Corso di Analisi Matematica T-B
Corso di Laurea in Ingegneria Gestionale
Anno Accademico 2013/14

Esercizi

A) Stabilire se i seguenti integrali generalizzati sono convergenti.

1. $\displaystyle\int_0^1 \frac{x}{1-x}\,dx$

11. $\displaystyle\int_1^{+\infty} \frac{\sqrt{x^2+2x}-\sqrt{x^2+2}}{(x-1)^{5/4}}\,dx$

2. $\displaystyle\int_0^2 \log\frac{x}{x+1}\,dx$

12. $\displaystyle\int_9^{+\infty} \frac{(\sqrt{x}-3)\,x\log x}{(x^2-9x)^{3/2}}\,dx$

3. $\displaystyle\int_0^{+\infty} \frac{x}{e^x}\,dx$

13. $\displaystyle\int_0^{+\infty} \frac{1}{e^{x+\frac{1}{x}}-e^x}\,dx$

4. $\displaystyle\int_1^{+\infty} \left(\exp\left(\frac{\sqrt{x}}{x^2+1}\right)-1\right)dx$

14. $\displaystyle\int_{-\infty}^{+\infty} \frac{e^x+x^4}{\cosh(2x)}\,dx$

5. $\displaystyle\int_2^{+\infty} x\log\frac{x+2}{x+3}\,dx$

15. $\displaystyle\int_{-\infty}^{+\infty} \frac{e^x+x^4}{\cosh x}\,dx$

6. $\displaystyle\int_2^{+\infty} \frac{\sqrt{4x-8}}{x\log(x/2)}\,dx$

16. $\displaystyle\int_{-\infty}^{+\infty} \frac{x^4\cosh x}{\cosh 2x}\,dx$

7. $\displaystyle\int_0^3 \frac{1}{x^{9/4}}\left(\sqrt{x^4+x^2}-x^2\right)dx$

17. $\displaystyle\int_0^{+\infty} \frac{(x+1)^3(e^x-2\sinh x)}{\sqrt{x}}\,dx$

8. $\displaystyle\int_1^3 \frac{1}{\log x}\left(\frac{x-1}{3-x}\right)^{2/3}dx$

18. $\displaystyle\int_1^{+\infty} \frac{1}{(x^2-1)\sqrt{\log x}}\,dx$

9. $\displaystyle\int_0^1 \frac{\sqrt{-\log x}}{1-\sqrt{x}}\,dx$

19. $\displaystyle\int_0^{+\infty} \frac{\log(x+1)}{x^2+2x^3}\,dx$

10. $\displaystyle\int_0^{+\infty} \frac{\sqrt{e^x-1}}{\sinh x}\,dx$

20. $\displaystyle\int_0^{+\infty} \frac{\log(1+x)}{\sqrt{x}\,\arctan x}\,dx$

B) Determinare per quali $a\in\mathbb{R}$ i seguenti integrali generalizzati sono convergenti.

1. $\displaystyle\int_0^{+\infty} \left(\exp\left(\frac{ax^2}{x^2+1}\right)-e\right)dx$

4. $\displaystyle\int_1^{+\infty} x^a\sin\left(\frac{1}{x^5}\right)\arctan(1+x^a)\,dx$

2. $\displaystyle\int_0^2 \frac{x^a}{x^{a+2}+x^3}\,dx$

5. $\displaystyle\int_1^{+\infty} \frac{x^2}{1+e^{ax}}\,dx$

3. $\displaystyle\int_1^{+\infty} \frac{\log\big(4\cosh(ax)\big)}{x^{-3a}+x^{4a}}\,dx$

6. $\displaystyle\int_0^1 \big(e^{(1+x)^{4a}}-e\big)\big(x^{6a}+x^{-3a}\big)\,dx$

C) Determinare per quali $a > 0$ i seguenti integrali generalizzati sono convergenti.

1. $\displaystyle\int_{1}^{+\infty} \frac{x^{-a}}{\sqrt{1 + ax^2} - 1}\, dx$

5. $\displaystyle\int_{0}^{1} \frac{\sin(x^{3a})}{x^{9a} + x^3}\, dx$

2. $\displaystyle\int_{1}^{+\infty} \frac{1}{x^a + x^{1/a}}\, dx$

6. $\displaystyle\int_{0}^{1} \frac{\log(1 + x^2)}{\sqrt{x^a + x^{14-a}}}\, dx$

3. $\displaystyle\int_{1}^{+\infty} \frac{\sqrt{x^{4a} + 1}}{x^{5a} + x^{3a}}\, dx$

7. $\displaystyle\int_{0}^{1} \frac{\left|\log(1 - x)\right|^{a+1}}{(x^2 - x^3)^a}\, dx$

4. $\displaystyle\int_{0}^{2} \frac{x^a + x^{2a}}{x^{4-a} + x^{4-2a}}\, dx$

8. $\displaystyle\int_{0}^{+\infty} \frac{1}{x^{2a+3} + x^{3a}} \log\frac{1 + 2x^9}{1 + x^9}\, dx$

D) Determinare per quali $a \neq 0$ i seguenti integrali generalizzati sono convergenti.

1. $\displaystyle\int_{0}^{1} \frac{1}{(1 + 2x)^a - (1 + x)^a}\, dx$

2. $\displaystyle\int_{0}^{+\infty} \frac{1}{(2 + x)^a - (1 + x)^a}\, dx$

Soluzioni

A)

1. Non convergente

2. Convergente

3. Convergente

4. Convergente

5. Non convergente

6. Non convergente

7. Non convergente

8. Convergente

9. Convergente

10. Convergente

11. Convergente

12. Convergente

13. Convergente

14. Convergente

15. Non convergente

16. Convergente

17. Convergente

18. Non convergente

19. Non convergente

20. Non convergente

B)

1. $a = 1$

2. $a \in \,]2, +\infty[$

3. $a \in \left] -\infty, -\dfrac{2}{3} \right[\cup \left] \dfrac{1}{2}, +\infty \right[$

4. $a \in \,]-\infty, 4[$

5. $a \in \,]0, +\infty[$

6. $a \in \left] -\dfrac{1}{3}, \dfrac{2}{3} \right[$

C)

1. $a \in \,]0, +\infty[$

2. $a \in \,]0, +\infty[\setminus \{1\}$

3. $a \in \left] \dfrac{1}{3}, +\infty \right[$

4. $a \in \,]0, 1[$

5. $a \in \left] 0, \dfrac{1}{6} \right[\cup \left] \dfrac{2}{3}, +\infty \right[$

6. $a \in \,]0, 6[\cup \,]8, +\infty[$

7. $a \in \,]0, 1[$

8. $a \in \left] 0, \dfrac{7}{2} \right[$

D)

1. Nessun a

2. $a \in \,]2, +\infty[$