Introduction to Mathematical General Relativity

The goal of this two weeks course is to introduce, from a mathematical vantage point, the many interesting problems in geometric analysis (differential geometry and partial differential equations) related to Einstein General Relativity. In particular we will discuss the basic features and properties of the space of solutions of the Einstein equations, with emphasis on the Einstein constraints. These latter form a system of non-linear , undetermined-elliptic partial differential equations with a richly structured space of solutions. The study of this system of PDEs is deeply connected to the geometry of scalar curvature on Riemannian manifolds , and gives rise to a very constructive synergy between geometric analysis and general relativity .

Week One. (Monday 14 – Saturday 19 September): **General Relativity and Einstein Equations.** Lecturer: *Mauro Carfora*

The first set of six lectures will introduce the necessary background from General Relativity and geometric analysis according to the following tentative plan:

- Survey of the Riemannian and Lorentzian differential geometry: a geometric analysis point of view.
- Introduction to General Relativity and Einstein's equations.
- Schwarzschild spacetime and black holes.
- 3+1 splitting of spacetime and the local Cauchy problem of Einstein's equations (ADM formalism)
- The constraint equations and the evolution equations.

Week Two (Monday 21 – Saturday 26 September): **Geometric Analysis and the Einstein Constraint Equations.** Lecturer: *Justin Corvino*

The second set of six lectures will survey the geometry of solutions to the Einstein Constraint Equations, emphasizing the role of the scalar curvature. We plan to discuss the following topics, in varying depth as time permits:

- Solving the constraints I: The basic formulation of the conformal method.
- Modeling isolated systems: asymptotically flat solutions of the constraint equations.
- Scalar curvature and the Positive Energy (Mass) Theorem.
- Solving the constraints II: Building solutions to the constraints by gluing techniques.