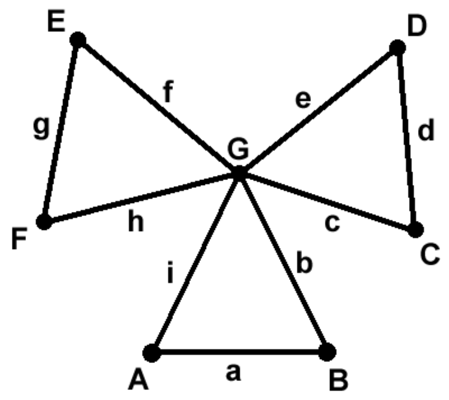
Mathematical Methods – 13 Jul. 2012 – Graph Theory
**EXAMPLE 2**




Let G be the graph drawn here:

1) *(1 pt.)* Adjacency matrix:

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| A | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| B | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| C | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| D | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| E | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| F | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| G | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

2) *(1 pt.)* Incidence matrix:

| | a | b | c | d | e | f | g | h | i |
|---|---|---|---|---|---|---|---|---|---|
| A | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| B | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| D | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| E | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| F | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| G | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |

3) *(1 pt.)* Minimum degree $\delta =$ 2  Maximum degree $\Delta =$ 6

4) *(1 pt.)* Connectivity $\kappa =$ 1  Edge-connectivity $\kappa' =$ 2

5) *(1 pt.)* Is G bipartite? Why? (If answer is “yes”, list the two vertex sets of the bipartition)
No. It has odd cycles.

6) *(1 pt.)* Does G have an Euler tour? Why? (If answer is “yes”, write the edge sequence of one)
Yes. All of its vertices have even degree. abcdefghi

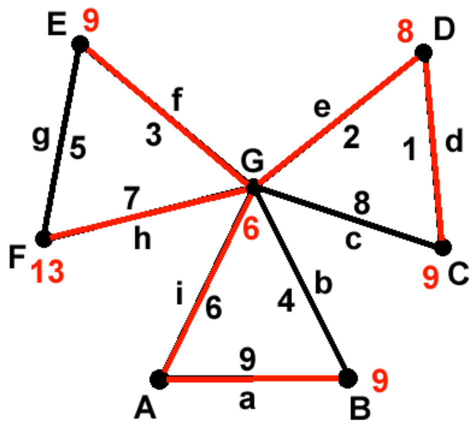
7) *(1 pt.)* Does G have an Euler trail with distinct origin and terminus? Why? (If answer is “yes”, write the edge sequence of one)
No. It does not have two vertices of odd degree.

8) *(1 pt.)* Does G have a Hamilton path? (If answer is “yes”, write the vertex sequence of one)
No.

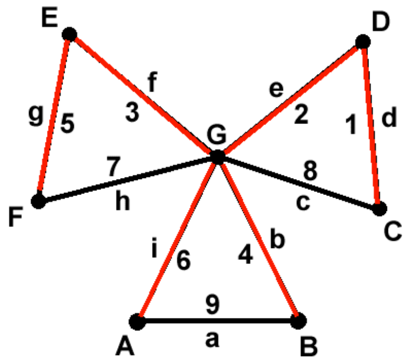
9) *(1 pt.)* List the edge set of a maximum matching. Is it a perfect matching?
adg. No.

Now the vertices represent towns and the edge weights represent distances.

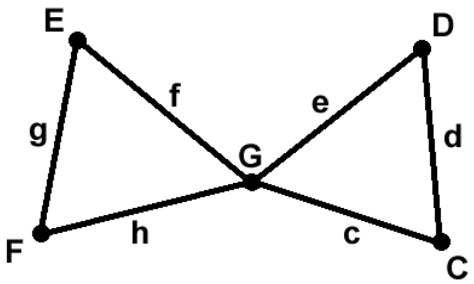
10) *(2 pts.)* Use Dijkstra’s algorithm to find minimal routes from A to all other vertices.



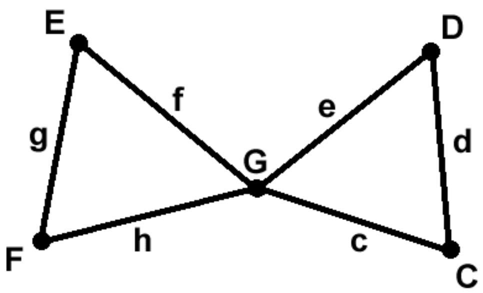
11) *(2 pts.)* Use Kruskal's algorithm to find a spanning tree with minimum total weight (an optimal connector of the towns).




12) *(3 pts.)* Use the recursive formula to compute τ (# of spanning trees) of this graph (passages not shown here, but in test you are supposed to show them): 9




13) *(4 pts.)* Use logic operations to find all minimal coverings and all maximal independent sets of this graph (please show all passages).




(C+DG)(D+CG)(E+FG)(F+EG)(G+CDEF)

(C+DG)(D+CG) = CD+CCG+DGD+DGCG

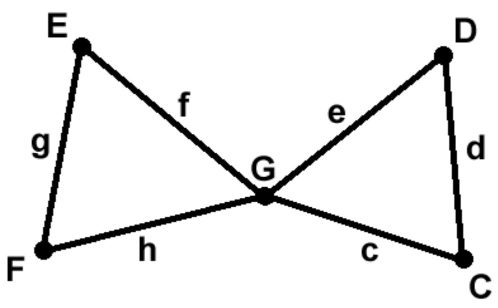
(CD+CG+DG)(E+FG) = CDE+CDFG+CGE+CGFG+DGE+DGFG

(CDE+CGE+CGF+DGE+DGF)(F+EG) =
= CDEF+CDEEG+CGEF+CGEEG+CGFF+CGFEG+DGEF+DGEEG+DGFF+DGFEG

(CDEF+CGE+CGF+DGE+DGF)(G+CDEF) = CDEFG+CDEFCDEF+CGEG+
+CGECDEF+CGFG+CGFCDEF+DGEG+DGECDEF+DGFG+DGFCDEF =
= CDEF+CGE+CGF+DGE+DGF

Minimal coverings: {C,D,E,F}, {C,G,E}, {C,G,F}, {D,G,E}, {D,G,F}
Maximal independent sets: {G}, {D,F}, {D,E}, {C,F}, {C,E}

14) *(4 pts.)* Compute the chromatic polynomial of this graph (passages not shown here, but in test you are supposed to show them).




$k(k-1)^2(k-2)^2 =$
$= k^5-6k^4+13k^3-12k^2+4k$