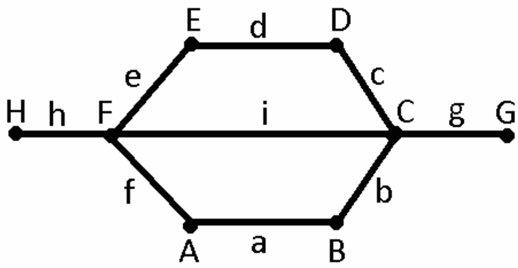
Mathematical Methods – 21 Dec. 2021 – Graph Theory
UniBo matriculation number:
(no name, please)

Let G be the graph drawn here:



1) *(1 pt.)* Adjacency matrix:

	A	B	C	D	E	F	G	H
A	0	1	0	0	0	1	0	0
B	1	0	1	0	0	0	0	0
C	0	1	0	1	0	1	1	0
D	0	0	1	0	1	0	0	0
E	0	0	0	1	0	1	0	0
F	1	0	1	0	1	0	0	1
G	0	0	1	0	0	0	0	0
H	0	0	0	0	0	1	0	0

2) *(1 pt.)* Incidence matrix:

	a	b	c	d	e	f	g	h	i
A	1	0	0	0	0	1	0	0	0
B	1	1	0	0	0	0	0	0	0
C	0	1	1	0	0	0	1	0	1
D	0	0	1	1	0	0	0	0	0
E	0	0	0	1	1	0	0	0	0
F	0	0	0	0	1	1	0	1	1
G	0	0	0	0	0	0	1	0	0
H	0	0	0	0	0	0	0	1	0

3) *(1 pt.)* Minimum degree $\delta = 1$ Maximum degree $\Delta = 4$

4) *(1 pt.)* Connectivity $\kappa = 1$ Edge-connectivity $\kappa' = 1$

5) *(1 pt.)* Is G bipartite? Why? (If answer is "yes", list the two vertex sets of the bipartition)
Yes. It contains no odd cycles. {A,C,E,H}, {B,D,F,G}

6) *(1 pt.)* Does G have an Euler tour? Why? (If answer is "yes", write the edge sequence of one)
No. It contains vertices of odd degree.

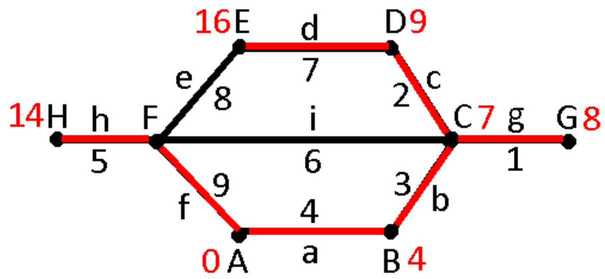
7) *(1 pt.)* Does G have an Euler trail? Why? (If answer is "yes", write the edge sequence of one)
Yes. It contains only two vertices of odd degree. hfabiedcg

8) *(1 pt.)* Does G have a Hamilton path? (If answer is "yes", write the vertex sequence of one)
No.

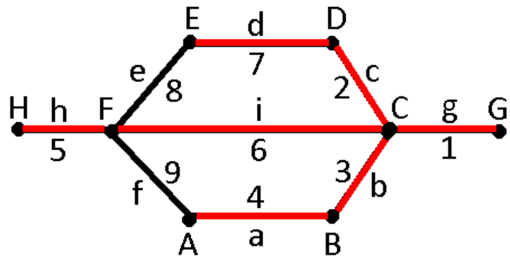
9) *(1 pt.)* List the edge set of a maximum matching. Is it a perfect matching?
{a,d,g,h} Yes.

Now the vertices represent towns and the edge weights represent distances.

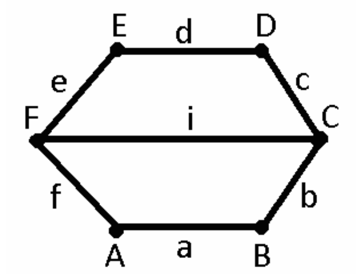
10) *(2 pts.)* Use Dijkstra's algorithm to find minimal routes from A to all other vertices.



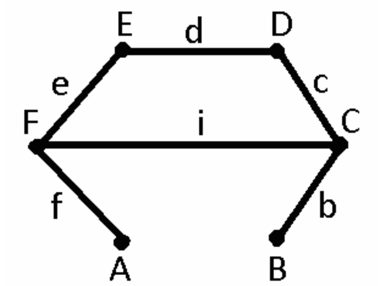
11) *(2 pts.)* Use Kruskal's algorithm to find a spanning tree with minimum total weight (an optimal connector of the towns).



12) *(3 pts.)* **Use the recursive formula** to compute τ (# of spanning trees) of this graph (passages not shown here, but in test you are supposed to show them): 15



13) *(4 pts.)* Use logic operations to find all minimal coverings and all maximal independent sets of this graph (please show all passages).

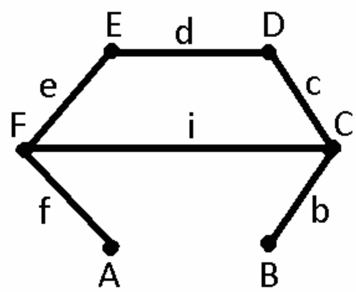


(A+F)(B+C)(C+BDF)(D+CE)(E+DF)(F+ACE) =
= (AB+AC+BF+CF)(C+BDF)( ) =
= (ABC+ABBDF+ACC+ACBDF+BFC+BFBDF+CFC+CFBDF)( ) =
= (AC+BDF+CF)(D+CE)( ) = (ACD+ACCE+BDFD+BDFCE+CFD+CFCE)( ) =
= (ACD+ACE+BDF+CDF+CEF)(E+DF)( ) =
= (ACDE+ACDDF+ACEE+ACEDF+BDFE+BDFDF+CDFE+CDFDF+CEFE+CEFDF)( ) =
= (ACD+ACE+BDF+CDF+CEF)(F+ACE) =
= ACDF+ACDACE+ACEF+ACEACE+BDFF+BDFACE+CDFF+CDFACE+CEFF+CEFACE =
= ACE+BDF+CDF+CEF

Minimal coverings: {A,C,E}, {B,D,F}, {C,D,F}, {C,E,F}

Maximal independent sets: {B,D,F}, {A,C,E}, {A,B,E}, {A,B,D}

14) *(4 pts.)* Compute the chromatic polynomial of this graph (passages not shown here, but in test you are supposed to show them).



$k^6-6k^5+15k^4-19k^3+12k^2-3k$