1

$$y''+y'+2y=e^{\frac{1}{2}x}\sin\left(\frac{\sqrt{7}}{2}x\right)+3$$

$$y''+y'+2y=0 \longrightarrow \lambda^2+\lambda+2=0 \longrightarrow \lambda_{1,2}=\frac{-1\pm\sqrt{1-8}}{2}=\frac{-1\pm i\sqrt{7}}{2}=\begin{cases}\frac{-1+\sqrt{7}\,i}{2}\\ \frac{-1-i\sqrt{7}}{2}\end{cases}$$

L'integrale generale di $y''+y'+2y=0$ è $V_2=\mathrm{span}\left\{e^{-\frac{1}{2}x}\cos\left(\frac{\sqrt{7}}{2}x\right), e^{-\frac{1}{2}x}\sin\left(\frac{\sqrt{7}}{2}x\right)\right\}$.

Cerchiamo una soluzione di $y''+y'+2y=\varphi$ con $\varphi=e^{\frac{1}{2}x}\sin\left(\frac{\sqrt{7}}{2}x\right)$

nella forma $\psi=e^{\frac{1}{2}x}\left(A\sin\left(\frac{\sqrt{7}}{2}x\right)+B\cos\left(\frac{\sqrt{7}}{2}x\right)\right)$

$$\psi'=\frac{1}{2}e^{\frac{1}{2}x}\left(A\sin\left(\frac{\sqrt{7}}{2}x\right)+B\cos\left(\frac{\sqrt{7}}{2}x\right)\right)+e^{\frac{1}{2}x}\left(\frac{\sqrt{7}}{2}A\cos\left(\frac{\sqrt{7}}{2}x\right)-\frac{\sqrt{7}}{2}B\sin\left(\frac{\sqrt{7}}{2}x\right)\right).$$

$$\psi''=\frac{1}{4}e^{\frac{1}{2}x}\left(A\sin\left(\frac{\sqrt{7}}{2}x\right)+B\cos\left(\frac{\sqrt{7}}{2}x\right)\right)+\frac{1}{2}e^{\frac{1}{2}x}\left(\frac{\sqrt{7}}{2}A\cos\left(\frac{\sqrt{7}}{2}x\right)-\frac{\sqrt{7}}{2}B\sin\left(\frac{\sqrt{7}}{2}x\right)\right)+\frac{1}{2}e^{\frac{1}{2}x}\left(\frac{\sqrt{7}}{2}A\cos\left(\frac{\sqrt{7}}{2}x\right)-\frac{\sqrt{7}}{2}B\sin\left(\frac{\sqrt{7}}{2}x\right)\right)$$
$$+e^{\frac{1}{2}x}\left(-\frac{7}{4}A\sin\left(\frac{\sqrt{7}}{2}x\right)-\frac{7}{4}B\cos\left(\frac{\sqrt{7}}{2}x\right)\right).$$

Sostituendo in $y''+y'+2y=\varphi$ otteniamo

$$\frac{1}{4}e^{\frac{1}{2}x}\left(A\sin\left(\frac{\sqrt{7}}{2}x\right)+B\cos\left(\frac{\sqrt{7}}{2}x\right)\right)+e^{\frac{1}{2}x}\left(\frac{\sqrt{7}}{2}A\cos\left(\frac{\sqrt{7}}{2}x\right)-\frac{\sqrt{7}}{2}B\sin\left(\frac{\sqrt{7}}{2}x\right)\right)+e^{\frac{1}{2}x}\left(-\frac{7}{4}A\sin\left(\frac{\sqrt{7}}{2}x\right)-\frac{7}{4}B\cos\left(\frac{\sqrt{7}}{2}x\right)\right)$$
$$+\frac{1}{2}e^{\frac{1}{2}x}\left(A\sin\left(\frac{\sqrt{7}}{2}x\right)+B\cos\left(\frac{\sqrt{7}}{2}x\right)\right)+e^{\frac{1}{2}x}\left(\frac{\sqrt{7}}{2}A\cos\left(\frac{\sqrt{7}}{2}x\right)-\frac{\sqrt{7}}{2}B\sin\left(\frac{\sqrt{7}}{2}x\right)\right)$$
$$+2e^{\frac{1}{2}x}\left(A\sin\left(\frac{\sqrt{7}}{2}x\right)+B\cos\left(\frac{\sqrt{7}}{2}x\right)\right)=e^{\frac{1}{2}x}\sin\left(\frac{\sqrt{7}}{2}x\right)$$

$$\begin{cases}\frac{1}{4}A-\frac{\sqrt{7}}{2}B-\frac{7}{4}A+\frac{1}{2}A-\frac{\sqrt{7}}{2}B+2A=1\\ \frac{1}{4}B+\frac{\sqrt{7}}{2}A-\frac{7}{4}B+\frac{1}{2}B+\frac{\sqrt{7}}{2}A+2B=0\end{cases}\qquad \begin{cases}A-\sqrt{7}B=1\\ \sqrt{7}A+B=0\end{cases}\qquad A=\frac{\det\begin{bmatrix}1&-7\\0&1\end{bmatrix}}{\det\begin{bmatrix}1&-\sqrt{7}\\ \sqrt{7}&1\end{bmatrix}}=\frac{1}{8}$$

$$B=\frac{\det\begin{bmatrix}1&1\\ \sqrt{7}&0\end{bmatrix}}{\det\begin{bmatrix}1&-\sqrt{7}\\ \sqrt{7}&1\end{bmatrix}}=-\frac{\sqrt{7}}{8},\quad \text{Quindi } \psi=e^{\frac{1}{2}x}\left(\frac{1}{8}\sin\left(\frac{\sqrt{7}}{2}x\right)-\frac{\sqrt{7}}{8}\cos\left(\frac{\sqrt{7}}{2}x\right)\right)$$

Cerchiamo ora una soluzione di $y''+y'+2y=3$ nella forma $\eta=\text{costante}=(k)$

$\eta'=0$ $\eta''=0$ e sostituendo: $2k=3$, da cui $k=\frac{3}{2}$ e $\eta=\frac{3}{2}$

L'integrale generale di $y''+y'+2y=e^{\frac{1}{2}x}\sin\left(\frac{\sqrt{7}}{2}x\right)+3$ è:

$$LV_2=\operatorname{span}\left\{e^{-\frac{1}{2}x}\sin\left(\frac{\sqrt{7}}{2}x\right),\ e^{-\frac{1}{2}x}\cos\left(\frac{\sqrt{7}}{2}x\right)\right\}+\psi+\eta.$$

\# 2 Calcolare in $\mathbb{C}$ $(z^6+2+3i)\left(z^2+(2+i\sqrt{2}+3i)z+(2i-\sqrt{2})3\right)=0$

$z^6+2+3i=0 \iff z^6=-2-3i$. Allora $z_k=(13)^{\frac{1}{12}}e^{i\theta_k}$, $k=0,1,2,3,4,5$,

con $\theta_k=\dfrac{\arctan\left(\frac{3}{2}\right)+\pi+2k\pi}{6}$.

Mentre

$z^2+(2+i\sqrt{2}+3i)z+(2i-\sqrt{2})3=0 \iff (z+2+i\sqrt{2})(z+3i)=0 \iff$

$z=-2-i\sqrt{2}$, $z=-3i$.

\# 3 $\displaystyle\int_1^{+\infty}\frac{\left(\sin\left(\frac{t}{2}\right)+2\right)t^{-\frac{1}{2}}}{(t^2-1)^{\frac{\alpha}{2}}\left(\frac{1}{2}+t\right)^{\frac{\alpha}{2}}}dt$; $\alpha>0$; $\displaystyle\frac{\left(\sin\left(\frac{t}{2}\right)+2\right)t^{-\frac{1}{2}}}{(t-1)^{\frac{\alpha}{2}}(t+1)^{\frac{\alpha}{2}}\left(\frac{1}{2}+t\right)^{\frac{\alpha}{2}}}=\frac{\left(\sin\left(\frac{t}{2}\right)+2\right)t^{-\frac{1}{2}}}{(t^2-1)^{\frac{\alpha}{2}}\left(\frac{1}{2}+t\right)^{\frac{\alpha}{2}}}$.

Se $t_0>1$, $t\in(1,t_0]$

$$\frac{c_1(\alpha,t_0)}{(t-1)^{\frac{\alpha}{2}}}\le\frac{t^{-\frac{1}{2}}}{(t-1)^{\frac{\alpha}{2}}(t_0+1)\left(\frac{1}{2}+t_0\right)^{\frac{\alpha}{2}}}\le\frac{\left(\sin\left(\frac{t}{2}\right)+2\right)t^{-\frac{1}{2}}}{(t-1)^{\frac{\alpha}{2}}(t+1)^{\frac{\alpha}{2}}\left(\frac{1}{2}+t\right)^{\frac{\alpha}{2}}}\le\frac{3t^{-\frac{1}{2}}}{2^{\frac{\alpha}{2}}\left(\frac{3}{2}\right)^{\frac{\alpha}{2}}}\frac{1}{(t-1)^{\frac{\alpha}{2}}}\le C_2(\alpha)\frac{1}{(t-1)^{\frac{\alpha}{2}}}$$

Quindi $\displaystyle\int_1^{t_0}\frac{\left(\sin\left(\frac{t}{2}\right)+2\right)t^{-\frac{1}{2}}}{(t^2-1)^{\frac{\alpha}{2}}\left(\frac{1}{2}+t\right)^{\frac{\alpha}{2}}}dt$ converge se e solo se $\frac{\alpha}{2}<1 \iff \alpha<2$

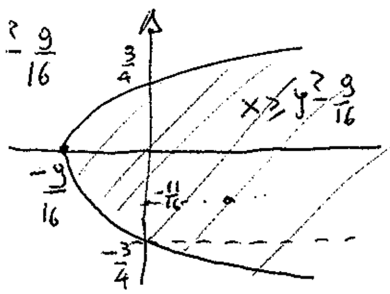
Se $t\ge t_0$, allora $\displaystyle\frac{C_4(\alpha)}{t^{\frac{1}{2}+\frac{3}{2}\alpha}}\le\frac{\left(\sin\left(\frac{t}{2}\right)+2\right)t^{-\frac{1}{2}}}{(t-1)^{\frac{\alpha}{2}}(t+1)^{\frac{\alpha}{2}}\left(\frac{1}{2}+t\right)^{\frac{\alpha}{2}}}\le\frac{C_3(\alpha)}{t^{\frac{1}{2}+\frac{3}{2}\alpha}}$.

Quindi $\displaystyle\int_{t_0}^{+\infty}\frac{\left(\sin\left(\frac{t}{2}\right)+2\right)t^{-\frac{1}{2}}}{(t^2-1)^{\frac{\alpha}{2}}\left(\frac{1}{2}+t\right)^{\frac{\alpha}{2}}}dt$ converge se e solo se $\frac{1}{2}+\frac{3}{2}\alpha>1 \iff \frac{3}{2}\alpha>\frac{1}{2}$

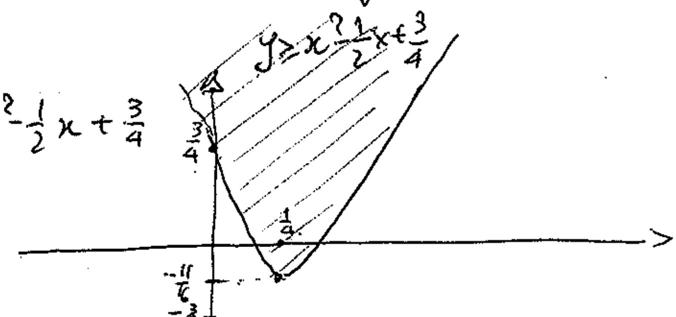
cioè se e solo se $\alpha>\frac{1}{3}$. Pertanto $\displaystyle\int_1^{+\infty}\frac{\left(\sin\left(\frac{t}{2}\right)+2\right)t^{-\frac{1}{2}}}{(t^2-1)^{\frac{\alpha}{2}}\left(\frac{1}{2}+t\right)^{\frac{\alpha}{2}}}dt$ converge

se e solo se $\frac{1}{3}<\alpha<2$.

#4 $\displaystyle\iint_A y\,dx\,dy$, $A=\left\{(x,y)\in\mathbb{R}^2 : x\geq y^2-\frac{9}{16}\ ,\ y\geq x^2-\frac{1}{2}x+\frac{3}{4}\right\}$

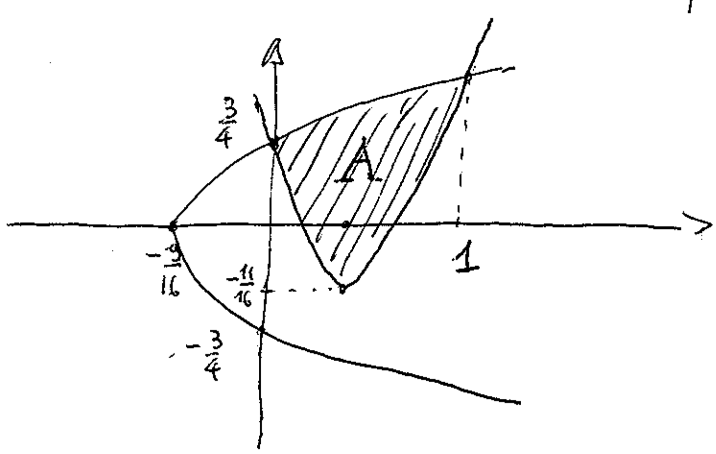


;



$V=\left(-\frac{b}{2a},-\frac{\Delta}{4a}\right)=\left(\frac{1}{4},-\frac{11}{16}\right)$

Quindi



Conviene vedere il dominio come insieme y-semplice. Determiniamo pertanto le intersezioni tra le due parabole. Si osservi come una soluzione sia già nota e sia 0.

$$\begin{cases} x=y^2-\frac{9}{16} \\ y=x^2-\frac{1}{2}x+\frac{3}{4}\end{cases} \iff \begin{cases} x=\left(x^2-\frac{1}{2}x+\frac{3}{4}\right)^2-\frac{9}{16} \\ y=x^2-\frac{1}{2}x+\frac{3}{4}\end{cases} \iff \begin{cases} x=x^4+\underline{\frac{1}{4}x^2}+\cancel{\frac{9}{16}}-x^3+\underline{\frac{3}{2}x^2}-\frac{3}{4}x-\cancel{\frac{9}{16}} \\ / \end{cases} \iff$$

$$\begin{cases} x=x^4-x^3+\frac{7}{4}x^2-\frac{3}{4}x \\ / \end{cases} \iff \begin{cases} 0=x\left(x^3-x^2+\frac{7}{4}x-\frac{7}{4}\right) \\ y=x^2-\frac{1}{2}x+\frac{3}{4}\end{cases} \iff \begin{cases} x=0 \\ y=\frac{3}{4}\end{cases} \vee \begin{cases} x^3-x^2+\frac{7}{4}x-\frac{7}{4}=0 \\ y=x^2-\frac{1}{2}x+\frac{3}{4}\end{cases}$$

$$\iff \begin{cases} x=0 \\ y=\frac{3}{4}\end{cases} \vee \begin{cases} x^2(x-1)+\frac{7}{4}(x-1)=0 \\ / \end{cases}$$

$$\Updownarrow$$

$$\begin{cases} \left(x^2+\frac{7}{4}\right)(x-1)=0 \\ / \end{cases} \longleftrightarrow \begin{cases} x=1 \\ y=\frac{5}{4}\end{cases}$$

$$\iff \begin{cases} x=0 \\ y=\frac{3}{4}\end{cases} \vee \begin{cases} x=1 \\ y=\frac{5}{4}\end{cases}$$

Pertanto

$$\iint_A y\,dx\,dy = \int_0^1 \left(\int_{x^2-\frac{1}{2}x+\frac{3}{4}}^{\sqrt{x+\frac{9}{16}}} y\,dy \right) dx = \int_0^1 \left[\frac{1}{2} y^2 \right]_{y=x^2-\frac{1}{2}x+\frac{3}{4}}^{y=\sqrt{x+\frac{9}{16}}} dx$$

$$= \int_0^1 \frac{1}{2}\left(x+\frac{9}{16}\right) - \frac{1}{2}\left(x^2-\frac{1}{2}x+\frac{3}{4}\right)^2 dx = \frac{1}{2}\int_0^1 \left(x+\frac{9}{16}-x^4-\frac{1}{4}x^2-\frac{9}{16}+x^3-\frac{3}{2}x^2+\frac{3}{4}x\right)dx$$

$$= \frac{1}{2}\int_0^1 \left(-x^4+x^3-\frac{7}{4}x^2+\frac{7}{4}x\right)dx = \frac{1}{2}\left[-\frac{x^5}{5}+\frac{x^4}{4}-\frac{7}{12}x^3+\frac{7}{8}x^2\right]_{x=0}^{x=1}$$

$$= \frac{1}{2}\left(-\frac{1}{5}+\frac{1}{4}-\frac{7}{12}+\frac{7}{8}\right) = \frac{1}{2}\left(\frac{1}{20}+\frac{-14+21}{24}\right) = \frac{1}{2}\left(\frac{1}{20}+\frac{7}{24}\right) = \frac{1}{2}\,\frac{6+35}{120} = \frac{41}{240}$$

#5

$$\iiint_A f(x,y,z)\,dx\,dy\,dz = \int_a^b \left(\iint_{A(z)} f(x,y,z)\,dx\,dy \right) dx\,dy$$

$$A = \left\{ (x,y,z) \in \mathbb{R}^3 : \ z \geq 2x^2+3y^2 \ ; \ z \geq 2-\sqrt{2x^2+3y^2} \ ; \ z \leq 6 \right\}$$

$$A_z = \left\{ (x,y) \in \mathbb{R}^2 : \ 2x^2+3y^2 \leq z \ ; \ \sqrt{2x^2+3y^2} \geq 2-z \right\} \quad \text{con} \quad z \leq 6$$

$$\Pi_3(A) = \left\{ z \in \mathbb{R} : \ A_z \neq \phi \right\}$$

Affinché $A_z \neq \phi$ bisogna che $\rho^2 \leq z$ e $\rho \geq 2-z$ con $\rho = \sqrt{2x^2+3y^2}$, da cui $-\sqrt{z} \leq \rho \leq \sqrt{z}$ (se $z \geq 0$) e $\rho \geq 2-z$, mentre se $z<0$ $A_z = \phi$. Inoltre se $2-z \leq 0$ allora $\rho \geq 2-z$ è sempre soddisfatta e $\rho \leq \sqrt{z}$. Quindi se $z \in [0,2]$ bisogna che $2-z \leq \sqrt{z}$ affinché $A_z \neq \phi$; pertanto

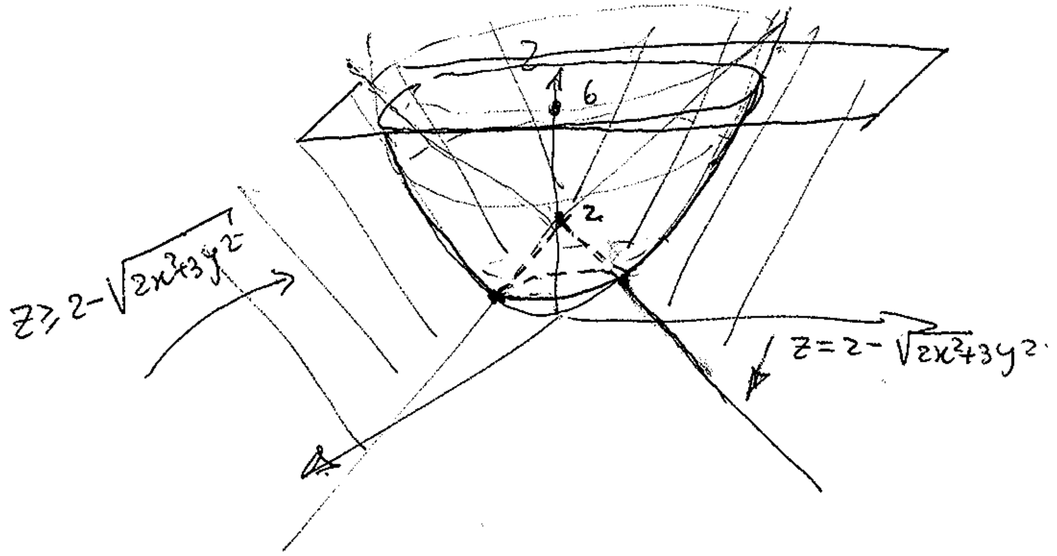
$$\begin{cases} 4-4z+z^2 \leq z \\ z \in [0,2] \end{cases} \quad \begin{cases} z^2-5z+4 \leq 0 \\ z \in [0,2] \end{cases} \quad \begin{cases} (z-4)(z-1) \leq 0 \\ z \in [0,2] \end{cases} \Longleftrightarrow \begin{cases} 1 \leq z \leq 4 \\ z \in [0,2] \end{cases} \Longleftrightarrow z \in [1,2]$$

In tal caso $(z \in [1,2])$ $A_z = \left\{ (x,y) \in \mathbb{R}^2 : (2-z)^2 \leq 2x^2+3y^2 \leq z \right\}$

Se $z \in [2,6]$; $A(z) = \{(x,y) \in \mathbb{R}^2 : 2x^2+3y^2 \leq z\}$

Quindi $a=1$ e $b=6$ e

$$\iiint_A f\,dxdydz = \int_1^2 \Big(\iint_{\{(x,y)\in\mathbb{R}^2:\ (2-z)^2 \leq 2x^2+3y^2 \leq z\}} f(x,y,z)\,dxdy \Big) dz + \int_2^6 \Big(\iint_{\{(x,y)\in\mathbb{R}^2:\ 2x^2+3y^2 \leq z\}} f(x,y,z)\,dxdy \Big) dz$$



#6 $\quad f(x,y) = (2x^2+3y^2)(x^2-y^2-5)$

$$\frac{\partial f}{\partial x} = 4x(x^2-y^2-5) + 2(2x^2+3y^2)x \quad ; \quad \frac{\partial f}{\partial y} = 6y(x^2-y^2-5) + (2x^2+3y^2)(-2y)$$

$$\begin{cases} \frac{\partial f}{\partial x} = 0 \\ \frac{\partial f}{\partial y} = 0 \end{cases} \Longleftrightarrow \begin{cases} 2x\left(2(x^2-y^2-5) + 2x^2+3y^2\right) = 0 \\ 2y\left(3(x^2-y^2-5) - (2x^2+3y^2)\right) = 0 \end{cases} \Longleftrightarrow \begin{cases} x = 0 \\ y\left(3(x^2-y^2-5) - (2x^2+3y^2)\right) = 0 \end{cases} \text{I}$$

$$\vee \begin{cases} 2(x^2-y^2-5) + 2x^2+3y^2 = 0 \\ y\left(3(x^2-y^2-5) - (2x^2+3y^2)\right) = 0 \end{cases} \text{II}$$

$$\text{I} \quad \begin{cases} x=0 \\ y(-3y^2-15-3y^2) = 0 \end{cases} \Longleftrightarrow \begin{cases} x=0 \\ y=0 \end{cases} \vee \begin{cases} x=0 \\ -6y^2-15=0 \end{cases} \Longleftrightarrow \begin{cases} x=0 \\ y=0 \end{cases}$$

II $\begin{cases} 2x^2+3y^2=-2(x^2-y^2-5) \\ y\left(3(x^2-y^2-5)+2(x^2-y^2-5)\right)=0 \end{cases} \Longleftrightarrow \begin{cases} / \\ y=0 \end{cases}$

$\vee \begin{cases} / \\ 5(x^2-y^2-5)=0 \end{cases} \Longleftrightarrow \begin{cases} 2x^2=-2(x^2-5) \\ y=0 \end{cases} \vee \begin{cases} 2x^2+3y^2=0 \\ 5(x^2-y^2-5)=0 \end{cases}$

$\Longleftrightarrow \begin{cases} 2x^2=5 \\ y=0 \end{cases} \Longleftrightarrow \begin{cases} x=\sqrt{\frac{5}{2}} \\ y=0 \end{cases} \vee \begin{cases} x=-\sqrt{\frac{5}{2}} \\ y=0 \end{cases}$ $\quad \emptyset$

I punti critici sono $(0,0)$, $\left(\sqrt{\frac{5}{2}},0\right)$, $\left(-\sqrt{\frac{5}{2}},0\right)$.

$$\frac{\partial^2 f}{\partial x^2}=4(x^2-y^2-5)+8x^2+2(2x^2+3y^2)+8x^2$$

$$\frac{\partial^2 f}{\partial y\partial x}=-8xy+12xy \;;\; \frac{\partial^2 f}{\partial y^2}=6(x^2-y^2-5)-12y^2-12y^2-2(2x^2+3y^2)$$

$$Hf(0,0)=\begin{bmatrix} -20 & , & 0 \\ 0 & , & -30 \end{bmatrix}$$ quindi $(0,0)$ è p.to di massimo

$$Hf\left(\pm\sqrt{\tfrac{5}{2}},0\right)=\begin{bmatrix} 4\left(\frac{5}{2}-5\right)+8\cdot\frac{5}{2}+4\cdot\frac{5}{2}+8\cdot\frac{5}{2} & , & 0 \\ 0 & , & 6\left(\frac{5}{2}-5\right)-2\left(2\cdot\frac{5}{2}\right) \end{bmatrix}$$

$$=\begin{bmatrix} -\cancel{10}+20+\cancel{10}+20 & , & 0 \\ 0 & , & -3-10 \end{bmatrix}=\begin{bmatrix} 40 & 0 \\ 0 & -13 \end{bmatrix}$$

quindi $\left(\sqrt{\frac{5}{2}},0\right)$ e $\left(-\sqrt{\frac{5}{2}},0\right)$ sono punti di sella.

#7 $f: \mathbb{R}^3 \to \mathbb{R}^2$, $g \in C^1(\mathbb{R},\mathbb{R})$ $\phi \in C^1(\mathbb{R}^2;\mathbb{R})$

$$Jf(x_0,y_0,z_0)=\begin{bmatrix} \frac{\partial f_1}{\partial x}(x_0,y_0,z_0), & \frac{\partial f_1}{\partial y}(x_0,y_0,z_0), & \frac{\partial f_1}{\partial z}(x_0,y_0,z_0) \\ \frac{\partial f_2}{\partial x}(x_0,y_0,z_0), & \frac{\partial f_2}{\partial y}(x_0,y_0,z_0), & \frac{\partial f_2}{\partial z}(x_0,y_0,z_0) \end{bmatrix}$$

con $(x_0,y_0,z_0)=(2,1,1)$

dove $f_1(x,y,z)=\sin^5(x^2y+z^7)$ e

$$f_2=\phi(\underbrace{x^7+g(5x^3yz)}_{u}, \underbrace{h_1(y^5+3x^3z^2)}_{v}).$$

Quindi

$$\frac{\partial f_1}{\partial x}(x,y,z)=y\,5\sin^4(x^2y+z^7)2x\cos(x^2y+z^7); \quad \frac{\partial f_1}{\partial y}=5\sin^4(x^2y+z^7)\cos(x^2y+z^7)2x^2$$

$$\frac{\partial f_1}{\partial z}(x,y,z)=5\sin^4(x^2y+z^7)\cos(x^2y+z^7)\cdot 7z^6.$$ Pertanto

$$\frac{\partial f_1}{\partial x}(2,1,1)=5\sin^4(5)\cdot 4\cdot\cos 5=20\sin^4 5\cdot\cos 5; \quad \frac{\partial f_1}{\partial y}=40\sin^4(5)\cdot\cos(5)$$

$$\frac{\partial f_1}{\partial z}(2,1,1)=35\sin^4(5)\cos(5).$$

Analogamente

$$\frac{\partial f_2}{\partial x}(x,y,z)=\frac{\partial\phi}{\partial u}(x^7+g(5x^3yz),h_1(y^5+3x^3z^2))(7x^6+g'(5x^3yz)\cdot 15x^2)+\frac{\partial\phi}{\partial v}(x^7+g(5x^3yz),h_1(y^5+3x^3z^2))\cdot 9x^2z^2\cdot h_1'(y^5+3x^3z^2).$$

$$\frac{\partial f_2}{\partial y}(x,y,z)=\frac{\partial\phi}{\partial u}(x^7+g(5x^3yz),h_1(y^5+3x^3z^2))\cdot g'(5x^3yz)\cdot z+\frac{\partial\phi}{\partial v}(x^7+g(5x^3yz),h_1(y^5+3x^3z^2))\cdot h_1'(y^5+3x^3z^2)\cdot 5y^4$$

$$\frac{\partial f_3}{\partial z}(x,y,z)=\frac{\partial\phi}{\partial u}(x^7+g(5x^3yz),h_1(y^5+3x^3z^2))\cdot g'(5x^3yz)y+\frac{\partial\phi}{\partial v}(x^7+g(5x^3yz),h_1(y^5+3x^3z^2))\cdot h_1'(y^5+3x^3z^2)\cdot 6x^3z.$$

Pertanto
$$\frac{\partial f_2}{\partial x}(2,1,1)=\frac{\partial\phi}{\partial u}(2^7+g(40+1),h_1(25))\cdot(7\cdot 2^6+g'(41)\cdot 60)+\frac{\partial\phi}{\partial v}(2^7+g(41),h_1(25))\,36\cdot h_1'(25)$$

$$\frac{\partial f_2}{\partial y}(2,1,1)=\frac{\partial\phi}{\partial u}(2^7+g(41),h_1(25))\cdot g'(41)+\frac{\partial\phi}{\partial v}(2^7+g(41),h_1(25))\cdot h_1'(25)\cdot 5$$

$$\frac{\partial f_2}{\partial z}(2,1,1)=\frac{\partial\phi}{\partial u}(2^7+g(41),h_1(25))g'(41)+\frac{\partial\phi}{\partial v}(2^7+g(41),h_1(25))h_1'(25)\cdot 48.$$