A Diffusive Strategy in Group Competition

E. Agliari

Dipartimento di Fisica, Università degli Studi di Parma,
viale Usberti 7/A, 43100 Parma, Italy and
Theoretische Polymerphysik, Universität Freiburg,
*Hermann-Herder-Str. 3, D-79104 Freiburg, Germany**

R. Burioni

Dipartimento di Fisica, Università degli Studi di Parma,
viale Usberti 7/A, 43100 Parma, Italy and
INFN, Gruppo collegato di Parma, viale Usberti 7/A, 43100 Parma, Italy

P. Contucci

Dipartimento di Matematica, Università di Bologna, 40127, Bologna, Italy

Abstract

We propose and study a model for the dynamics of two groups in competition, which includes diffusive effects and strategic choices. This model is able to mimic some phenomena taking place during marketing or political campaigns. Using a statistical mechanics approach on the simplest random interaction environment (Erdös-Renyi graph), we find, by numerical simulations, that a well defined stationary state is reached and we compare the final state to the one obtained with standard dynamics by means of total magnetization and magnetic susceptibility. Our results show that the diffusive strategic dynamics has a critical interaction parameter strictly lower than the standard one.

*Electronic address: elena.agliari@fis.unipr.it

I. INTRODUCTION

In the past few years the application of statistical mechanics to social phenomena gave rise to interesting models, which where able to capture some general mechanism in opinion forming [1–5]. In these models, the relations between people in a group are represented by a network with a given topology, where sites are people, links model interactions and the opinion of a single agent is typically represented by a discrete variable on the corresponding site.

One of the most important aspects in opinion forming within a community is the dynamics through which information, able to influence opinion and to orient the community, propagates throughout the network. In general, one can build a cost function for the configurations with some parameters measuring the interaction between people in the community. When one expects that the final state reached by the system will obey an equilibrium Boltzmann distribution based on this cost function, the dynamics is usually implemented according to a standard Monte Carlo algorithm, with sequential or random updating, characterized by a detailed balance condition, which is quite unrealistic in real social communities.

As a matter of fact, the spreading of information influencing opinion in social networks often exhibits a "strategic" character: agents propagating the information can decide to chose the best "move" according to a local strategy, based on the observation of what happens to their neighbours.

A common situation (typical of the so-called Majority Games [5]) is the following: People try to convince their neighbours in order to share the same opinion or trait because of ideological reasons or, also, because this translates in some economical advantage. To fix ideas let us suppose that the opinion or the trait considered can be described by a dichotomic variable. For example, during an electoral campaign or before a referendum, people try to convince their acquaintances to support a given candidate or a given position. In a different context we can think about a community of people where each agent has made a subscription to a phone company. Let us suppose that only two different companies, A and B, exist so that we can distinguish between A-users and B-users. Now, fares for phone-calls are different according to whether the call occurs between customers of the same company or between customers of different companies, being higher in the latter case. As a result, for an A-user (B-user) the optimal situation is when all his acquaintances are also A-users (B-users) as he can then enjoy low fares. Hence, each agent would like to induce his neighbours to adopt the same company.

In both situations cited above, strategies are possible if an agent knows the neighbourhood of his acquaintances.

As an example, if I want to increase the number of, say, A-users, among my acquaintances, I can either pick up a friend of mine randomly among those who are B-users and try to convince him to become an A-user, or select among my friends the B-user whose acquaintances are mostly A-users. The latter strategy is of course expected to be more effective.

Another important aspect in dynamics is that it must contain a stochastic character, reproducing the randomness typical of social interactions. It is not reasonable that phone calls, mail exchanges and other contacts among agents occur according to a deterministic rule. More likely, they can exhibit a diffusive feature which must be captured by the dynamics.

In this paper, we will study the equilibrium reached by the system endowed with a dynamics which takes into account these two aspects: strategy and diffusive character. As we intend to focus on the dynamical process, we will model the social network by a Erdös-Renyi random graph. This graph provides a stochastic network, able to capture some aspects of a real community, and allowing for some exact calculations [6–8], although it does not take into account some well known topological aspects of real social networks [9–12]. Interestingly, the results we obtained appear to be robust, as they hold also on more general finite dimensional structures [13, 14] and on scale free graphs [15]. Moreover, we adopt as a cost function the ferromagnetic Ising Hamitonian which, being one of the simplest model mimicking interactions amongst the agents of social systems, allows us to focus on the dynamical process. In particular, the interaction parameter J here represents the "imitation strength" and it measures how important it is for two nearest-neighbours to agree. For instance, in the phone companies example, a large value of J corresponds to a situation where a phone call between A-A or B-B users is much cheaper than a phone call between A-B users.

We find that after a suitable time the values of the global observables of the system display a well-defined value independently of the initial conditions, indicating that a stationary situation is reached. We also recover the phase diagram expected for the Ising model on the Erdös-Renyi random graph, and this implies that there exists an interaction parameter J_c

such that if $J < J_c$ the number of A-users equals, on the average, the number of B-users, while if $J > J_c$ a prevailing group is formed. However, we evidence a remarkable difference: With respect to the case of a non-strategic dynamics, the critical region is shifted to a lower value of the interaction parameter J. In the example of the competition between the two phone companies, this means that, once the price policy has been set by the companies, if the equilibrium is reached by a strategic dynamics, the extent of the prevailing community is larger.

The paper is organized as follows: in Sec. II we present our model and we describe the topology it is embedded in. Then, Sec. III is devoted to the description of the strategic diffusive dynamics introduced and in Sec. IV we show our results. Finally, Sec. V is left for conclusions and outlook.

II. MODEL AND NOTATIONS

A social network is meant as a (typically large) set of people or groups of people, also called "agents", with some pattern of interactions between them. This can be efficiently envisaged by means of a graph whose nodes represent agents and links between two of them represent the existence of a relationship (which could be acquaintanceship, friendship, etc.). Therefore, each agent i is connected with a set of "nearest-neighbours", whose number α_i is referred to as the "degree" of the node i.

Now, several kinds of graphs have been proposed in the past as model able to mimic the features displayed by a real population. Among these the random graph introduced by Erdös and Rényi (ER) [16] is one of the most studied since it combines a stochastic character with an easy definition which allows to calculate exactly many interesting quantities [8].

The ER random graph can be defined as follows: given a number N of nodes, we introduce connections between them in such a way that each pair of vertices i, j has a connecting link with independent probability p. Moreover, we can calculate the probability p_k that a node in a random graph has degree exactly equal to k; this is given by the binomial distribution

$$p_k = \binom{N-1}{k} p^k (1-p)^{N-1-k}, \tag{1}$$

which, if p is chosen to be $p = \alpha/(N-1)$, can be rewritten as

$$p_k = \binom{N-1}{k} \left[\frac{\alpha}{N-1-\alpha}\right]^k \left[1 - \frac{\alpha}{N-1}\right]^{N-1} \approx \frac{\alpha^k}{k!} e^{-\alpha}, \tag{2}$$

where the last approximate equality becomes exact in the limit of large N. Notice that the distribution appearing in the r.h.s. is the Poisson distribution, i.e. a large random graph has a Poisson degree distribution, with average α.

We now outline a general framework for modeling interactions among agents. First of all, we associate to each agent i a binary variable $s_i = \pm 1$, representing the two possible forms of the considered opinion or trait. For example, $s_i = +1$ might indicate that the i-th agent does support the current government or is an A-user, while $s_i = -1$ that he does not support the current government or that he is a B-user. The whole community, described by the set $\{s\}$, will therefore be characterized by the mean value

$$m = \frac{1}{N} \sum_k s_k \tag{3}$$

which can be measured by, say, a referendum vote or a survey.

We assume that agents do not possess any a priori bias towards $+1$ or -1 state, but they move towards a given trait as a result of the interaction with their nearest-neighbours. More precisely, we introduce a cost function H_{ik} which quantifies the cost for individual i to agree with individual k as [17]

$$H_{ik}(s_i, s_k) = -J_{ik} s_i s_k, \tag{4}$$

where J_{ik} represents the strength of interaction between agents i and k. When i and k agree ($s_i s_k = 1$) we have a cost $H_{ik} = -J_{ik}$, while when they disagree ($s_i s_k = -1$) we have $H_{ik} = J_{ik}$. Thus, the interaction works in such a way that, when $J_{ik} > 0$, then i and k tend to imitate each others assuming the same trait and vice versa when $J_{ik} < 0$. The magnitude of J_{ik} gives how important it is for i to agree or disagree with k.

For the whole population we have the total cost function

$$H(\mathbf{s}, \mathbf{J}) = \sum_{k \sim i} H_{ik} = -\sum_{k \sim i} J_{ik} s_i s_k, \tag{5}$$

where the sum is extended over all the couples of nearest-neighbour agents denoted as $k \sim i$.

The cost function of Eq. 5 is just the well-known Ising Hamiltonian (see e.g. [18]) and it is treated by statistical tools. It must be underlined that the cost function $H(\mathbf{s}, \mathbf{J})$ does not lead to any natural dynamics and it is a very interesting matter of investigation to define a proper dynamic able to make the system evolve towards an "equilibrium" configuration $\{\tilde{s}(\mathbf{J})\}$. This can be achieved in several ways: Apart from exact analytical approaches, available only for special structure topologies (one dimensional and two dimensional lattices) and mean-field solutions, a number of approximate techniques have been developed, including series expansions, field theoretical methods and computational methods.

Here we adopt Monte Carlo (MC) numerical techniques in order to simulate the evolution of the system from a given initial configuration $\{s_0\}$ to the stationary state $\{\tilde{s}^D(\mathbf{J})\}$, which in general does depend on the evolutionary dynamics D we choose. From $\{\tilde{s}^D(\mathbf{J})\}$ we then calculate the average trait $m^D(\mathbf{J})$ which provides very interesting information about the overall behaviour of the population as a function of the parameters J_{ik}.

Notice that $\mathbf{J}$ can be chosen to be directed and group dependent and this may account, in the examples discussed in the introduction, for different influences and fares inter and intra different groups. For instance, if, say, the A company applies very high costs for phone calls between different users, we expect the pertaining interaction strengths to be very large. On the other hand, if for a B user costs for phone calls towards A users are only slightly more expensive then the relevant interaction strengths are small.

III. DIFFUSIVE STRATEGIC DYNAMICS

In order to simulate the evolution of the population described by the cost function in Eq. 4 several different algorithms have been introduced. Among them a well-established one is the so-called single-flip algorithm which makes the system evolve by means of successive opinion-flips, where we call "flip" on the node j the transformation $s_j \to s'_j = -s_j$.

More precisely, the algorithm is made up of two parts: first we need a rule according to which select an agent to be updated, then we need a probability distribution which states how likely the opinion-flip is.

As for the latter, we adopt the well-known Glauber probability: Given a configuration $\mathbf{s}$, then an opinion-flip on the j-th node is accepted with probability

$$p(\mathbf{s}, s'_j, \mathbf{J}) = \frac{1}{1 + e^{\Delta H(\mathbf{s}, s'_j, \mathbf{J})}}. \tag{6}$$

where $\Delta H(\mathbf{s}, s'_j, \mathbf{J}) = H(\mathbf{s}'_j, \mathbf{J}) - H(\mathbf{s}, \mathbf{J})$ is the variation in the cost function due to the flip $s_j \to s'_j$.

Notice that, for single-flip dynamics the cost variation ΔH consequent to an opinion-flip only depends on the opinion of a few agents, viz. the j-th one undergoing the flipping process and its α_j nearest-neighbours. This can be shown by spelling out the cost function variation appearing in Eq. (6):

$$H(\mathbf{s}'_j, \mathbf{J}) - H(\mathbf{s}, \mathbf{J}) = (s_j - s'_j) \sum_{i \sim j} J_{ij} s_i.$$

Interestingly, as can be derived from Eq. 6, each opinion-flip is the result of a stochastic process featuring a competition between an energetic and an entropic term: the lower the cost of the opinion-flip and the more likely its occurrence. The external parameter $\mathbf{J}$ tunes the probability for an energetically unfavourable event to happen: For very low values of $\mathbf{J}$ any event is equally likely to happen independently of the magnetic configuration, conversely, for high values of $\mathbf{J}$, when the agent j is surrounded by agents sharing the same opinion, the flipping of s_j gets a rare event.

As already recalled, the opinion-flip probabilities just described can determine a dynamics only after a prescription for updating the system has been introduced. In other words, we first need a selection rule according to which extract agents, then the opinion of the selected agent will be possibly updated according to $p(\mathbf{s}, s'_j, \mathbf{J})$. There exist several different choices for the first procedure, ranging from purely random to deterministic. For example, we can pick a single agent randomly or follow a particular, fixed sequence.

Now, the most popular algorithms select nodes to be updated according to a sequential order which, though computationally efficient, appears rather artificial in a social network. Indeed, unless no predetermined strategies are at work, the random updating ($D = \mathcal{R}$) seems to be the most plausible. In this case the probability that the current configuration $\mathbf{s}$ changes into $\mathbf{s}'_j$ due to the flip $s_j \to s'_j$, reads

$$\mathcal{P}^{\mathcal{R}}(\mathbf{s}, s'_j; \mathbf{J}) = \frac{1}{N} p(\mathbf{s}, s'_j, \mathbf{J}). \qquad (7)$$

The dynamics generated by $\mathcal{P}^{\mathcal{R}}$ has been intensively studied in the past (see e.g. [19]) and it has been shown to lead the system to the equilibrium (canonical) distribution.

Here we want to explore different relaxation dynamics ($D = \mathcal{S}$) featuring a realistic selection rule and a proper strategy. First of all, it is reasonable to assume that an opinion-flip occurs as a result of a direct interaction (phone call, mail exchange, etc.) between two neighbours and if the i-th agent has just undergone an opinion-flip he will, in turn, prompt one out of his α_i neighbours to change opinion. In this way, the selection rule exhibits a diffusive characters: the sequence of sites selected for the updating can be thought of as the the path of a random walk moving on the graph.

Now, the α_i neighbours are not equally likely to be chosen and it is just such a choice to determine the strategy. In fact, the arbitrary agent i aims to be surrounded by neighbours j sharing his own opinion (being this a cultural trait or a phone subscription), i.e. $s_i s_j = 1$, because this translates in some advantage for the agent i. Consequently, in our dynamics, amongst the α_i neighbours, the most likely to be chosen is also the most likely to undergo a spin-flip, namely the one which maximizes $\Delta H(\mathbf{s}, s_j', \mathbf{J})$. This corresponds to a local strategic choice of agent i, as it obtain the maximum effect from his move though keeping a stochastic character.

All this can be formalized as follows: Being i the newest updated agent, then the agent j is updated realizing the magnetic configuration $\mathbf{s}_j'$ according to the normalized probability:

$$\mathcal{P}^{\mathcal{S}}(\mathbf{s}, s_j'; i, j; \mathbf{J}) = \frac{p(\mathbf{s}, s_j', \mathbf{J})}{\sum_{j \in \mathcal{M}} p(\mathbf{s}, s_j'', \mathbf{J})}. \tag{8}$$

where $p(\mathbf{s}, s_j', \mathbf{J})$ (see Eq. 6), is the probability that the current configuration $\mathbf{s}$ changes due to a flip on the j-th site and $\mathcal{M}$ represents the set of i's nearest-neighbours who disagree with i itself ($s_j s_i = -1$); notice that the last condition can be relaxed (and the index j run through the α_i nearest-neighbours of the starting site i) without any qualitative change in the results.

Some remarks are in order now. First of all, according to Eq. 8 the configuration of the system can remain unchanged during a unit step. Moreover, by comparing Eq. 7 and Eq. 8, we notice that the latter depends on a larger set of variables. This point has some important consequences: the analytical approach to the master equation is extremely difficult and the *detailed balance condition* usually implemented in standard dynamics and leading to a standard Boltzmann distribution, is explicitly violated (see [14] for more details). This does not contradict our dynamic intent: it is not meant to recover a canonical Bolzmann equilibrium, but rather to model some possible strategies making the system evolve.

IV. NUMERICS

As mentioned before, the analysis of the diffusive dynamics has been carried out mainly from the numerical point of view by means of extensive Monte Carlo simulations [19]. Here we focus on the particular case of interaction parameters J_{ik} independent of the particular couple of agents considered, i.e. $J_{ik} \equiv J$, for any i, k; this allows to highlight the role of the dynamics leading the system to a stationary state and to understand how it possibly affects $\{\tilde{s}^D(J)\}$ and the average trait $m^D(\mathbf{J})$.

Now, before describing our results it is worth explaining how the Erdös-Renyi random graph is constructed. We consider a set of N sites and we introduce a bond between each pair of sites with probability $p = \alpha/(N-1)$, in such a way that the average coordination number per node is just α. Clearly, when $p = 1$ the complete graph is recovered.

In the simulation, once the network has been defined, we place a dichotomic variable s_i on each node i and allow it to interact with its nearest-neighbors. Once the external parameter J is fixed, the system is driven by the single-flip dynamics and it eventually relaxes to a stationary state characterized by well-defined properties. More precisely, after a suitable time lapse t_0 and for sufficiently large systems, measurements of a (specific) physical observable $x(\mathbf{s}, \alpha, J)$ fluctuate around an average value only depending on the external parameters J and α.

We also verified that, for a system (α, J) of a given finite size N, the extent of such fluctuations scales as $N^{-\frac{1}{2}}$ (see also [13, 14]), as indicated by standard statistical mechanics for a system in equilibrium. The estimate of the a given observable $\langle x \rangle$ is therefore obtained as an average over a suitable number of (uncorrelated) measurements performed when the system is reasonably close to the equilibrium regime. The estimate is further improved by averaging over different realizations of the underlying random graph with fixed α. In summary,

$$\langle x(\mathbf{s}, \alpha, J) \rangle = \mathbb{E}\left[\frac{1}{M}\sum_{n=1}^{M} x(\mathbf{s}(t_n))\right], \quad t_n = t_0 + n\mathcal{T}$$

where $\mathbf{s}(t)$ denotes the configuration of the system at time step t and $\mathcal{T}$ is the decorrelation parameter (i.e. the time, in units of spin flips, needed to decorrelate a given magnetic arrangement from the initial state); the symbol $\mathbb{E}$ denotes the average over different realizations of the graph.

In general, during a MC run in a given sample we find statistical errors which are significantly smaller than those arising from the ensemble averaging (see also [20]).

We stress once again that the final state obtained with the diffusive dynamics is stable, well-defined and, in particular, it does not depend on the initial conditions., i.e. it has all the

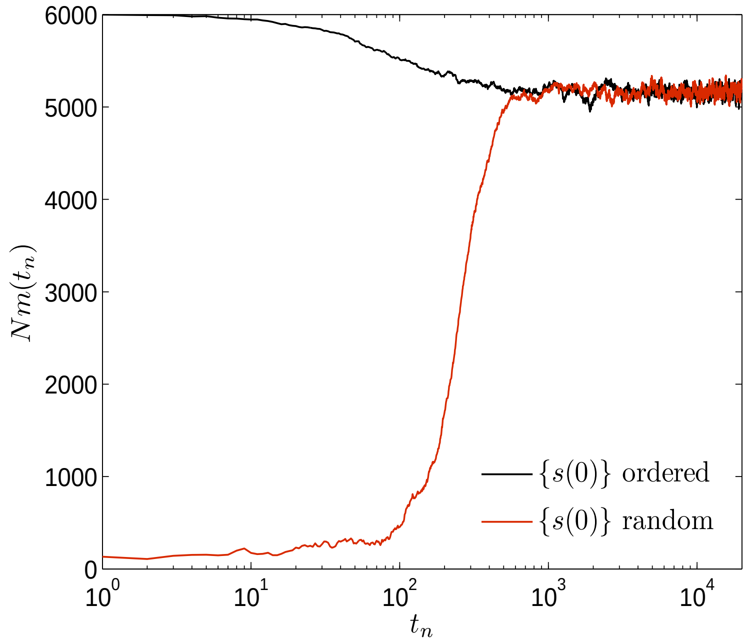

FIG. 1: Termalization of a system made up of $N = 6000$ agents and $\alpha = 30$. Two dramatically different initial configurations are considered and compared: an ordered configuration with $s_i = 1$ for any i (black) and a random configuration with $s_i = 1$ ($s_i = -1$) with probability 1/2 (red).

properties of an equilibrium state. This is of course well-established for standard dynamics and it was also verified for our diffusive dynamics. An example is shown in Fig. 1 where, for a given (α, J), the specific value around which $m(t_n)$ eventually fluctuates does not depend on the choice of the initial configuration selected for the simulation. To this aim we plotted $m(s(t_n))$ obtained starting with a completely ordered configuration ($m_0 = 1$) and with a completely random one.

In the following we focus on systems of sufficiently large size so to discard effects related to small N. For a wide range of interaction constants J and average coordinations α, we measure the average magnetization $\langle m \rangle$ and the cost function $\langle e \rangle$, as well as the susceptibility χ, calculated as

$$\chi(\alpha, J) \equiv JN \left[\langle m^2 \rangle - \langle m \rangle^2 \right] . \tag{9}$$

This quantity measures, at equilibrium, the reactivity of the system to a small external perturbation. Moreover, we compare results obtained for our dynamics with those obtained through a well-established algorithm, i.e. the Glauber heath-bath with random updating,

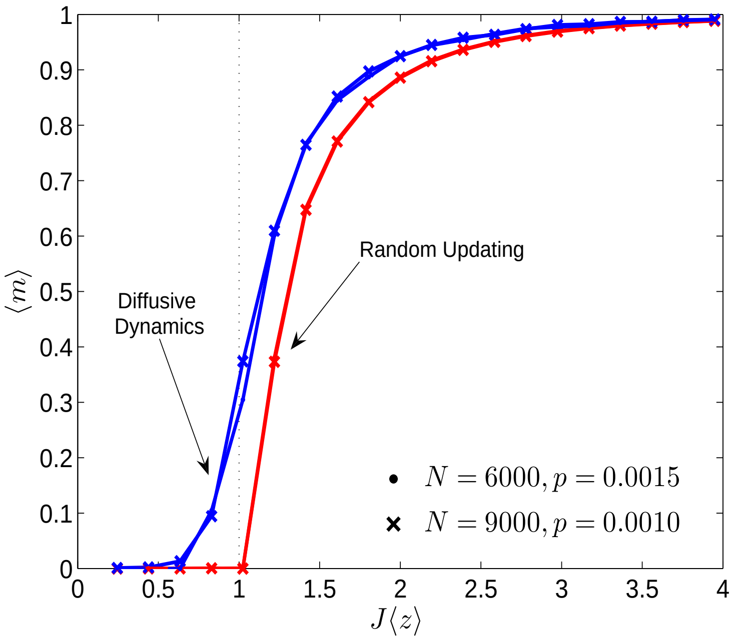

FIG. 2: Average opinion $\langle m \rangle$ for a population of $N = 6000$ ($\bullet$) and $N = 9000$ ($\times$) agents with $p = 0.0015$ and $p = 0.0010$, respectively; the average number of nearest-neighbours is therefore the same for both systems, $\alpha = 9$. Results obtained with a heath-bath dynamics ($\mathcal{R}$) and the strategic dynamics ($\mathcal{S}$) are compared: for the latter a smaller critical parameter J_c is found.

which is known to lead the system to a canonical steady state.

In Fig. 2 we show results pertaining to different system sizes ($N = 6000$, $N = 9000$), but keeping the average coordination number fixed ($\alpha = 9$ corresponding to $p = 0.0015$ and $p = 0.0010$, respectively). Their profiles display the typical behaviour expected for the Ising model on a random graph [8] and, consistently with the theory, highlight a phase transition at a well defined value $J_c^{\mathcal{S}}(\alpha)$. Otherwise stated, there exists a critical value of the parameter J below which the system is spontaneously ordered.

Note however that $J_c^{\mathcal{S}}(\alpha)$ is appreciably smaller than the critical value $J_c(\alpha) \approx 1/\alpha$ expected for the canonical Ising model defined on the ER random graph [8]. Remarkably, similar diffusive dynamics have been shown to lead an analogous increase of the critical interaction parameter on regular structures [13] and also for the spin-1 Ising model [14, 21, 22]. In these cases it was proved that a simple rescaling of the interaction constant J can not account for the differences between results produced by the diffusive dynamics and a heath-bath dynamics. This feature constitutes a first signature of the fact that the equilibria

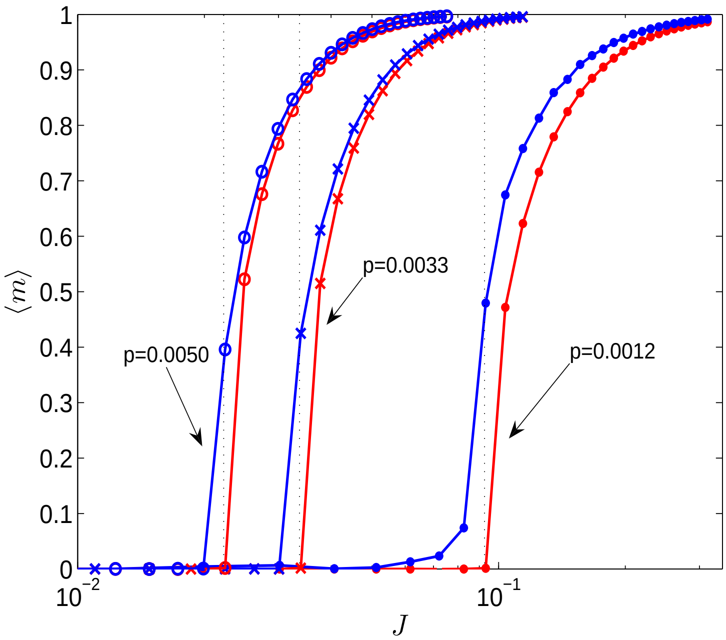

FIG. 3: Average opinion $\langle m \rangle$ for a population of $N = 9000$ agents with $\alpha = 11$ ($\bullet$), $\alpha = 30$ ($\times$) and $\alpha = 45$ (open circles). Results obtained with a standard dynamics ($\mathcal{R}$) and the strategic dynamics ($\mathcal{S}$) are compared: for the latter a smaller critical parameter J_c is found.

generated by the diffusive dynamics are not governed by the Boltzmann distribution.

As mentioned above, the critical value J_c depends on the system size and on the probability p, through their product α, i.e. $J_c \approx 1/\alpha$. In order to check if a similar behaviour also holds for the dynamics $\mathcal{S}$ we now fix the size of the system and make α vary; results for $N = 9000$ with $\alpha = 11, 30, 45$ are reported in Fig. 3. Indeed also for $J_c^{\mathcal{S}}$ we evidence a monotonic increase with α, however, in order to establish the actual relation between J and the set of parameters p, α further extensive simulations are necessary.

Similar to what happens with the usual dynamics the relaxation time needed to drive the system sufficiently close to the equilibrium situation is found to depend on the parameter J. More precisely, we experience the so called critical slowing down: the closer J to its critical value, the longer the relaxation time.

We now turn to the susceptibility defined in Eq. 9; results are shown in Fig. 4. In the thermodynamic limit, at a critical point J_c, the susceptibility diverges, while for finite sizes the susceptibility is expected to display a peak at J_c. This kind of behaviour is found also when the diffusive dynamics is applied and χ just peaks at $J_c^{\mathcal{S}}$. An important point is that

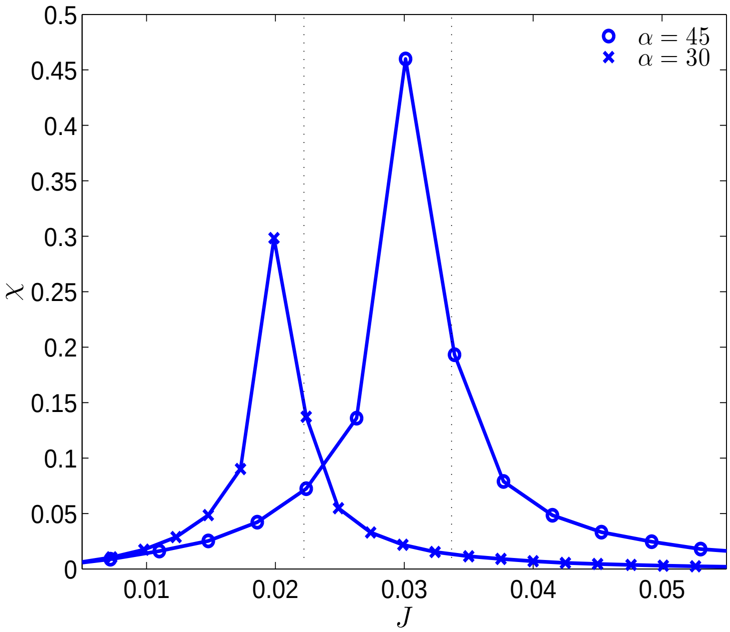

FIG. 4: Average susceptibility χ for a population of $N = 9000$ agents with $\alpha = 30$ ($\times$) and $\alpha = 45$ (o) obtained with the strategic dynamics. Notice that the function peaks at a value smaller than the critical J_c expected for standard dynamics (dotted lines).

the shape of the curve is not modified, indicating that the reaction of the system to an external perturbation is conserved, with respect to the usual equilibrium, in the vicinity of the critical point. The social system is therefore expected to behave in the same way. Hence, we have further evidence of the fact that the diffusive strategic dynamics recovers the phase transition typical of the Ising model, though providing a lower value for the critical interaction parameter.

V. CONCLUSIONS AND PERSPECTIVES

In this paper, we have introduced a dynamics for social systems displaying diffusive and strategic character. This dynamics has been shown to relax the system to thermodynamically well-behaved steady states. This means that, after a suitable time, the values of the global observables of the system display a well-defined value independently of the initial conditions (which can, at least, affect the orientation of the asymptotic arrangement). The magnetization, representing the average trait reached by the system as a function of the interaction parameter J, features a transition at a value of J which is strictly lower than the one obtained with a non-strategic random choice for the opinion flips. The shape of the susceptibility near the transition point is conserved, indicating that the reaction of the social system to a small external perturbation in the stationary state is not modified.

This picture indicates that with a strategic (local) choice in opinion flips a full-consensus configuration is obtained for lower values of the interaction parameter J, namely it is "easier" to obtain a community with an oriented opinion. Differently stated, in a society where a given value of the interaction J is present, the number of people with an oriented opinion is higher if the equilibrium is reached by a strategic opinion flip.

We expect this results to hold also in the case of different underlying topologies, more suitable to describe real social networks, such as scale free random networks with hubs and small world features [9]. Indeed, our dynamics allows a number of generalizations concerning, for instance, the topology describing the social network, the number of possible cultural traits or competitive companies admitted (namely the magnitude of the spin variable) [14], the possible presence of an external magnetic field (representing the effect of external biases such as advertisement) or a more complicated set of constant parameters J_{ij}. In particular, $\mathbf{J}$ could be a directed, block matrix and this would account for different fares inter and intra distinct groups; in this case it would be very interesting to understand the conditions, in terms of $\mathbf{J}$ elements, for the realization of an oriented (i.e. magnetized) system.

Acknowledgment

The authors are grateful to Adriano Barra for interesting suggestions and stimulating discussion. EA thanks the Italian Foundation "Angelo della Riccia" for financial support.

---

[1] Galam S and Moscovici S, 1991 *European Journal of Social Psychology* **21** 49

[2] Borghesi C and Bouchaud J P, 2007 *Quality and Quantity* **41** 557

[3] Gallo I and Contucci P, 2008 *Math-Ph. Electr. Journ.* **14**

[4] Castellano C Vilone D and Vespignani A, 2003 *Europhysics Letters* **63** 153

[5] Coolen A C C, 2005 *The Mathematical Theory of Minority Games* Oxford Finance Series

[6] Newman M E J, Watts D J and Strogatz S H, 2002 *PNAS* **99** 2566

[7] Bovier A and Gayrard V, 1993 *J. Stat. Phys.* **72** 643

[8] Agliari E, Barra A and Camboni F, 2008 *J. Stat. Mech.* 10003

[9] Albert R and Barabasi A L, 2002 *Rev. Mod. Phys.* **74** 47

[10] Watts D J and Strogatz S H, 1998 *Nature* **393** 409

[11] Newman M E J, Watts D J, Barabasi A L, 2006 *The Structure and Dynamics of Networks*, Princeton University Press

[12] Caldarelli G and Vespignani A, 2007 *Large scale structure and dynamics of complex networks*, World Scientific Publishing

[13] Buonsante P, Burioni R, Cassi D and Vezzani A, 2002 *Phys. Rev. E* **66** 036121

[14] Agliari E, Burioni R, Cassi D and Vezzani V, 2005 *Eur. Phys. J. B* **46** 109

[15] Agliari E, Burioni R and Contucci P, in preparation.

[16] Erdös P and Renyi A, 1959 *Publications Mathematicae* **6** 290

[17] Contucci P and Ghirlanda S, 2007 *Quality and Quantity* **41** 569

[18] Brush S G, 1967 *Rev. Mod. Phys.* **39** 883

[19] Newman M E J and Barkema G T, 2001 *Monte Carlo methods in Statistical Physics*, Oxford University Press

[20] Szalma F and Iglói F, 1999 *J. Stat. Phys.* **95** 763

[21] Agliari E, Burioni R, Cassi D and Vezzani A, 2006 *Eur. Phys. J. B* **49** 119

[22] Agliari E, Burioni R, Cassi D and Vezzani A, 2006 *Eur. Phys. J. B* **48** 529