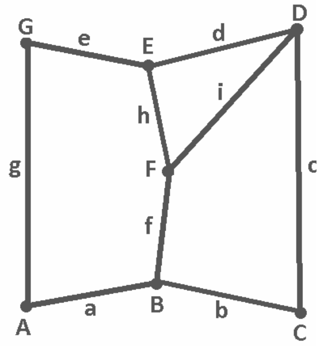
Mathematical Methods – 7 Jan. 2022 – Graph Theory
UniBo matriculation number:
(no name, please)

Let G be the graph drawn here:



1) *(1 pt.)* Adjacency matrix:

	A	B	C	D	E	F	G
A	0	1	0	0	0	0	1
B	1	0	1	0	0	1	0
C	0	1	0	1	0	0	0
D	0	0	1	0	1	1	0
E	0	0	0	1	0	1	1
F	0	1	0	1	1	0	0
G	1	0	0	0	1	0	0

2) *(1 pt.)* Incidence matrix:

	a	b	c	d	e	f	g	h	i
A	1	0	0	0	0	0	1	0	0
B	1	1	0	0	0	1	0	0	0
C	0	1	1	0	0	0	0	0	0
D	0	0	1	1	0	0	0	0	1
E	0	0	0	1	1	0	0	1	0
F	0	0	0	0	0	1	0	1	1
G	0	0	0	0	1	0	1	0	0

3) *(1 pt.)* Minimum degree δ = 2   Maximum degree Δ = 3

4) *(1 pt.)* Connectivity κ = 2   Edge-connectivity κ' = 2

5) *(1 pt.)* Is G bipartite? Why? (If answer is "yes", list the two vertex sets of the bipartition)
No. It contains odd cycles.

6) *(1 pt.)* Does G have an Euler tour? Why? (If answer is "yes", write the edge sequence of one)
No. It contains vertices of odd degree.

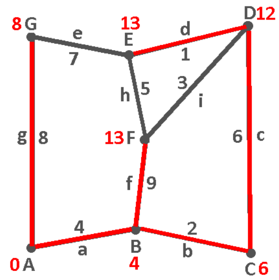
7) *(1 pt.)* Does G have an Euler trail? Why? (If answer is "yes", write the edge sequence of one)
No. It contains more than two vertices of odd degree.

8) *(1 pt.)* Does G have a Hamilton path? (If answer is "yes", write the vertex sequence of one)
Yes. ABCDFEG

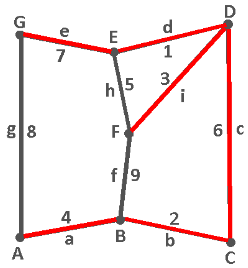
9) *(1 pt.)* List the edge set of a maximum matching. Is it a perfect matching?
{a,e,i} No.

Now the vertices represent towns and the edge weights represent distances.

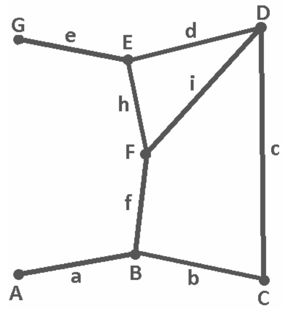
10) *(2 pts.)* Use Dijkstra's algorithm to find minimal routes from A to all other vertices.



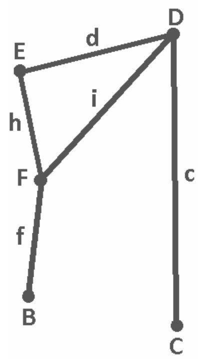
11) *(2 pts.)* Use Kruskal's algorithm to find a spanning tree with minimum total weight (an optimal connector of the towns).



12) *(3 pts.)* **Use the recursive formula** to compute τ (# of spanning trees) of this graph (passages not shown here, but in test you are supposed to show them): 11



13) *(4 pts.)* Use logic operations to find all minimal coverings and all maximal independent sets of this graph (please show all passages).

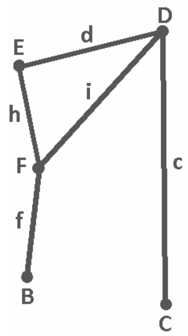


(C+D)(D+CEF)(E+DF)(F+BDE)(B+F) = (CD+CCEF+DD+DCEF)( ) =
= (CEF+D)(E+DF)( ) = (CEFE+CEFDF+DE+DDF)( ) = (CEF+DE+DF)(F+BDE)( ) =
= (CEFF+CEFBDE+DEF+DEBDE+DFF+DFBDE)( ) =
= (CEF+BDE+DF)(B+F) = CEFB+CEFF+BDEB+BDEF+DFB+DFF =
= CEF+BDE+DF

Minimal coverings: {C,E,F}, {B,D,E}, {D,F}

Maximal independent sets: {B,D}, {C,F}, {B,C,E}

14) *(4 pts.)* Compute the chromatic polynomial of this graph (passages not shown here, but in test you are supposed to show them).



$k^5-5k^4+9k^3-7k^2+2k$