Corso di Analisi Matematica T-B
Corso di Laurea in Ingegneria Meccanica
Anno Accademico 2012/13

Esercizi

A) Calcolare $\displaystyle\int_A f(x,y)\,dx\,dy$ con:

1. $A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; x \geq 0, y \geq -1, x+y \leq 2 \right\}$, $\quad f(x,y) = x^2y + 3y^2$

2. $A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; y \geq 0, x \leq y, x-2y \geq -1 \right\}$, $\quad f(x,y) = \sin(x+y)$

3. $A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; x^4 + y^4 \leq 16, x \geq 0 \right\}$, $\quad f(x,y) = x^3$

4. $A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; 9x^2 + y^2 \leq 16, 3x^2 + y \leq 4 \right\}$, $\quad f(x,y) = y$

5. $A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; x+y \leq 1, y-2x \leq 1, x-2y \leq 1 \right\}$, $\quad f(x,y) = 1$

6. $A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; y \geq 3x^2, x+y \leq 2 \right\}$, $\quad f(x,y) = x + 2y$

7. $A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; x^2 + y^2 \leq 4, x+y \leq 2 \right\}$, $\quad f(x,y) = 5x + y$

8. $A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; x^2 + 4y^2 \leq 20, 3x-2y \leq 10 \right\}$, $\quad f(x,y) = x$

9. $A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; x^2 + 4y^2 \leq 5, y \leq x^2 \right\}$, $\quad f(x,y) = y$

10. $A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; x^2 + 4y^2 \leq 9, x \geq 0, y \geq 0 \right\}$, $\quad f(x,y) = x + y$

11. $A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; x^2 + y^2 \leq 4, x \leq |y| \right\}$, $\quad f(x,y) = \sqrt{x^2 + y^2 + 4}$

12. $A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; 1 \leq x^2 + y^2 \leq 4, x - y \leq 0 \right\}$, $\quad f(x,y) = \dfrac{1}{x^2 + y^2 + 2}$

13. $A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; 9x^2 + 4y^2 \leq 1, x + y \geq 0 \right\}$, $\quad f(x,y) = x^2\sqrt{9x^2 + 4y^2}$

14. $A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; x^2 + y^2 + 4y \leq 0 \right\}$, $\quad f(x,y) = y$

15. $A = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; (x-1)^2 + (y-1)^2 \leq 4, 3x + y \leq 4 \right\}$, $\quad f(x,y) = x + y$

Soluzioni

A)

1. $\displaystyle\int_0^3 \int_{-1}^{2-x} (x^2y + 3y^2)\,dy\,dx = \frac{81}{20}$

2. $\displaystyle\int_0^1 \int_{2y-1}^{y} \sin(x+y)\,dx\,dy = -\frac{1}{6}\sin 2 + \frac{1}{3}\sin 1$

3. $\displaystyle\int_{-2}^2 \int_0^{\sqrt[4]{16-y^4}} x^3\,dx\,dy = \frac{64}{5}$

4. $\displaystyle\int_{-\sqrt{5/3}}^{\sqrt{5/3}} \int_{-\sqrt{16-9x^2}}^{4-3x^2} y\,dy\,dx = -\frac{10}{3}\sqrt{\frac{5}{3}}$

5. $\displaystyle\int_{-1}^0 \int_{(x-1)/2}^{1+2x} 1\,dy\,dx + \int_0^1 \int_{(x-1)/2}^{1-x} 1\,dy\,dx = \frac{3}{2}$

6. $\displaystyle\int_{-1}^{2/3} \int_{3x^2}^{2-x} (x+2y)\,dy\,dx = \frac{625}{108}$

7. $\displaystyle\int_{-2}^0 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (5x+y)\,dy\,dx + \int_0^2 \int_{-\sqrt{4-x^2}}^{2-x} (5x+y)\,dy\,dx = -8$

8. $\displaystyle\int_{-2\sqrt{5}}^2 \int_{-\sqrt{20-x^2}/2}^{\sqrt{20-x^2}/2} x\,dy\,dx + \int_2^4 \int_{(3x-10)/2}^{\sqrt{20-x^2}/2} x\,dy\,dx = -10$

9. $\displaystyle\int_{-\sqrt{5}}^{-1} \int_{-\sqrt{\frac{5-x^2}{4}}}^{\sqrt{\frac{5-x^2}{4}}} y\,dy\,dx + \int_{-1}^1 \int_{-\sqrt{\frac{5-x^2}{4}}}^{x^2} y\,dy\,dx + \int_1^{\sqrt{5}} \int_{-\sqrt{\frac{5-x^2}{4}}}^{\sqrt{\frac{5-x^2}{4}}} y\,dy\,dx = -\frac{29}{30}$

10. $\displaystyle\int_0^3 \int_0^{\pi/2} \frac{\rho^2}{2}\Big(\cos\theta + \frac{1}{2}\sin\theta\Big)\,d\theta\,d\rho = \frac{27}{4}$

11. $\displaystyle\int_0^2 \int_{\pi/4}^{7\pi/4} \rho\sqrt{\rho^2+4}\,d\theta\,d\rho = \big(8\sqrt{2}-4\big)\pi$

12. $\displaystyle\int_1^2 \int_{\pi/4}^{5\pi/4} \frac{\rho}{\rho^2+2}\,d\theta\,d\rho = \frac{\log 2}{2}\,\pi$

13. $\displaystyle\int_0^1 \int_{-\arctan(2/3)}^{\pi-\arctan(2/3)} \frac{1}{54}\,\rho^4\cos^2\theta\,d\theta\,d\rho = \frac{1}{540}\,\pi$

14. $\displaystyle\int_0^2 \int_0^{2\pi} \rho(\rho\sin\theta - 2)\,d\theta\,d\rho = -8\pi$

15. $\displaystyle\int_0^1 \int_{\pi-\arctan 3}^{2\pi-\arctan 3} \rho(\rho\cos\theta + \rho\sin\theta + 2)\,d\theta\,d\rho = -\frac{2}{3}\big(\cos(\arctan 3) + \sin(\arctan 3)\big) + \pi$