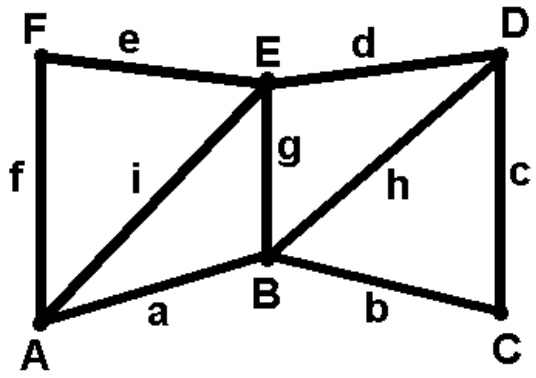
Mathematical Methods – Graph Theory
EXAMPLE 1



Let G be the graph drawn here:

1) *(1 pt.)* Adjacency matrix:

	A	B	C	D	E	F
A	0	1	0	0	1	1
B	1	0	1	1	1	0
C	0	1	0	1	0	0
D	0	1	1	0	1	0
E	1	1	0	1	0	1
F	1	0	0	0	1	0

2) *(1 pt.)* Incidence matrix:

	a	b	c	d	e	f	g	h	i
A	1	0	0	0	0	1	0	0	1
B	1	1	0	0	0	0	1	1	0
C	0	1	1	0	0	0	0	0	0
D	0	0	1	1	0	0	0	1	0
E	0	0	0	1	1	0	1	0	1
F	0	0	0	0	1	1	0	0	0

3) *(1 pt.)* Minimum degree δ = 2 Maximum degree Δ = 4

4) *(1 pt.)* Connectivity κ = 2 Edge-connectivity κ' = 2

5) *(1 pt.)* Is G bipartite? Why? (If answer is "yes", list the two vertex sets of the bipartition)
No: It has odd cycles.

6) *(1 pt.)* Does G have an Euler tour? Why? (If answer is "yes", write the edge sequence of one)
No: It has vertices with odd degree.

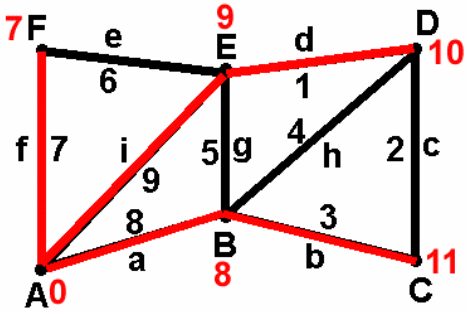
7) *(1 pt.)* Does G have an Euler trail with distinct origin and terminus? Why? (If answer is "yes", write the edge sequence of one)
Yes: it has just two vertices with odd degree. abcdefigh

8) *(1 pt.)* Does G have a Hamilton path? (If answer is "yes", write the vertex sequence of one)
ABCDEF

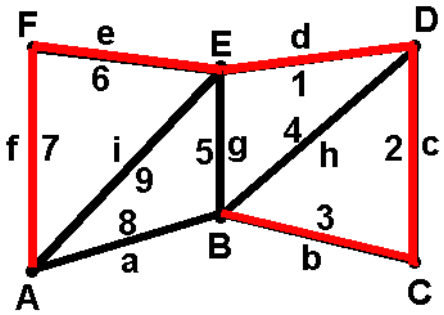
9) *(1 pt.)* List the edge set of a maximum matching. Is it a perfect matching?
cgf Yes.

Now the vertices represent towns and the edge weights represent distances.

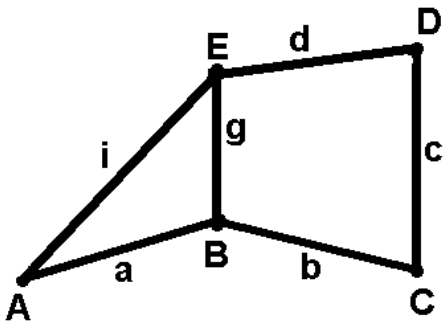
10) *(2 pts.)* Use Dijkstra's algorithm to find minimal routes from A to all other vertices.



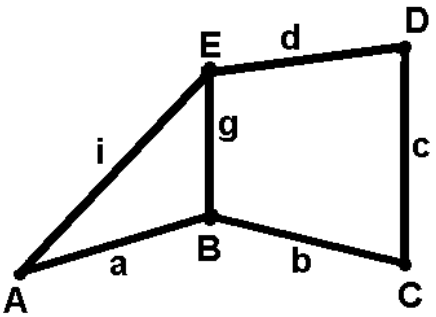
11) *(2 pts.)* Use Kruskal's algorithm to find a spanning tree with minimum total weight (an optimal connector of the towns).



12) *(3 pts.)* **Use the recursive formula** to compute τ (# of spanning trees) of this graph (passages not shown here, but in test you are supposed to show them): 11



13) *(4 pts.)* Use logic operations to find all minimal coverings and all maximal independent sets of this graph (please show all passages).



(A+BE)(B+AEC)(C+BD)(D+CE)(E+ABD)
(A+BE)(B+AEC) = AB+AAEC+BEB+BEAEC
(AB+AEC+BE)(C+BD) =
= ABC+AECC+BEC+ABBD+AECBD+BEBD
(ABC+AEC+BEC+ABD+BED)(D+CE) =
= ABCD+AECD+BECD+ABDD+BEDD+ABCCE+AECCE+BECCE+ABCDCE+BEDCE
(ABD+BED+AEC+BEC)(E+ABD) =
= ABDE+BEDE+AECE+BECE+ABDABD+BEDABD+AECABD+BECABD =
= BED+AEC+BEC+ABD

Minimal coverings: {B,E,D}, {A,E,C}, {B,E,C}, {A,B,D}
Maximal independent sets: {A,C}, {B,D}, {A,D}, {C,E}

14) *(4 pts.)* Compute the chromatic polynomial of this graph (passages not shown here, but in test you are supposed to show them).

$k^5 - 6k^4 + 14k^3 - 15k^2 + 6k$