FAUSTO FERRARI

Contents

1. Curriculum vitae

Born in Reggio nell' Emilia (Italy) in 1963.

- Since 2006 October 1st **Associate Professor in Mathematics** (Mathematical Analysis), at University of Bologna, Italy (with tenure) on service at the Faculty of Engineering.
- From 1994 July 16th to 2006 September 30th **Assistant Professor in Mathematics** (Ricercatore confermato di Analisi Matematica) at the University of Bologna, Italy, (with tenure) on service at the Faculty of Engineering.
- In 1997 **Dottore di ricerca in Matematica** (PhD in Mathematics), Milan University (Italy).Title of PhD-thesis: Teoremi di confronto di tipo Harnack per funzioni armoniche in domini con frontiera Hölderiana. Advisor: prof. Sandro Salsa.
- From 1992 to 1996 **Scuola di Dottorato di Ricerca,** (PhD School in Mathematics) Milan University (Italy), Corso di dottorato in Matematica (VIII ciclo) Milan University and Milan Polytechnic.
- From 1987 to 1994 **Math Teacher in the secondary school state** , on the staff from 1991 September 1-st 1994 July 15-th 15/7/1994.
- From 1987 to 1988 **Military service**, (from 1987 April 1st to 1988 June 30th).
- **Laurea in Mathematics** got with first class degree at Bologna University (Italy) 28-th November 1986. Title of the thesis: Soluzioni variazionali per un'equazione delle onde non lineare. Advisor prof. Ermanno Lanconelli.

2. Personal informations

Married since 1994. I am father of a daughter.

3. Short description of the scientific activity

In my scientific activity I studied problems associated with the regularity of the solutions of elliptic second order partial differential equations and parabolic partial differential equations. The techniques I used, are mainly based on tools like Harnack inequality and boundary Harnack comparison theorems.

In my PhD thesis, [34], and subsequently in [30], I studied the boundary regularity of the solutions of PDE's in Hölder domains giving an analytic proof of a boundary Harnack principle. This result provides the estimate of the ratio of two positive harmonic functions vanishing in a subset of the boundary.

Successively, starting from the theory developed by Bony, I extended this research to the case of degenerate operators satisfying the Hörmander rank condition. The case of the Laplace Kohn operator in the Heisenberg group provides a simple example of the operators I dealt with. In this framework, the behavior of the solutions near to the boundary depend both on the smoothness of the boundary and on the characteristic points. The characteristic points of a smooth surface are those points whose normal is orthogonal to the left invariant vector fields that appear in the definition of the Laplace-Kohn operator.

Notice that even on smooth C^∞ surface, e.g the boundary of a given set, some irregular points for the Dirichlet problem could exist.

The degenerate character of these operators produce, near to the characteristic points, a behavior similar to that of the classical harmonic functions in domains with cusps.

In the papers [28] and [27] (joint works with Bruno Franchi) I studied these problems for Grushin operators, see [28], and for operators given as sums of the squares of some vector fields. I proved a doubling formula of the harmonic measure and a boundary Harnack principle, see [27], introducing a new class of domains called $\phi-$Harnack domains, see [27]. Such domains are defined starting from the following fundamental property: the existence of a chains of metric balls, connecting any couple of points, such that the center of each of these balls belongs to the previous one ball and to the next one ball of the chain.

The length of these chains is given by the function ϕ and determines the main properties of the domain.

The existence of boundary Harnack type theorems for non-negative solutions of second order elliptic or parabolic PDE's is useful for studying free boundary problems with one or two phases.

In particular I proved, in a joint work with Cristina Cerutti and Sandro Salsa, see [26], a regularity result for the free boundary of the solutions of a two-phase problem for a class of elliptic operators in non-divergence form with variable coefficients of class $C^{0,\alpha}$. More precisely if the free boundary is the graph of a Lipschitz continuous function, we are able to prove that the regularity of the free boundary has to be $C^{1,\gamma}$ indeed. This paper generalizes some results first obtained by Luis Caffarelli for the Laplace operator.

There are several geometric aspects that enter in this proof. In particular the Harnack internal inequality produces the opening of the monotonicity cone of the solutions in a neighborhood of the free boundary with a gain of the width that it is transmitted until the free boundary. This effect produces a regularization of the free boundary.

To do this, some nice sub-solutions of the problems have to be built. This can be done considering the maximum of the given solution on balls of radius depending on a function that have to satisfy a nonlinear partial differential inequality.

Further improvements of this type of results can be found in [22] where a class of fully nonlinear operators uniformly elliptic, containing Bellman and Isaac operators, with $C^{0,\alpha}$ coefficients is investigated.

The main difficulties here depend on the fact that the solutions are not, in general, $C^{2,\alpha}$. Indeed the operators considered could be non-convex operators.

In [17] the regularity assumption on the free boundary has been weakened. In fact in the paper has been proved that, if the free boundary is near enough to a Lipschitz graph then, as in the case of the Laplace operator, the free boundary of the solutions of a two-phase problem associated to a linear uniformly elliptic operator with variable coefficients $C^{0,\alpha}$ in the principal parts and with L^∞ coefficients for the terms of order one is $C^{1,\gamma}$ regular.

The key idea relies on the notion of $\epsilon-$ monotonicity along the cone. If the free boundary is in a ϵ- neighborhood of a graph of a Lipschitz continuous function then there exists $\epsilon-$ monotonicity of the solutions, and the $\epsilon-$ monotonicity implies the full monotonicity in the complementar set of a neighborhood of a Lipschitz graph (of size proportional to ϵ^k, with $k < 1$ and κ depending on the $C^{0,\alpha}$ regularity of the coefficients), see for the details [17].

The case of the operators in divergence form with Lipschitz continuous coefficients is described in [19], while the case with $C^{0,\alpha}$ coefficients is still open.

Further developments following these techniques have been obtained for fully nonlinear operators in the same class studied in [22] assuming hypothesis of (strong) $\epsilon-$monotonicity, are described in [13].

In the framework of the Heisenberg group, in [8], we started to study free boundary problems for degenerate operators. In particular we prove some density estimates for local minima of a fluid jet functional.

Very recent developments in the case of nonhomogeneous case for linear elliptic operators in nondivergence form are described in [37]. In that paper we obtain regularity results for the free boundary even when the two-phase problem is associated with a nonhomogenuous equation. We implemented new techniques that avoid to use families of subsolutions constructed taking the maximum of solutions in balls of variable radii. Further developments are in order, in particular for fully nonlinear nonhomogeneous operators. In that case a fundamental role it seems to be played from a transmission problem. I already studied transmission problem for operators in divergence form in [6], obtaining sharp regularity results.

As far as the parabolic case with variable coefficients there are the recent results contained in [10] and [2]. In particular in [10] we proved the Lipschitz regularity of solutions of parabolic two-phase problems, assuming Lipschitz continuity in space time variables of the graph of the free boundary assuming hypotheses of nondegeneracy in spacial variables of the free boundary itself. In [2] remove the nondegeneracy assumption and we prove the smoothness of the front.

Another field of research is associated with the notion of curvature.

For example the mean curvature of a smooth surface imbedded in $\mathbb{R}^n$ is the symmetric elementary function of order one of the $n-1$ principal curvatures. Analogously the symmetric elementary function of order $n-1$ of the $n-1$ principal curvatures is the Gauss curvature.

Analogously, considering the k-th eigenvalues of the Hessian matrix of a function u defined in $\mathbb{R}^n$, we find the $k-$th Hessian operator. Among the $k-$th Hessian operators there is the Laplace operator, $(k = 1)$, and the determinant of the Hessian matrix, namely the n-th symmetric elementary function of the eigenvalues of the Hessian matrix of u.

I studied this operators in [32]. I gave sufficient conditions for the existence of ground state for $k-$th Hessian equations.

In collaboration with Bruno Franchi and Guozhen Lu, see [24], we prove some results about a relative Alexandrov-Fenchel inequality.

In this framework I would like to cite the following recent works [14], [12] and [11] where curvatures play a fundamental role. More precisely in [14], a Sobolev-Poincaré inequality with weights has been proved (here the weights are associated with curvatures of the level surfaces of stable solutions of semilinear equations). From this type of equations follow some results on the properties of symmetry of the solutions of semilinear problems (see the Heisenberg case in [15], the Grushin case in [12], and the hyperbolic case, see [11]). Recent results about this approach are contained in [9] and in [3] where we analyze the case of Carnot groups. In particular

we put in evidence as for Carnot groups that are not of type star the approach in [11] is not easyly applicable. Indeed, for the Engel group, that is not a group of type star, we give some nonexistence results of symmetric solutions for semilinear equations using a different approach based on the informations arising on the vector fields with constant intrinsic norm.

Concerning the existence of some inequalities connecting the $k-$th Hesssian operators, the fractional Laplace operators and the p-Laplace operator, there is the recent paper [7] obtained with Bruno Franchi and Igor Verbitsky. Moreover, I point out the several interesting properties of the fractional subharmonic functions with respect to the $k-$th Hessian operators discovered in [5] and the results in the paper [39], where the Caffarelli-Silvestre approach to prove Harnack inequality for nonlocal operator is generalized to the framework of Carnot groups.

In the project [35] we would like to better understand the nature of functions that satisfy equations determined as sum of $k-$Hessian operators, or a linear operator and a $k-$Hessian operators. This resarch is related to some equations used in the description the physical formation of crystals.

I studied the Harnack inequality for nonlinear operators, see [25], for p-Laplace degenerate operators defined starting from vector fields and matrices satisfying some A_p conditions.

In this case I proved that the Harnack constant depends on a ratio of the measure induced by the weights on the balls of radius r to the exponent $1/p$.

I wrote a paper about the degenerate obstacle problem for operators in divergence form, see [29]. In particular I proved, assuming a degenerate condition determined by some weights, an apriori estimate of the measure of the contact set and some further L^∞ estimates of the solutions using some symmetrisation techniques. Very recently I wrote a research paper, [4], about some a priori estimates for linear and nonlinear degenerate operators modelled on the field of the Heisenberg group where the symmetrization techniques still continue to be applied.

I also studied the notion of *reach* and the notion of *curvature* of a surface in the Heisenberg group, see the joint papers with Nicola Arcozzi, [20] and [16] in order to understand the correct meaning of this notions in the Heisenberg framework.

We introduced the notion of the metric normal far from the characteristic points. In [20] we proved that real part of the Laplace - Kohn operator of the intrinsic distance from a regular surface is the intrinsic divergence of the intrisic normal vector field to the surface, namely the intrinsic mean curvature of the surface. In this research we define the notion of *imaginary curvature* a geometric invariant that plays a role in the description of the horizontal Hessian matrix of the distance from a surface, see also [16]. The notion of imaginary curvature plays a role also the characterization of p-harmonic functions in the Heisenberg group, see [38], and in the regularity of the distance from a surface in Carnot groups of step 2, see [40].

Further researches concerning the geometric properties of the surfaces in the Heisenberg group can be found the following recent papers: [18], [41], [49] and [40]. In this 'metric stream' I put the project in [48] where we want to better describe, from a mathematical point of wiev, how to focalize two different rays with two lens in some prescribed directions.

In continuity with my researches in [38], we proved a characterization of the meaning of $p-$harmonicity function in the framework of the Heisenberg group using appropriate mean formulas. In [1],[47] we study some important properties of axysimmetric function in the Heisenberg group with respect to the flux of mean curvature, like existence and uniqueness of viscosity solutions of level-set equation.

4. Recent Meetings and talks

- International Workshop "Advances in Nonlinear Science", Department of Mathematics, University of Pittsburgh, Pittsburgh, March 14-16, 2013.

  Title: (Fausto Ferrari) *Geometric inequalities and semilinear equations in Carnot groups*

http://www.math.pitt.edu/ lewicka/Semester_GamesPDE_13/adv_non_sci_workshop.html

- Geometry and Analysis Seminar, Department of Mathematics Columbia University, New York, March 7, 2013
  Title: (Fausto Ferrari) *Fractional sub-Laplacians in Carnot groups.*
  http://www.math.columbia.edu/event/geometry-and-analysis-seminar-11-13/
- PDE and Analysis Seminars, Department of Mathematics, University of Pittsburgh, Pittsburgh, March 19, 2012 Title: (Fausto Ferrari) *Some relations between fractional Laplace operators and Hessian operators.*
  http://www.pitt.edu/ hajlasz/Analysis%20Seminar/seminar.html
- Geometry and Analysis Seminar, Department of Mathematics Columbia University, New York, March 22, 2012 Title: (Fausto Ferrari) *On the regularity of the solutions of two-phase problems for parabolic operators.*
  http://www.math.columbia.edu/event/geometry-and-analysis-seminar-6/
- XIX Congresso UMI, Bologna 12-17 Settembre 2011. Short communication in the section: Equazione alle derivate parziali. Title: (Fausto Ferrari) *Sulla regolarità delle soluzioni di problemi a due fasi per operatori parabolici.*
- PDE and Geometric Analysis, Department of Mathematics, Wisconsin University, Madison, September 13, 2010 Title: (Fausto Ferrari)*Semilinear PDEs and some symmetry properties of stable solutions.*
  https://www.math.wisc.edu/wiki/index.php/Previous_PDE/GA_seminars#Fausto_Ferrari_.28Bolo
- Colloquium at the Department of Mathematics, Missouri University, Comumbia, September, 2, 2010 (Fausto Ferrari) Title: *Symmetry Properties of Stable Solutions of Some Semilinear PDEs*
  http://www.math.missouri.edu/events/calendar.php?view=view_month&month=9&year=2010
- Indam school of symmetry for elliptic PDEs (30 years after a conjecture of De Giorgi and related problems) Rome May 25-29, 2009 Invited speaker. Title: (Fausto Ferrari) *Some inequalities associated with semilinear elliptic equations with variable coefficients and applications.*
- Seminario di Equazioni Differenziali del Dipartimento di Matematica dell'Università di Roma la Sapienza. Title: (Fausto Ferrari) *Disuguaglianze di tipo Poincaré con pesi e soluzioni stabili di equazioni semilineari nel gruppo di Heisenberg" 23 marzo 2009.*
- XVIII Congresso UMI, Bari 24-29 Settembre 2007. Short communication in the section : Equazione alle derivate parziali.
  Title (Fausto Ferrari): *Un risultato di regolarità delle frontiere libere piatte per problemi a due fasi.*
- International Conference on PDEs and Harmonic Analysis Beijing, June 25-29, 2007
  Title (Fausto Ferrari):
  The metric normal and its applications in the Heisenberg group.
  http://math.bnu.edu.cn/l̃ijunfeng/ICPHA/
- 1022nd AMS Meeting University of Arkansas Fayetteville, Arkansas. University of Arkansas, Fayetteville, AR (2006 Fall Southeastern Section Meeting) November 3-4, 2006.
  Title (Fausto Ferrari): *Metric normal and curvatures in the Heisenberg group.*
- Meeting on suelliptic PDE's and application to geometry and finance, Cortona June 12-17, 2006
  Title (Fausto Ferrari): *A Steiner formula in the Heisenberg group for Carnot-Charathodory balls*
- Workshop: Geometric inequalities, Firenze 16-20 May 2005
  Title (Fausto Ferrari): *Distance function, metric normal and sets of positive reach in the Heisenberg group.*

- XXV Conferenza di Analisi Armonica 6-9 Aprile 2005, Bologna
  Title (Fausto Ferrari): *Il problema del commesso viaggiatore nel gruppo di Heisenberg.*
- 1002nd AMS Meeting University of Pittsburgh, Pennsylvania November 6-7, 2004 Special Session on Geometric Analysis and Partial Differential Equations in Subelliptic Structures.
  Title (Fausto Ferrari): *The horizontal Hessian matrix of the distance function in the Heisenberg group.*
- Viscosity, metric and control theoretic methods in nonlinear PDE's, Serapo (Italy) September 27-th October 1-st 2004.
  Title (Fausto Ferrari): *Distance function and metric normal in the Heisenberg group.*
- XVII Congresso UMI: short communication in the section: Calcolo delle variazioni, teoria del controllo e ottimizzazione.
  Title (Fausto Ferrari): *Su una classe di disuguaglianze isoperimetriche relative per le misure di curvatura di Federer.*
- Analisi e Convessità (Progetto GNAMPA 2003 Aspetti analitici della Convessità) Cremona, 23-24 Gennaio 2004.
  Title (Fausto Ferrari): *On relative isoperimetric inequalities for curvature measures.*

5. Meetings (organizer)

- Organizer of the Meeting: *Mathematics and Applications*,
  Milan, 2010, January 11 (Politecnico di Milano),
  Cagliari, 2010, June, 3-5 (Università di Cagliari);
  Further details can be found to the following web page: http://www.mate.polimi.it/mathapp/
- Organizer of the Meeting: *Seminari di Analisi Matematica*,
  giornata di lavoro in ricordo di Bruno Pini, Bologna 27/11/2009, Accademia delle Scienze e Dipartimento di Matematica di Bologna.
  A tribute to the memory and the work of Bruno Pini, Bologna, 2009 November 27.
  Further details can be found to the following web page :
  http://www.dm.unibo.it/%7Eferrari/pini/pini.html
- Organizer of the Meeting in memory of Bruno Pini, Bologna il 26 Novembre 2010.
  Title: *Seminari di Analisi Matematica*, giornata di lavoro in ricordo di Bruno Pini, Bologna 26/11/2010, Dipartimento di Matematica di Bologna.
  Further details:
  http://www.dm.unibo.it/%7Eferrari/pini10/pini.html
- Organizer of: 'Topics in Mathematics" for the PhD in Mathematics, University of Bologna A.A. 2010/2011: Alcune idee forti in Matematica 15 Aprile 2011, 24 Giugno 2011.
  Further details:
  http://www.dm.unibo.it/dottorato/topics in mathematics/
- Organizer of the Meeting in memory of Bruno Pini, Bologna il 25 Novembre 2011.
  Ttle: *Seminari di Analisi Matematica*, giornata di lavoro in ricordo di Bruno Pini, Bologna 25/11/2011, Dipartimento di Matematica di Bologna.
  Further details:
  http://www.dm.unibo.it/%7Eferrari/pini2011/pini.html

6. Invitations from foreign Universities

- **In 2002** Invited by Prof. Luis Caffarelli, I visited the Dpt. of Mathematics at Austin of the Texas University (USA).
  Invited by Prof. Guozhen Lu I visited the Dpt. of Mathematics at Detroit of the Wayne State University (USA).

- **In 2003** Invited by Prof. Luis Caffarelli, I visited the Dpt. of Mathematics at Austin of the Texas University (USA).
- **In 2005** Invited by Prof. Mikhail Feldman, I visited the Dpt. of Mathematics at Madison of Wisconsin University (USA).
- **In 2006** Invited by Prof. Hervé Pajot, I visited the Dpt. of Mathematics of the University of Grenoble (Francia).
- **In 2006** Invited by Prof. M. Feldman I visited the Dpt. of Mathematics at Madison of Wisconsin University (USA).
- **In 2010** Invited by Prof. I. Verbitsky I visited the Dpt. of Mathematics at Columbia, Missouri University, Columbia (USA).
- **In 2010** Invited by Prof. M. Feldman I visited the Dpt. of Mathematics at Madison of Wisconsin University (USA).
- **In 2011** Invited by Prof. I.Peral I visited the Dipartimento di Matematica dell'Università Autonoma di Madrid, Madrid (Spagna).
- **In 2012** Invited by Prof. J. Manfredi I visited the Dpt. of Mathematics of the Pittsburgh University, Pittsburgh PA (USA).
- **In 2012** Invited by Prof. D. De Silva I visited the Dpt. of Mathematics of the Columbia University, New York (USA).
- **In 2012** Invited by Prof. Prof. C. Gutierrez I visited the Dpt. of Mathematics of the Temple University, Philadelphia PA (USA).
- **In 2013** Invited by Prof. D. De Silva I visited the Dpt. of Mathematics of the Columbia University, New York (USA).
- **In 2013** Invited by Prof. J. Manfredi I visited the Dpt. of Mathematics of the Pittsburgh University, Pittsburgh PA (USA).

7. Collaborations and grants

- Scientific collaboration with prof. Juan J. Manfredi University of Pittsburgh, (PA) USA on the properties of p-laplace operator in the Heisenberg group.
- Scientific collaboration with prof. Daniela De Silva Columbia University of New York (NY) USA on the regularity of solutions of nonhomogeneous problems in the Heisenberg group.
- Scientific collaboration with prof. Daniela De Silva Columbia University of New York (NY) USA and prof. Sandro Salsa of the Politecnico di Milano about the regularity of solutions of nonhomogeneous problems: linear operators with variable coefficients, fully nonlinear operators and parabolic operators.
- Scientific collaboration with prof. Cristian Gutierrez Temple University of Philadelphia (PA) USA on refraction and refracted rays light.
- Member of the "EPSILON"-team (Elliptic Pdes and Symmetry of Interfaces and Layers for Odd Nonlinearities, PE1), ERC starting grant 2011 coordinato da E. Valdinoci.
- Scientific collaboration with prof. Ireneo Peral, Universida Autonoma de Madrid, Madrid (Spain) on the existence of solutions to equations given by the sums of Hessian operators and linear operators.
- Scientific collaboration with prof. Igor Verbitsky dell'Università del Missouri, Columbia on the properties of the Hessian operators and laplacian fractional.
- Member of GNAMPA project "Equazioni nonlineari su varietà: proprietà qualitative e classificazione delle soluzioni" with Enrico Valdinoci and Isabeau Birindelli.
- Collaboration with Bruno Franchi (Dipartimento di Matematica dell' Università di Bologna) and Prof. Hervé Pajot (Departement de Matematique de l'Université de Grenoble). We studied sufficient conditions about the existence of rectificable paths of finite lenght connecting points in a given sets in the Heisenberg group.

- Collaboration with Prof. Sandro Salsa (Dipartimento di Matematica del Politecnico di Milano). Our project follows three main lines:
  - Free boundary regularity in Stefan problem: the case of variable coefficients.
  - Free boundary problems: monotonicity formulas for elliptic and and parabolic operators and application to optimal regularity results of solutions and of free boundary for two-phase problems with variable coefficients in divergence and nondivergence form.
  - Free boundary problems for fully nonlinear parabolic operators with one or two phases: extention, for operators with coefficients indipendent to time of the regularity results of the free boundary already proved for the free boundary in the elliptic framework.
- Prof. Mikhail Feldman (Dipartimento di Matematica dell'Università del Wisconsin, Madison). Collaboration about free boundary problems and Monge Ampere problem in the Heisenberg group.
- Member of the European project: European Science Foundation. Global and geometric aspects of nonlinear partial differential equations.
- Member of the research group Neuromathematics and Visual Cognition, University of Bologna.

8. Referee activity

I belong to:" Elenco dei revisori del MIUR.'

I served as a referee for the following journals: Advances in Mathematics, Annales de l'Institut Fourier, Arabian Journal of Mathematics (AJM), Mathematische Zeitschrift, Mathematische Nachrichten, Journal of Mathematical Analysis and Applications, Nonlinear Differential Equations and Applications, Calculus of Variations and PDE's, Analysis and PDE's.

List of publications

[1] F. Ferrari, Q. Liu, J.J Manfredi, *On the horizontal mean curvature flow for axisymmetric surfaces in the Heisenberg group* Accepted paper, to appear in Communications in Contemporary Mathematics, DOI: 10.1142/S0219199713500272

[2] F. Ferrari, S. Salsa, *Two-phase free boundary problems for parabolic operators: smoothness of the front,* (2013) Accepted paper, to appear in CPAM

[3] F. Ferrari, A. Pinamonti, *Nonexistence results for semilinear equations in Carnot groups,* Analysis and Geometry in Metric Spaces, 1,(2013)130-146 DOI: 10.2478/agms-2013-0001

[4] F. Ferrari,*Some a priori estimates for a class of operators in the Heisenberg group* (2012) Accepted paper, to appear in Annali di Matematica Pura e Applicata, DOI 10.1007/s10231-012-0313-7

[5] F. Ferrari, I. E. Verbitsky,*Radial fractional Laplace operators and Hessian inequalities,* J. Differential Equations 253 (2012), no. 1, 244-272

[6] G. Citti, F. Ferrari, A sharp regularity result of solutions of a transmission problem Proc. Amer. Math. Soc. 140 (2012), 615-620

[7] F. Ferrari, B. Franchi, I. Verbitsky, Hessian inequalities and the fractional Laplacian J. reine und angew. Mathematik 667 (2012) 133-148 DOI 10.1515/crelle.2011.116

[8] F. Ferrari, E. Valdinoci, *Density estimates for a fluid jet model in the Heisenberg group*, doi:10.1016/j.jmaa.2011.04.057,J. Math. Anal. Appl. 382, no. 1, 448-468 (2011)

[9] F. Ferrari, *Some inequalities associated with semilinear elliptic equations with variable coefficients and applications*, Symmetry for elliptic PDEs, 81-104 , Contemp. Math., 528, Amer. Math. Soc., Providence, RI, 2010.

[10] F. Ferrari, S.Salsa, Regularity of the solutions for parabolic two-phase free boundary problems, Comm. Part. Diff. Equations, 35, 6, 1095-1129, (2010).

[11] I. Birindelli, F. Ferrari, E. Valdinoci,*Semilinear PDEs in the Heisenberg group: the role of the right invariant vector fields*, accettato per la pubblicazione apparirà in Nonlinear Analysis Series A: Theory, Methods & Applications, 72, 987–997 (2010)); doi:10.1016/j.na.2009.07.039

[12] F. Ferrari, E. Valdinoci, *Geometric PDEs in the Grushin Plane: Weighted Inequalities and Flatness of Level Sets,* International Mathematics Research Notices, 22, 4232-4270 (2009); doi: 10.1093/imrn/rnp088

[13] R. Argiolas, F.Ferrari, *Flat free boundaries regularity in two-phase problems for a class of fully nonlinear elliptic operators with variable coefficients*, Interfaces Free Bound. 11, no. 2, 177-199 (2009).

[14] F. Ferrari, E.Valdinoci, *Some weighted Poincaré inequalities*, Indiana Univ. Math. J. Volume 58, Issue 4, 1619-1638 (2009).

[15] F. Ferrari, E. Valdinoci, *A geometric inequality in the Heisenberg group and its applications to stable solutions of semilinear problems*, Math. Ann. 343, 2, 351–370 (2009).

[16] N. Arcozzi, F. Ferrari, *The Hessian of the distance from a surface in the Heisenberg group,* Ann. Acad. Sci. Fenn. Math. 33,1, 35–63 (2008).

[17] F. Ferrari, S. Salsa, *Regularity of the free boundary in two-phase problems for linear elliptic operators*, Adv. in Math. 214, 1, 288-322 (2007).

[18] F. Ferrari, *A Steiner formula in the Heisenberg group for Carnot-Charathéodory balls.* Subelliptic PDE's and applications to geometry and finance, 133–143, Lect. Notes Semin. Interdiscip. Mat., 6, Semin. Interdiscip. Mat. (S.I.M.), Potenza, 2007.

[19] F. Ferrari, S. Salsa, *Subsolutions of elliptic operators in divergence form and application to two-phase free boundary problems*, Bound. Value Probl. 2007, Art. ID 57049, 21 pp.

[20] N. Arcozzi, F. Ferrari, *Metric normal and distance function in the Heisenberg group*, Math. Z. 256, 3, 661-684 (2007).

[21] F. Ferrari, B. Franchi, Hervé Pajot, *Courbure et sous-ensembles de courbes rectifiables dans le groupe de Heisenberg* in Equations aux Dérivés Partielles. 2005–2006, Exp. No. XII, 12 pp., Sémin. Équ. Dériv. Partielles, École Polytech., Palaiseau, (2006).

[22] F. Ferrari, *Two-phase problems for a class of fully nonlinear elliptic operators. Lipschitz free boundaries are $C^{1,\gamma}$*, Amer. J. Math. 128, 3, 541–571 (2006).

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[24] *On a relative Alexandrov-Fenchel inequality for convex bodies in Euclidean spaces,* Forum Math. 18, 907-921 (2006).

[25] F. Ferrari, *Harnack inequality for two-weight subelliptic p-Laplace operators,* Math. Nachr., 279, 8, 815-830 (2006).

[26] M.C.Cerutti, F. Ferrari & S. Salsa, *Two-phase problems for linear elliptic operators with variable coefficients: Lipschitz free boundaries are $C^{1,\gamma}$,* Arch. Rational Mech. Anal. 171, 329-348 (2004).

[27] F. Ferrari, B. Franchi, *A local doubling formula for the harmonic measure associated with subelliptic operators and applications,* Communication in Partial Differential Equations (vol. 28) (n. 1,2) pp 1–60 (2003).

[28] F. Ferrari, B. Franchi, *Geometry of the boundary and doubling property of the harmonic measure for Grushin type operators,* Rend. Sem. Mat. Univ. Politec. Torino, vol 58 n. 3, pp. 281- 299 (2000).

[29] F. Ferrari, *Rearrangements and an obstacle problem for degenerate variational inequalities,* Ricerche di Matematica, XLVII, n. , pp. 257-275 (1998).

[30] F. Ferrari, *On the boundary behavior of harmonic functions in Hölder domains* J. Fourier Anal. Appl., 4, pp. 447-461 (1998).

[31] F. Ferrari, *Teoremi di confronto di tipo Harnack per funzioni armoniche in domini con frontiera hölderiana*, Boll. Un. Mat. Ital., 8 1A, pp. 113-116 (1998).

[32] F. Ferrari, *Un teorema di confronto per funzioni armoniche in domini con frontiera hoelderiana* Equazioni Differenziali (Ferrara, 3-6 Giugno 1996) pp. 175-181, Ann. Univ. Ferrara, Supplemento al Volume XLI, Ferrara, 1996 .

[33] F. Ferrari, *Ground state solutions for k-th Hessian operators*, Boll. Un. Mat. Ital. B (7) 9 (1995), no. 3, 553–586.

Tesi di Dottorato di Ricerca

[34] F. Ferrari, *Teoremi di confronto di tipo Harnack per funzioni armoniche in domini con frontiera Hölderiana,* VIII ciclo, Università di Milano e Politecnico di Milano (1997).

Lavori sottomessi

[35] F. Ferrari, M. Medina, I. Peral, *Biharmonic elliptic problems involving 2-Hessian nonlinearities and zero order perturbations,* (2013) submitted.

[36] D. De Silva, F. Ferrari, S. Salsa, *Free boundary regularity for fully nonlinear non-homogeneous two-phase problems,* arXiv:1304.0406 (2013) submitted.

[37] D. De Silva, F. Ferrari, S. Salsa, *Two-phase problems with distributed source: regularity of the free boundary,* arXiv:1210.7226 (2012) submitted.

[38] F. Ferrari, Q. Liu, J.J Manfredi, *On the characterization of p−harmonic functions on the Heisenberg group by mean value properties* arXiv:1210.2881v1 (2012) submitted.
[39] F. Ferrari, B. Franchi, *Harnack inequality for fractional sub-Laplacians in Carnot groups* arXiv:1206.0885v3 (2012) submitted.
[40] N. Arcozzi, F. Ferrari, F. Montefalcone, *CC-distance and metric normal of smooth hypersurfaces in sub-Riemannian, Carnot groups,* arXiv:0910.5648 (2009) submitted.

Preprints

[41] F. Ferrari,*Curvature formulae of noncharacteristic smooth sets in the Heisenberg group.*
[42] F. Ferrari, A. Pinamonti,*Characterization by approximated mean formula of p−harmonic functions in Carnot groups.* (2013)

In progress

[43] F. Ferrari, *Maximum principle for axisymmetric solutions to nonlinear operators* (2013).
[44] F. Ferrari, *Nonlocal operators characterised by mean formulas* (2013).
[45] D. De Silva, F. Ferrari, S. Salsa, *Existence of solutions for nonhomogeneous two-phase problems in divergence form and regularity of their free boundaries* (2013).
[46] D. De Silva, F. Ferrari, *Nonhomogeneous two-phase problems in the Heisenberg group* (2013).
[47] F. Ferrari, Q. Liu, J.J Manfredi, *Axisymmetric surfaces and mean curvature in the Heisenberg group* (2012)
[48] F. Ferrari, C. Gutierrez, *Refracted rays* (2012).
[49] F. Ferrari, *A Steiner type formula for Carnot-Charathéodory balls in the Heisenberg group,* (2007).
[50] N. Arcozzi, F. Ferrari, *A variational approximation of the perimeter with second order penalization in the Heisenberg group,* (2007).

Seminars of the Department of Mathematics of Bologna (with proceedings partially available to the following web address: http://www.dm.unibo.it/seminario-pini/)

[i] Fausto Ferrari: *Su alcune relazioni tra operatori frazionari del laplaciano e operatori hessiani,* 9 Giugno 2011.
[ii] Fausto Ferrari: *Sulla simmetria delle soluzioni stabili di alcune equazioni semilineari,* 8 Aprile 2010.
[iii] Fausto Ferrari: *Soluzioni stabili di equazioni semilineari e disuguaglianze di tipo Poincaré con curvature nel gruppo di Heisenberg,* 19 marzo 2009. Seminario di analisi Matematica "Bruno Pini" A.A. 2008/2009, Tecnoprint- Bologna 43-60.
[iv] Fausto Ferrari: *Regolarità delle frontiere libere piatte in problemi a due fasi per operatori ellittici,* Seminario di Analisi Matematica 05/06 (Tecnoprint-Bologna in corso di stampa).
[v] Fausto Ferrari: *Una versione geometrica del problema del commesso viaggiatore nel gruppo di Heisenberg,* Seminario di Analisi Matematica 04/05 (Tecnoprint-Bologna in corso di stampa).
[vi] Fausto Ferrari: *La disuguaglianza di Alexandrov-Fenchel relativa*, Seminario di Analisi Matematica 03/04 (in corso di stampa).
[vii] Fausto Ferrari: *Problemi ellittici a due fasi: regolarità della frontiera libera*, Seminario di Analisi Matematica 02/03 Tecnoprint-Bologna pp 59–67.
[viii] Fausto Ferrari: *Proprietà di regolarità della misura armonica per operatori subellittici,* Seminario di Analisi Matematica Dipartimento di Matematica dell'Università di Bologna, A.A. 1999/2000 pp. 33–48 (2000).
[ix] Fausto Ferrari: *Simmetrizzazioni e disequazioni variazionali per operatori ellittici,* Seminario di Analisi Matematica 96/97 121–136 Tecnoprint-Bologna.
[x] Fausto Ferrari: *Teoremi di tipo Harnack per funzioni armoniche domini con frontiera hölderiana,* Seminario di Analisi Matematica 94/95, 149–159 Tecnoprint-Bologna.
[xi] Fausto Ferrari: *Stati fondamentali per operatori hessiani,* Seminario di Analisi Matematica 93/94 135–146 Tecnoprint-Bologna.

9. Public data base

Google Scholar profile:

http://scholar.google.it/citations?user=oLu2w8wAAAAJ&hl=it

I am on Research Gate

9.1. **Comments on bibliometric datas.**

10. Teaching

I spent my didactic activity teaching and also always covering the role of member or president of grading commissions (when the responsibility of the course was mine) for the following subjects (Analisi Matematica I, II, III and moreover Analisi Matematica L-A, Analisi Matematica L-B, Analisi Matematica L-C, Complementi di Analisi Matematica ed Elementi di Calcolo delle Probabilità, Complementi di Analisi Matematica L-S (laurea specialistica), Complementi di Analisi Matematica L-M (laurea magistrale) at the Faculty of Engineering of the University of Bologna at Bologna, since 1994.

At the Faculty of Engineering of the University of Bologna at Bologna, to each credit the professor delivers ten hours of lessons to the students, namely $\#CFU \times 10$. *So that (*$9CFU$ *at the Faculty of Engineering reads as* 90 *hours of lessons to the students) . Since 13/14, the value of a credit in teaching activity correspond to* 8 *hours by teaching delivered by the professor to a very large number of courses in Laurea Magistrale.*

At the Faculty of Science (MMFFNN) of the University of Bologna at Bologna, to each credit the professor delivers height hours of lessons to the students, $\#CFU \times 8$. *(*$3CFU$ *at the Faculty of Engineering reads as* 24 *hours of lessons to the students).*

Scheduled activity during 2013/2014 (teaching period) as Associate professor at "Scuola di Ingegneria e Architettura" University of Bologna

- Analisi Matematica T: Ingegneria Civile (A-E) (9CFU) and Analisi Matematica T1: Ingegneria per l'Ambiente e il Territorio (9CFU).
- Complementi di Analisi Matematica ed Elementi di Calcolo delle Probabilità: Ingegneria Civile (a module 3CFU dedicated to Calcolo delle Probabilità (Probability) among 9CFU of the course)

Extra teaching activity

- Complementi di Analisi Matematica L-M: Laurea Magistrale in Ingegneria Chimica e di Processo (4CFU). (4CFU=32 h.)

Scheduled activity during 2013/2014 (teaching period) as Associate professor at "Scuola di Scienze" University of Bologna

- Analisi Superiore 2: II year students Laurea Magistrale, Curriculum Generale e Applicativo A.A. 2013-2014 (a module 3 CFU (24 h.) among the 6 CFU (48 h.).

Courses delivered as Associate Professor to the Faculty of Engineering of the University of Bologna at Bologna:

A.A. 12/13
- Activity scheduled from September 2012 to Dicember 2013: Analisi Matematica T: Ingegneria Civile (A-K) (9CFU) and
  Analisi Matematica T1: Ingegneria per l'Ambiente e il Territorio (9CFU).
- Complementi di Analisi Matematica L-M: Laurea Magistrale in Ingegneria Chimica e di Processo (4CFU).
- Scheduled activity from March 2013 to June 2013: Complementi di Analisi Matematica ed Elementi di Calcolo delle Probabilità: Ingegneria Civile (a module 3CFU dedicated to Calcolo delle Probabilità (Probability) among 9CFU of the course).

A.A. 11/12
- Analisi Matematica T: Ingegneria Civile (A-K) and
  Analisi Matematica T-1: Ingegneria per l'Ambiente e il Territorio (9CFU).
- Complementi di Analisi Matematica ed Elementi di Calcolo delle Probabilità: Ingegneria Civile (a module 3CFU dedicated to Calcolo delle Probabilità (Probability) among 9CFU of the course).
  Complementi di Analisi Matematica L-M: Laurea Magistrale in Ingegneria Chimica e di Processo (4CFU).

Substitute at the Faculty of Science, Scienze MM.FF.NN.) of Bologna A.A. 11/12 degree in Matematica :
Calcolo delle Variazioni (i) (second module, 3 credits (24 ore) , of a course of 6 credits (48 ore)).

A.A. 10/11
- Analisi Matematica T-1: Ingegneria dell'Automazione and Ingegneria per l'Ambiente e il Territorio (9CFU).
- Complementi di Analisi Matematica L-M: Laurea Magistrale in Ingegneria Chimica e di Processo (4CFU).

Substitute at the Faculty of Science, Scienze MM.FF.NN.) of Bologna A.A. 11/12 degree in Matematica :
Calcolo delle Variazioni (i) (second module, 3 credits (24 ore) , of a course of 6 credits (48 ore)).

A.A. 09/10
- Analisi Matematica T-A: Ingegneria Civile (A-K), (9CFU)
- Complementi di Analisi Matematica L-M: Laurea Magistrale in Ingegneria Chimica e di Processo (6CFU).

Substitute at the Faculty of Science, Scienze MM.FF.NN.) of Bologna A.A. 11/12 degree in Matematica :
Calcolo delle Variazioni (i) (second module, 3 credits (24 ore) , of a course of 6 credits (48 ore)).

A.A. 08/09
- Complementi di Analisi Matematica L-M : Laurea Magistrale in Ingegneria Chimica e di Processo and
  and Complementi di Analisi Matematica L-S: Laurea in Ingegneria specialistica per l'Ambiente e il Territorio. (6CFU).
- Analisi Matematica T-A: Ingegneria Civile (A-K) and
  Analisi Matematica T-1: Ingegneria per l'Ambiente e il Territorio (9CFU).

A.A. 07/08
- Complementi di Analisi Matematica L-S (6 CFU) :Laurea specialistica in Ingegneria Chimica e di Processo e Ingegneria per l'Ambiente e il Territorio.
- Analisi Matematica L-B (6 CFU): laurea in Ingegneria per l'Ambiente e il Territorio, and laurea in Ingegneria Chimica.
- Analisi Matematica L-A (6 CFU):laurea in Ingegneria per l'Ambiente e il Territorio and laurea in Ingegneria Chimica .

A.A. 06/07
- Analisi Matematica L-A (6 CFU):laurea in Ingegneria per l'Ambiente e il Territorio e Ingegneria Chimica.
- Analisi Matematica L-B (6 CFU):laurea in Ingegneria per l'Ambiente e il Territorio e Ingegneria Chimica.
- Complementi di Analisi Matematica L-S (6 crediti) per i corsi di Laurea specialistica in Ingegneria Civile and Ingegneria per l'Ambiente e il Territorio.

Courses delivered as Ricercatore to the Faculty of Engineering of the University of Bologna at Bologna:

A.A. 05/06
- Corso di accoglienza per le matricole (20 ore).
- Analisi Matematica L-A: Laurea in Ingegneria Elettronica and Laurea in Ingegneria Automatica (6CFU).

A.A. 04/05
- Corso di accoglienza per le matricole (20 ore).

– Analisi Matematica L-A: Laurea in Ingegneria per l'Ambiente e il Territorio and Laurea in Ingegneria delle Telecomunicazioni (N-Z) (6CFU).

A.A. 03/04 – Corso di accoglienza per le matricole (40 ore).
– Analisi Matematica L-B: Laurea in Ingegneria Energetica and Laurea in Ingegneria Meccanica (N-Z) (6CFU).

A.A. 02/03 Corso di accoglienza per le matricole (40 ore).

A.A. 01/02 – Analisi Matematica L-B: Laurea in Ingegneria Informatica (A-K)and Laurea in Ingegneria Elettrica. (6CFU).
– Corso di accoglienza per le matricole (40 ore).

A.A. 99/00 Matematica A: Diploma in Ingegneria dell'Ambiente e delle Risorse (55 ore).

A.A. 98/99 Matematica B: Diploma in Ingegneria dell'Ambiente e delle Risorse (55 ore).

A.A. 97/98 Matematica A: Diploma in Ingegneria dell'Ambiente e delle Risorse (55 ore).

Activity as assistant professor (Ricercatore) to the Facoltà di Ingegneria di Bologna:

A.A. 94/95 : Analisi Matematica I e II (CdL Elettronica), 6 h. per week.
A.A. 95/96 : Analisi Matematica I e II (CdL Elettronica), 6 h. per week.
A.A. 96/97 : Analisi Matematica I e II (CdL Elettronica e Gestionale), 9 h. per week.
A.A. 97/98 : due corsi di Analisi Matematica I (CdL Meccanica e Gestionale), 6 h. per week.
A.A. 98/99 : Analisi Matematica I (CdL Edile e Civile) 5 h. per week.
A.A. 99/00 : Analisi Matematica I e II (CdL Civile e Gestionale), 3 h. per week.
A.A. 00/01 : Analisi Matematica I e II (CdL Gestionale), 5 h. per week.
A.A. 01/02 : Analisi matematica L-C, 2 h. per week.
A.A. 02/03 : Analisi Matematica L-A e L-B, 5 h. per week.

10.1. **Teaching in other University.**

A.A. 05/06 - Equazioni differenziali ordinarie: Laurea in Ingegneria Civile (L-Z), Faculty of Engineering, Politecnico di Milano, (5CFU).

10.2. **Teaching in 'Dottorato di Ricerca in Matematica' (Professor in PhD Schools in Mathematics).**

A.A. 07/08 - Dottorato di Ricerca in Matematica del Dipartimento di Matematica dell'Università di Bologna. Title of the subject: Introduzione ai problemi di frontiera libera (30 h).

A.A. 2010/2011 : 'Topics in Mathematics" for the PhD in Mathematics, University of Bologna A.A. 2010/2011: Alcune idee forti in Matematica 15 Aprile 2011, 24 Giugno 2011.
Further details:
http://www.dm.unibo.it/dottorato/topics in mathematics/

10.3. **Didactic publications and projects:**

- Simonetta Abenda, Fausto Ferrari, Silvano Matarasso, *Prove di Analisi Matematica 1*, editore Progetto Leonardo Bologna.
- I participate to the Springer project: "Encyclopedia of Thermal Stresses", edited by Professor Richard Hetnarski, with the following contribution: *Fourier transform*