3) $z = \dfrac{4i}{i+\sqrt{3}} = \dfrac{4i\,(\sqrt{3}-i)}{(\sqrt{3}+i)(\sqrt{3}-i)} = \dfrac{4+4\sqrt{3}i}{4}$

$= 1 + i\sqrt{3} \quad \Rightarrow \quad \mathrm{Im}(z) = \sqrt{3}$

$w = \sqrt{2}\left(\cos\left(\frac{\pi}{5}\right) + i \sin\left(\frac{\pi}{5}\right)\right) \Rightarrow w^6 = (\sqrt{2})^6 \left(\cos\left(\frac{6\pi}{5}\right) + i \sin\left(\frac{6\pi}{5}\right)\right)$

Dunque $|w^6| = 2^3 = 8$

Oppure bastava dire che $|w| = \sqrt{2} \Rightarrow |w^6| = |w|^6 = 2^3 = 8$

4) $F(x) = \displaystyle\int_0^{x^3-x^2} \sqrt{1+t^4}\, dt$

$F'(x) = (3x^2 - 2x)\sqrt{1+(x^3-x^2)^4}$

$F'(x) \geq 0 \iff 3x^2 - 2x \geq 0 \iff x \leq 0 \ \text{o} \ x \geq \frac{2}{3}$

F cresce in $]-\infty, 0]$ e in $[\frac{2}{3}, +\infty[$

5) $\begin{cases} y' = t^2(1+2y) \\ y(-1) = 3 \end{cases}$ variabili separabili

$\displaystyle\int_3^{y(t)} \frac{du}{1+2u} = \int_{-1}^{t} s^2\, ds = \frac{t^3+1}{3}$

$\dfrac{t^3+1}{3} = \dfrac{1}{2}\log(1+2u)\Big|_{u=3}^{u=y} = \dfrac{1}{2}\left[\log(1+2y(t)) - \log(7)\right]$

$\Rightarrow \log\dfrac{1+2y(t)}{7} = \dfrac{2}{3}(t^3+1) \Rightarrow 1+2y(t) = 7\exp\left(\dfrac{2}{3}(t^3+1)\right)$

$\Rightarrow y(t) = \dfrac{1}{2}\left[7\exp\left(\dfrac{2}{3}(t^3+1)\right) - 1\right] = \dfrac{7}{2}e^{\frac{2}{3}(t^3+1)} - \dfrac{1}{2}$

— — —

$4y'' - 4y' + 5y = 0$ eq. caratteristica $4z^2 - 4z + 5 = 0 \to z = \frac{1}{2} \pm i$

Dunque la soluzione generale è

$y(t) = e^{t/2}(c_1 \cos t + c_2 \sin t)$

Dunque la soluzione generale è

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1) $\lim_{x\to+\infty} \frac{x^2 \arctan\left(\frac{1}{x^2}\right)}{3-e^{2/x}} = \frac{1}{2}\lim_{x\to+\infty} \frac{x^2 \arctan\left(\frac{1}{x^2}\right)}{1}$ ($x^2 \to +\infty$, $\arctan\left(\frac{1}{x^2}\right) \to 0$, $3-e^{2/x} \to 2$)

$= \frac{1}{2}\lim_{x\to+\infty} \frac{\arctan(1/x^2)}{x^{-2}} \overset{\text{Hop}}{=} \frac{1}{2}\lim \frac{\frac{1}{1+(1/x^2)^2}\cdot(-2x^{-3})}{-2x^{-3}} = \frac{1}{2}$

$\lim_{x\to 2} \frac{(x^2+x-6)\ln(1+x^2)}{x-\sqrt{2x}} = \log 5 \lim_{x\to 2} \frac{x^2+x-6}{x-\sqrt{2x}}$ $\left(\frac{0}{0}\right)$

$\overset{\text{Hop}}{=} \log 5 \lim_{x\to 2} \frac{2x+1}{1-\sqrt{2}\cdot\frac{1}{2\sqrt{x}}} = \log 5 \cdot \frac{5}{1-\frac{1}{2}} = 10\log(5)$

2) $\int_2^3 \frac{1+\ln x}{x}\,dx = \int_2^3 \frac{dx}{x} + \int_2^3 \frac{\ln x}{x}\,dx$

$= \left[\ln x + \frac{(\ln x)^2}{2}\right]_2^3 = \ln\frac{3}{2} + \frac{1}{2}\left((\ln 3)^2 - (\ln 2)^2\right)$

$\int_{-1}^0 x e^{-3x}\,dx = \left[-\frac{e^{-3x}}{3}x\right]_{-1}^0 + \int_{-1}^0 \frac{e^{-3x}}{3}\,dx$

$= \left[e^{-3x}\left(-\frac{x}{3}-\frac{1}{9}\right)\right]_{-1}^0 = -\frac{1}{9} + e^3\left(-\frac{1}{3}+\frac{1}{9}\right) = -\frac{1}{9} - \frac{2}{9}e^3$

3) $z = \frac{8i}{1+i\sqrt{3}} = \frac{8i(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})} = \frac{8i+8\sqrt{3}}{1+3} =$

$= 2i + 2\sqrt{3}$ Donque $\mathrm{Im}(z) = 2$

$\mathrm{Re}\, z = 2\sqrt{3}$

$w = \sqrt{3}\left(\cos\left(\frac{2}{5}\pi\right) + i\sin\left(\frac{2}{5}\pi\right)\right)$

$\Rightarrow |w| = \sqrt{3} \Rightarrow |w^6| = |w|^6 = \sqrt{3}^6 = 27$

(1) $\lim_{x\to+\infty} \dfrac{x^3 \arctan\left(\frac{1}{x^2}\right)}{1-2e^{2/x}} = -\lim_{x\to+\infty} x^3 \arctan\left(\frac{1}{x^2}\right) =$

($1-2e^{2/x} \to -1$; $x^3 \to +\infty$, $\arctan\left(\frac{1}{x^2}\right) \to 0$)

$$= -\lim_{x\to+\infty} \frac{\arctan\left(\frac{1}{x^2}\right)}{x^{-3}} \overset{\text{Hop}}{=} -\lim_{x\to+\infty} \frac{\frac{1}{1+\left(\frac{1}{x^2}\right)^2}\cdot(-2x^{-3})}{-3x^{-4}} = -\infty$$

$$\lim_{x\to 1} \frac{(2x^2-x-1)\ln(1+3x^2)}{\sqrt{x}-x} = \frac{0}{0} = \text{Hop}$$

($\ln(1+3x^2) \to \log 4$)

$$= \log 4 \lim_{x\to 1} \frac{2x^2-x-1}{\sqrt{x}-x} \overset{\text{Hop}}{=} \log 4 \lim_{x\to 1} \frac{4x-1}{\frac{1}{2\sqrt{x}}-1}$$

$$= \log 4 \cdot \frac{3}{-\frac{1}{2}} = -6\log 4$$

(2) $\displaystyle\int_2^3 \frac{1+\ln x}{x\ln x}\,dx = \int_2^3 \frac{dx}{x\ln x} + \int_2^3 \frac{dx}{x} = \left[\log(\log x) + \log x\right]$

Sia x che $\log x$ sono positivi in $[2,3]$

$$= \log\left(\frac{\log 3}{\log 2}\right) - \log\frac{3}{2}$$

4) $F(x) = \int_0^{x^3+x^2} \sqrt{1+t^4}\, dt$

$F'(x) = (3x^2+2x)\sqrt{1+(x^3+x^2)^4} \geq 0 \iff 3x^2+2x \geq 0$

$\iff x \leq -\frac{2}{3}$ o $x \geq 0$. Dunque F è crescente negli intervalli $]-\infty, -\frac{2}{3}]$ e $[0, +\infty[$

5) $y' = t^2(3+2y)$, $y(-1) = 2$ variabili separabili

$$\int_2^{y(t)} \frac{du}{3+2u} = \int_{-1}^{t} s^2\, ds = \frac{t^3+1}{3}$$

$$\frac{1}{2}\log(3+2u)\Big|_{u=2}^{u=y(t)} = \frac{1}{2}\log\left(\frac{3+2y(t)}{7}\right)$$

Dunque

$$\log\frac{3+2y(t)}{7} = \frac{2}{3}(t^3+1) \Rightarrow y(t) = \frac{1}{2}\left[7e^{\frac{2}{3}(t^3+1)} - 3\right]$$

$$= \frac{7}{2}e^{\frac{2}{3}(t^3+1)} - \frac{3}{2}$$

$9y'' - 6y' + 10y = 0$ Eq. caratteristica $9z^2 - 6z + 10 = 0$

Dunque $z = \frac{1}{3} \pm i$

Pertanto la sol.t. generale è $y(t) = e^{t/3}(c_1\cos(t) + c_2 \sin$