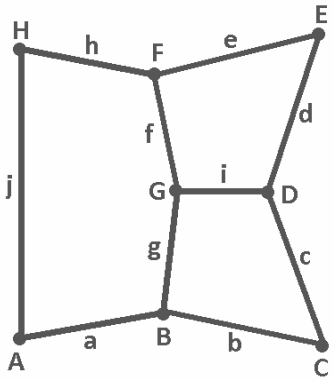
Mathematical Methods – 7 Jan. 2022 – Graph Theory
UniBo matriculation number:
(no name, please)



Let G be the graph drawn here:

1) *(1 pt.)* Adjacency matrix:

	A	B	C	D	E	F	G	H
A	0	1	0	0	0	0	0	1
B	1	0	1	0	0	0	1	0
C	0	1	0	1	0	0	0	0
D	0	0	1	0	1	0	1	0
E	0	0	0	1	0	1	0	0
F	0	0	0	0	1	0	1	1
G	0	1	0	1	0	1	0	0
H	1	0	0	0	0	1	0	0

2) *(1 pt.)* Incidence matrix:

	a	b	c	d	e	f	g	h	i	j
A	1	0	0	0	0	0	0	0	0	1
B	1	1	0	0	0	0	1	0	0	0
C	0	1	1	0	0	0	0	0	0	0
D	0	0	1	1	0	0	0	0	1	0
E	0	0	0	1	1	0	0	0	0	0
F	0	0	0	0	1	1	0	1	0	0
G	0	0	0	0	0	1	1	0	1	0
H	0	0	0	0	0	0	0	1	0	1

3) *(1 pt.)* Minimum degree $\delta = 2$ Maximum degree $\Delta = 3$

4) *(1 pt.)* Connectivity $\kappa = 2$ Edge-connectivity $\kappa' = 2$

5) *(1 pt.)* Is G bipartite? Why? (If answer is "yes", list the two vertex sets of the bipartition)
No. It contains odd cycles.

6) *(1 pt.)* Does G have an Euler tour? Why? (If answer is "yes", write the edge sequence of one)
No. It contains vertices of odd degree.

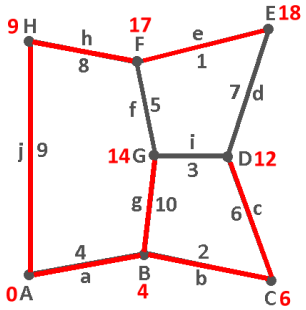
7) *(1 pt.)* Does G have an Euler trail? Why? (If answer is "yes", write the edge sequence of one)
No. It contains more than two vertices of odd degree.

8) *(1 pt.)* Does G have a Hamilton path? (If answer is "yes", write the vertex sequence of one)
Yes. HABCDGFE

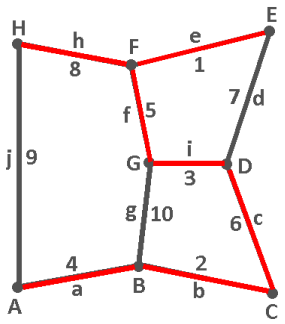
9) *(1 pt.)* List the edge set of a maximum matching. Is it a perfect matching?
{b,e,i,j} Yes.

Now the vertices represent towns and the edge weights represent distances.

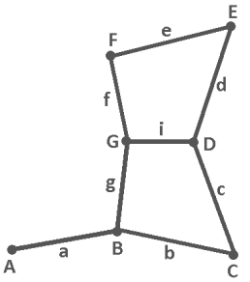
10) *(2 pts.)* Use Dijkstra's algorithm to find minimal routes from A to all other vertices.



11) *(2 pts.)* Use Kruskal's algorithm to find a spanning tree with minimum total weight (an optimal connector of the towns).

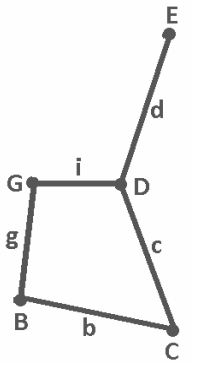


12) *(3 pts.)* **Use the recursive formula** to compute τ (# of spanning trees) of this graph (passages not shown here, but in test you are supposed to show them): 15



13) *(4 pts.)* Use logic operations to find all minimal coverings and all maximal independent sets of this graph (please show all passages).

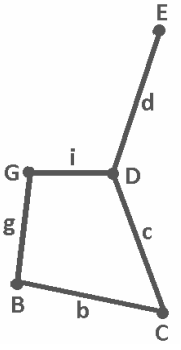
(B+CG)(C+BD)(D+CEG)(E+D)(G+BD) =
= (BC+BBD+CGC+CGBD)( ) = (BC+BD+CG)(D+CEG)( ) =
= (BCD+BCCEG+BDD+BDCEG+CGD+CDCEG)( ) =
= (BD+CGD+CGE)(E+D)( ) = (BDE+BDD+CGDE+CGDD+CGEE+CGED)( ) =
= (BD+CGD+CGE)(G+BD) = BDG+BDBD+CGDG+CGDBD+CGEG+CGEBD =
= BD+CGD+CGE



Minimal coverings: {B,D}, {C,D,G}, {C,E,G}

Maximal independent sets: {C,E,G}, {B,E}, {B,D}

14) *(4 pts.)* Compute the chromatic polynomial of this graph (passages not shown here, but in test you are supposed to show them).



$k^5-5k^4+10k^3-9k^2+3k$