{36}$$

$f: \mathbb{R}\setminus\{-7\} \longrightarrow \mathbb{R}$, $f(x) = e^{6-x}\frac{x}{|x+7|}$

f è derivabile in tutto il suo dominio di def. perché prodotto e composizione di funzioni derivabili. Inoltre per ogni $x \in \mathbb{R}\setminus\{-7\}$

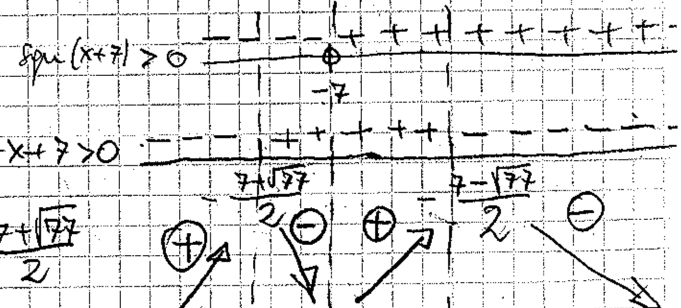
$$f'(x) = -e^{6-x}\frac{x}{|x+7|} + \frac{|x+7| - x\,\mathrm{sgn}(x+7)}{|x+7|^2}e^{6-x} = e^{6-x}\left(\frac{-|x+7|x + |x+7| - x\,\mathrm{sgn}(x+7)}{|x+7|^2}\right)$$

Pertanto

$$\begin{cases} f' > 0 \\ x \in \mathbb{R}\setminus\{7\} \end{cases} \Longleftrightarrow \begin{cases} -|x+7|x + |x+7| - x\,\mathrm{sgn}(x+7) > 0 \\ x \in \mathbb{R}\setminus\{7\} \end{cases} \Longleftrightarrow$$

$$\begin{cases} \mathrm{sgn}(x+7)\left(-(x+7)x + (x+7) - x\right) > 0 \\ x \in \mathbb{R}\setminus\{7\} \end{cases} \Longleftrightarrow \begin{cases} 0 < \mathrm{sgn}(x+7)\left(-x^2 - 7x + x + 7 - x\right) \\ x \in \mathbb{R}\setminus\{7\} \end{cases}$$

$$\Longleftrightarrow \begin{cases} \mathrm{sgn}(x+7)\left(-x^2 - 7x + 7\right) > 0 \\ x \in \mathbb{R}\setminus\{7\} \end{cases}$$



$$\frac{7 \pm \sqrt{49+28}}{-2} \quad \begin{matrix} -\frac{7+\sqrt{77}}{2} \\ -\frac{7-\sqrt{77}}{2} \end{matrix}$$

Quindi f è monotona strettamente $\nearrow$ in $\left(-\infty, -\frac{7+\sqrt{77}}{2}\right]$

f è " " $\nearrow$ in $\left(-7, -\frac{7-\sqrt{77}}{2}\right]$

Mentre f è monotona strett. $\searrow$ in $\left[-\frac{7+\sqrt{77}}{2}, -7\right)$

e f " " " $\searrow$ in $\left[-\frac{7-\sqrt{77}}{2}, +\infty\right)$

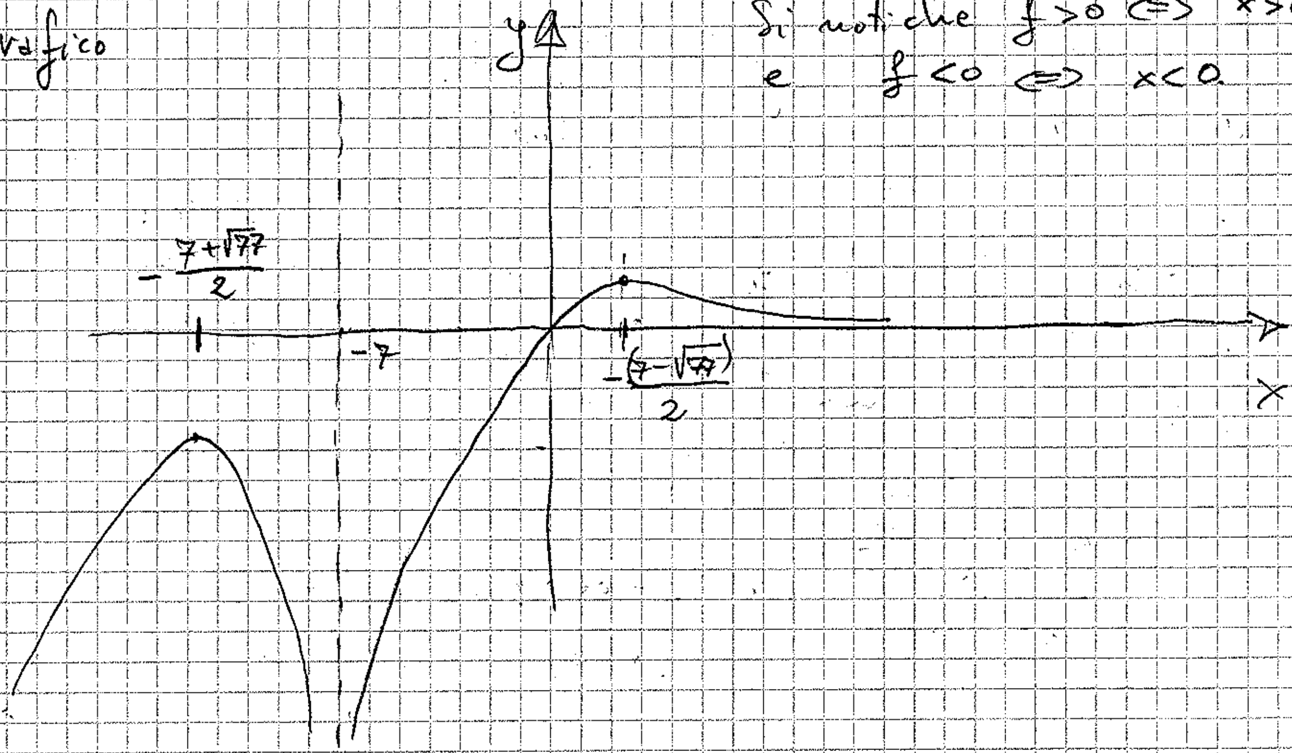
in $x = -\frac{7+\sqrt{77}}{2}$ si realizza un p.to di max rel; in $-\frac{7-\sqrt{77}}{2}$ si realizza un p.to di max ass.

$\lim_{x \to -\infty} f(x) = -\infty$, $\lim_{x \to +\infty} f(x) = 0$

$\lim_{x \to -7} f(x) = -\infty$

Grafico

Si noti che $f > 0 \iff x > 0$
e $f < 0 \iff x < 0$



Quindi esiste un massimo locale e un massimo assoluto
($x = -\frac{7+\sqrt{77}}{2}$ p.to max locale e $x = -\frac{(7-\sqrt{77})}{2}$ p.to di max ass.)

$f(\mathbb{R} \setminus \{7\}) = \left(-\infty, f\left(-\frac{7-\sqrt{77}}{2}\right)\right]$

Si noti che f non ha minimo.

$$\# \quad \int_0^{+\infty} \frac{\operatorname{senh}\left(\frac{\alpha x}{x^2+25}\right)}{x^{8\alpha}+5x^2}\,dx \qquad \alpha>0 \qquad 4$$

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$$\int_0^1 \frac{\operatorname{senh}\left(\frac{\alpha x}{x^2+25}\right)}{x^{8\alpha}+5x^2}\,dx \quad , \quad \alpha>0.$$

$$\frac{\operatorname{senh}\left(\frac{\alpha x}{x^2+25}\right)}{x^{8\alpha}+5x^2} \underset{x\to 0}{\sim} \frac{\frac{\alpha x}{x^2+25}}{x^{8\alpha}+5x^2} \sim \begin{cases} \frac{\alpha x}{25}\cdot\frac{1}{5x^2} \sim \frac{\alpha}{125x} & \text{se } 8\alpha\geq 2 \\ \frac{\alpha x}{25x^{8\alpha}} \sim \frac{\alpha}{25x^{8\alpha-1}} & \text{se } 8\alpha<2 \end{cases}$$

pertanto, se $8\alpha\geq 2$ l'integrale diverge e se $8\alpha<2$ converge quando $8\alpha-1<1$ cioè se è soddisfatto $\begin{cases} 8\alpha<2 \\ 8\alpha-1<1 \end{cases} \Longleftrightarrow$

$$\begin{cases} \alpha<\frac{1}{4} \\ 8\alpha<2 \end{cases} \qquad \begin{cases} \alpha<\frac{1}{4}. \\ \alpha<\frac{1}{4} \end{cases}$$

Quindi $\displaystyle\int_0^1 \frac{\operatorname{senh}\left(\frac{\alpha x}{x^2+25}\right)}{x^{8\alpha}+5x^2}\,dx < +\infty \Longleftrightarrow 0<\alpha<\frac{1}{4}$

$(\alpha>0)$

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$$\int_1^{+\infty} \frac{\operatorname{senh}\left(\frac{\alpha x}{x^2+25}\right)}{x^{8\alpha}+5x^2}\,dx$$

$$\frac{\operatorname{senh}\left(\frac{\alpha x}{x^2+25}\right)}{x^{8\alpha}+5x^2} \underset{x\to+\infty}{\sim} \frac{\frac{\alpha x}{x^2+25}}{2(x^{8\alpha}+5x^2)} \sim \frac{\alpha}{2x(x^{8\alpha}+5x^2)}$$

$$\sim \begin{cases} \frac{\alpha}{2x^{8\alpha+1}} & \text{se } 8\alpha>2 \\ \frac{\alpha}{10x^3} & \text{se } 8\alpha\leq 2 \end{cases}$$

Se $8\alpha > 2$ l'integrale convergerà se $8\alpha + 1 > 1$, cioè se e solo se $\begin{cases} 8\alpha > 2 \\ 8\alpha + 1 > 1 \end{cases} \iff \begin{cases} \alpha > \frac{1}{4} \\ \alpha > 0 \end{cases} \iff \alpha > \frac{1}{4}$

Se $8\alpha \leq 2$ l'int. converge per ogni $\alpha > 0$ perché $\int_1^{+\infty} \frac{\alpha}{10x^3} dx$ è convergente

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Pertanto $\int_0^{+\infty} \frac{\sinh\left(\frac{\alpha x}{x^2+25}\right)}{x^{8\alpha} + 5x^2} dx$ converge se e solo se $0 < \alpha < \frac{1}{4}$.

\# $f: [0, +\infty) \longrightarrow \mathbb{R}$, $f(x) = (x+2)^{\frac{\sin x}{7+\cos x}}$

$$f(x) = e^{\frac{\sin x}{7+\cos x} \log(x+2)}$$

$$f'(x) = \left\{ \frac{[\cos x(7+\cos x) + \sin^2 x]}{(7+\cos x)^2} \log(x+2) + \frac{\sin x}{(7+\cos x)(x+2)} \right\} (x+2)^{\frac{\sin x}{7+\cos x}}$$

$$f'\left(\frac{7}{2}\pi\right) = \left(\frac{1}{49} \log\left(\frac{7}{2}\pi + 2\right) - \frac{1}{7\left(\frac{7}{2}\pi + 2\right)} \right) \left(\frac{7}{2}\pi + 2\right)^{-\frac{1}{7}}$$

\# $\lim_{x \to 0} \frac{\left(\sinh(2x + 2x^2) - e^{2x} + 1\right) \sin(x+2)}{\cosh^2(49x^2) - \cos(49x^2)}$

$$N \underset{x\to 0}{\sim} \left(\cancel{2x} + \cancel{2x^2} + \cancel{\frac{(2x)^3}{3!}} + 4x^4 - \cancel{1} - \cancel{2x} - \cancel{\frac{4x^2}{2}} - \cancel{\frac{8x^3}{3!}} - \frac{16}{4!}x^4 + o(x^4)\right) \operatorname{sen} 2 .$$

$$D \sim \left(\cosh 49x^2 - \cos 49x^2\right)\left(\cosh 49x^2 + \cos 49x^2\right) \underset{x\to 0}{\sim}$$ (6)

$$\sim \left(\cancel{1} + \frac{(49x^2)^2}{2} - \cancel{1} + \frac{(49x^2)^2}{2} + o(x^3)\right) \cdot 2 .$$

Pertanto $\lim_{x\to 0} \frac{N}{D} = \lim_{x\to 0} \frac{\frac{10}{3}x^4 \operatorname{sen} 2}{2 \cdot 49^2 x^4} = \frac{5}{3} \cdot \frac{1}{49^2} \sin(2).$

$$\lim_{n\to+\infty} \frac{n^4 + 19 \cdot 4^n + 4 \cdot 19^{\frac{n}{2}}}{\left(n^{19} + 4^{n+1} + 19^{\frac{n}{2}+1}\right) \operatorname{sen} \frac{n}{n+4}}$$

$$= \lim_{n\to+\infty} \frac{4 \cdot 19^{\frac{n}{2}}}{19^{\frac{n}{2}+1} \operatorname{sen} \frac{n}{n+4}} = \frac{4}{19 \operatorname{sen}(1)}$$

Calcolare

$$\int_0^1 \frac{\operatorname{senh}(3x)}{4\operatorname{senh}^2(3x) + \cosh(3x) + 4}\, dx \quad ; \quad \operatorname{senh}^2 t - \cosh^2 t = -1$$

pos. $\cosh(3x) = t$; $dt = 3\operatorname{senh}(3x)dx$

$$= \frac{1}{3}\int_1^{\cosh(3)} \frac{dt}{4(t^2-1) + t + 4}\, dt = \frac{1}{3}\int_1^{\cosh(3)} \frac{dt}{4t^2+t}\, dt$$

$$= \frac{1}{3}\int_1^{\cosh(3)} \frac{1}{t(4t+1)}\, dt$$

$$= \frac{1}{3}\int_1^{\cosh(3)} \frac{1}{t}\,dt - \frac{4}{3}\int_1^{\cosh(3)} \frac{1}{4t+1}\,dt$$

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Perché:

$$\frac{A}{t} + \frac{B}{4t+1} = \frac{1}{t(4t+1)}$$

$$\frac{4At + A + Bt}{t(4t+1)} = \frac{1}{t(4t+1)}$$

$$\frac{(4A+B)t + A}{t(4t+1)} = \frac{1}{t(4t+1)} \qquad \begin{cases} 4A+B=0 \\ A=1 \end{cases} \Longleftrightarrow \begin{cases} B=-4 \\ A=1 \end{cases}$$

$$= \frac{1}{3}\Big[\log|t| - \log|4t+1|\Big]_{t=1}^{t=\cosh(3)}$$

$$= \frac{1}{3}\log(\cosh(3)) - \frac{1}{3}\log(4\cosh(3)+1) + \frac{1}{3}\log 5$$

Risolvere l'equazione in $\mathbb{C}$

$$\left((z+4)^5 + 4 - 5i\right)\left(z^2 + (4-5i)z - 20i\right) = 0$$

$$(z+4)^5 + 4 - 5i = 0 \quad \vee \quad z^2 + (4-5i)z - 20i = 0$$

$(z+4)^5 = -4+5i$ allora

$$z_k + 4 = (41)^{\frac{1}{10}} e^{i\theta_k} \quad \text{con } \theta_k = \frac{-\arctan\left(\frac{5}{4}\right) + \pi + 2k\pi}{5}, \quad k=0,1,2,3,4$$

mentre $z^2 + (4-5i)z - 20i = 0$ ha come sol $z=-4$ e $z=5i$

perché $(z+4)(z-5i) = z^2 + (4-5i)z - 20i$.