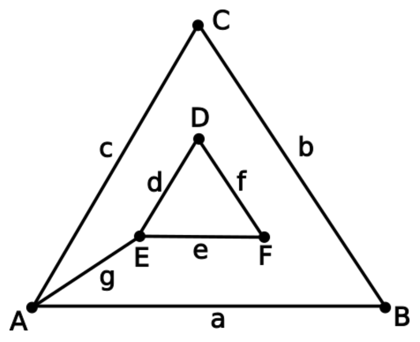
Mathematical Methods – 15 December 2023 – Graph Theory
UniBo matriculation number:
(no name, please)

Let G be the graph drawn here:



1) *(1 pt.)* Adjacency matrix:

	A	B	C	D	E	F
A	0	1	1	0	1	0
B	1	0	1	0	0	0
C	1	1	0	0	0	0
D	0	0	0	0	1	1
E	1	0	0	1	0	1
F	0	0	0	1	1	0

2) *(1 pt.)* Incidence matrix:

	a	b	c	d	e	f	g
A	1	0	1	0	0	0	1
B	1	1	0	0	0	0	0
C	0	1	1	0	0	0	0
D	0	0	0	1	0	1	0
E	0	0	0	1	1	0	1
F	0	0	0	0	1	1	0

3) *(1 pt.)* Minimum degree $\delta =$ **2** Maximum degree $\Delta =$ **3**

4) *(1 pt.)* Connectivity $\kappa =$ **1** Edge-connectivity $\kappa' =$ **1**

5) *(1 pt.)* Is G bipartite? Why? (If answer is "yes", list the two vertex sets of the bipartition)
No: It has odd cycles.

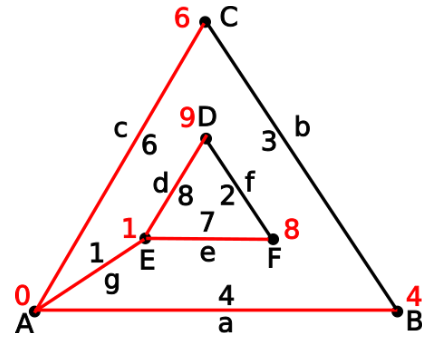
6) *(1 pt.)* Does G have an Euler tour? Why? (If answer is "yes", write the edge sequence of one)
No: It has vertices with odd degree.

7) *(1 pt.)* Does G have an Euler trail with distinct origin and terminus? Why? (If answer is "yes", write the edge sequence of one)
Yes: it has exactly two vertices with odd degree. abcgefd

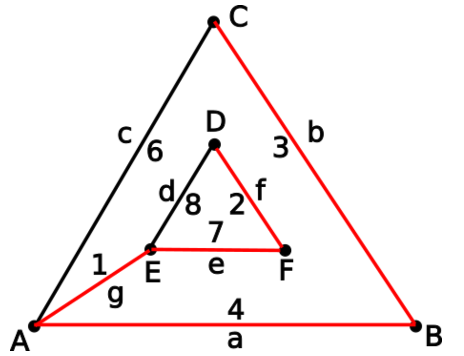
8) *(1 pt.)* Does G have a Hamilton path? (If answer is "yes", write the vertex sequence of one)
Yes. BCAEFD

9) *(1 pt.)* List the edge set of a maximum matching. Is it a perfect matching?
bfg. Yes.

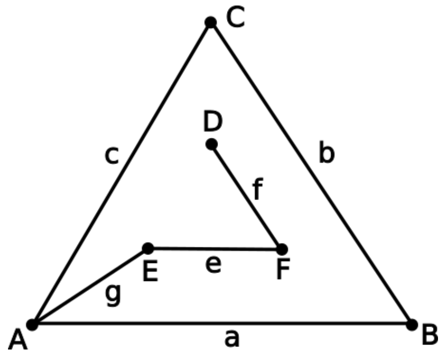
Now the vertices represent towns and the edge weights represent distances.
10) *(2 pts.)* Use Dijkstra's algorithm to find minimal routes from A to all other vertices.

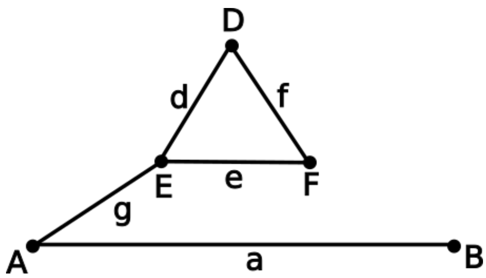


11) *(2 pts.)* Use Kruskal's algorithm to find a spanning tree with minimum total weight (an optimal connector of the towns).



12) *(3 pts.)* **Use the recursive formula** to compute τ (# of spanning trees) of this graph (passages not shown here, but in test you are supposed to show them): 3





13) *(4 pts.)* Use logic operations to find all minimal coverings and all maximal independent sets of this graph.

(A+BE)(B+A)(D+EF)(E+ADF)(F+DE) = (AB+AA+BEB+BEA)( ) =
= (A+BE)(D+EF)( ) = (AD+AEF+BED+BEEF)( ) =
= (AD+AEF+BED+BEF)(E+ADF)( ) =
= (ADE+ADADF+AEFE+AEFADF+BEDE+BEDADF+BEFE+BEFADF)( ) =
= (ADE+ADF+AEF+BED+BEF)(F+DE) =
= ADEF+ADEDE+ADFF+ADFDE+AEFF+AEFDE+BEDF+BEDDE+BEFF+BEFDE =
= ADE+ADF+AEF+BED+BEF

Minimal coverings : {A,D,E}, {A,D,F}, {A,E,F}, {B,D,E}, {B,E,F}
Maximal independent sets : {B,F}, {B,E}, {B,D }, {A,F}, {A,D}

14) *(4 pts.)* Compute the chromatic polynomial of this graph (passages not shown here, but in test you are supposed to show them).

$k^6-6k^5+14k^4-16k^3+9k^2-2k$