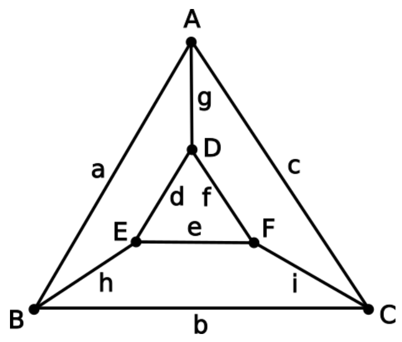
Mathematical Methods – 20 December 2022 – Graph Theory
UniBo matriculation number:
(no name, please)



Let G be the graph drawn here:

1) *(1 pt.)* Adjacency matrix:

	A	B	C	D	E	F
A	0	1	1	1	0	0
B	1	0	1	0	1	0
C	1	1	0	0	0	1
D	1	0	0	0	1	1
E	0	1	0	1	0	1
F	0	0	1	1	1	0

2) *(1 pt.)* Incidence matrix:

	a	b	c	d	e	f	g	h	i
A	1	0	1	0	0	0	1	0	0
B	1	1	0	0	0	0	0	1	0
C	0	1	1	0	0	0	0	0	1
D	0	0	0	1	0	1	1	0	0
E	0	0	0	1	1	0	0	1	0
F	0	0	0	0	1	1	0	0	1

3) *(1 pt.)* Minimum degree δ = **3** Maximum degree Δ = **3**

4) *(1 pt.)* Connectivity κ = **3** Edge-connectivity κ' = **3**

5) *(1 pt.)* Is G bipartite? Why? (If answer is "yes", list the two vertex sets of the bipartition)
No: It has odd cycles.

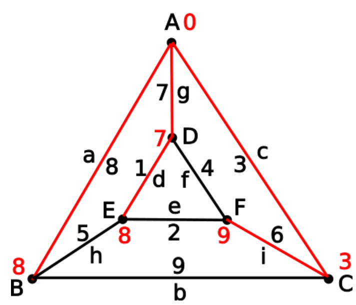
6) *(1 pt.)* Does G have an Euler tour? Why? (If answer is "yes", write the edge sequence of one)
No: It has vertices with odd degree.

7) *(1 pt.)* Does G have an Euler trail with distinct origin and terminus? Why? (If answer is "yes", write the edge sequence of one)
No: It has more than two vertices with odd degree.

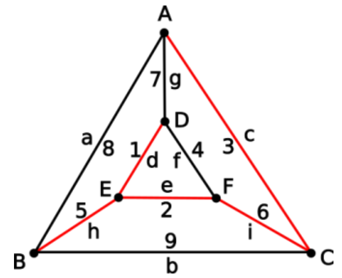
8) *(1 pt.)* Does G have a Hamilton path? (If answer is "yes", write the vertex sequence of one)
Yes: ABCFDE.

9) *(1 pt.)* List the edge set of a maximum matching. Is it a perfect matching?
ghi. Yes.

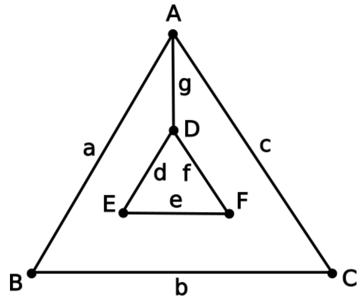
Now the vertices represent towns and the edge weights represent distances.
10) *(2 pts.)* Use Dijkstra's algorithm to find minimal routes from A to all other vertices.



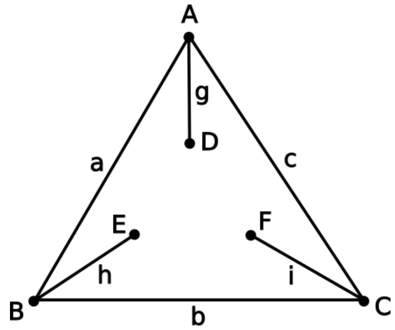
11) *(2 pts.)* Use Kruskal's algorithm to find a spanning tree with minimum total weight (an optimal connector of the towns).



12) *(3 pts.)* **Use the recursive formula** to compute τ (# of spanning trees) of this graph (passages not shown here, but in test you are supposed to show them): 9



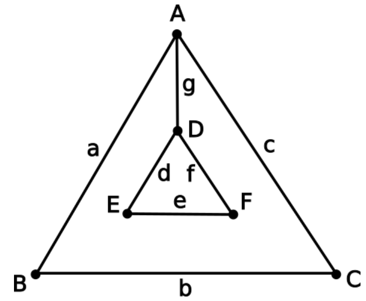
13) *(4 pts.)* Use logic operations to find all minimal coverings and all maximal independent sets of this graph.



(A+BCD)(B+ACE)(C+ABF)(D+A)(E+B)(F+C) =
= (AB+AACE+BCDB+BCDACE)( ) = (AB+ACE+BCD)(C+ABF)( ) =
= (ABC+ABABF+ACEC+ACEABF+BCDC+BCDABF)( ) =
= (ABC+ABF+ACE+BCD)(D+A)( ) =
= (ABCD+ABCA+ABFD+ABFA+ACED+ACEA+BCDD+BCDA)( ) =
= (ABC+ABF+ACE+BCD)(E+B)( ) =
= (ABCE+ABCB+ABFE+ABFB+ACEE+ACEB+BCDE+BCDB)( ) =
= (ABC+ABF+ACE+BCD)(F+C) = ABCF+ABCC+ABFF+ABFC+ACEF+ACEC+BCDE+BCDC =
= ABC+ABF+ACE+BCD

Minimal coverings : {A,B,C}, {A,B,F}, {A,C,E}, {B,C,D}
Maximal independent sets : {D,E,F}, {C,D,E}, {B,D,F}, {A,E,F}

14) *(4 pts.)* Compute the chromatic polynomial of this graph (passages not shown here, but in test you are supposed to show them).



$\mathbf{k^6-7k^5+19k^4-25k^3+16k^2-4k}$