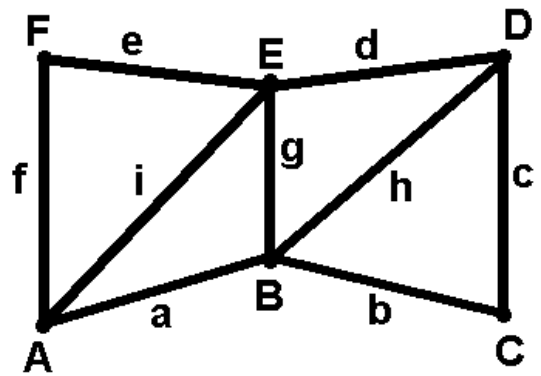Mathematical Methods – Graph Theory
EXAMPLE 1

Let G be the graph drawn here:

1) *(1 pt.)* Adjacency matrix:

2) *(1 pt.)* Incidence matrix:

3) *(1 pt.)* Minimum degree δ =   Maximum degree Δ =

4) *(1 pt.)* Connectivity κ =   Edge-connectivity κ' =

5) *(1 pt.)* Is G bipartite? Why? (If answer is "yes", list the two vertex sets of the bipartition)

6) *(1 pt.)* Does G have an Euler tour? Why? (If answer is "yes", write the edge sequence of one)

7) *(1 pt.)* Does G have an Euler trail with distinct origin and terminus? Why? (If answer is "yes", write the edge sequence of one)

8) *(1 pt.)* Does G have a Hamilton path? (If answer is "yes", write the vertex sequence of one)
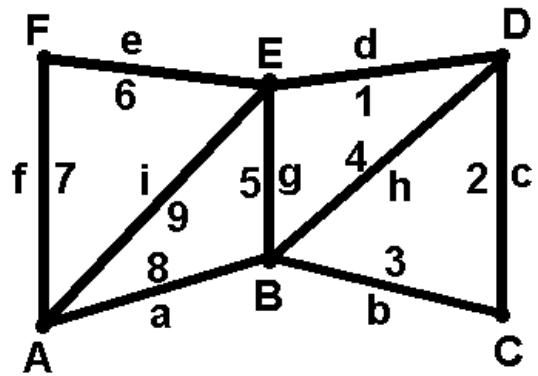
9) *(1 pt.)* List the edge set of a maximum matching. Is it a perfect matching?

Now the vertices represent towns and the edge weights represent distances.

10) *(2 pts.)* Use Dijkstra's algorithm to find minimal routes from A to all other vertices.
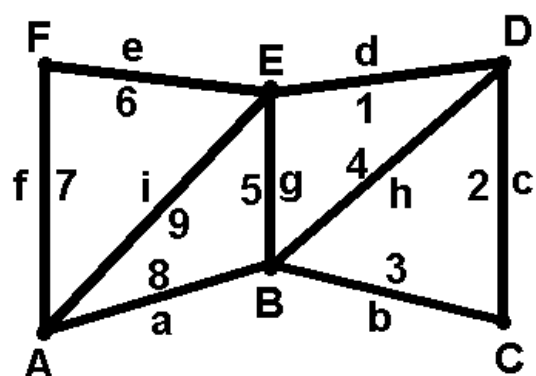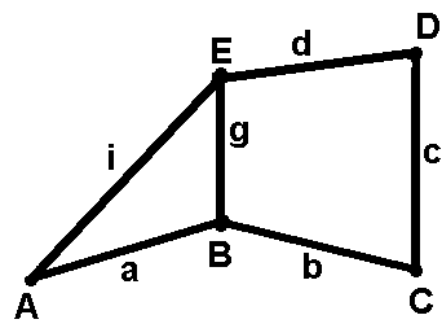
11) *(2 pts.)* Use Kruskal's algorithm to find a spanning tree with minimum total weight (an optimal connector of the towns).

12) *(3 pts.)* **Use the recursive formula** to compute τ (# of spanning trees) of this graph (please show all passages):
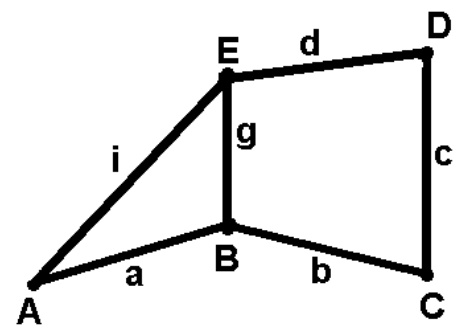
13) *(4 pts.)* Use logic operations to find all minimal coverings and all maximal independent sets of this graph (please show all passages).

14) *(4 pts.)* Compute the chromatic polynomial of this graph (please show all passages).