Asymptotic expansion of the solutions of an inverse problem

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We consider the abstract parabolic inverse problem

$$\begin{cases} u'(t) = A(t)u(t) + f(t)z(t) + h(t), & t \in [0, +\infty), \\ u(0) = u_0, & \\ \Phi(u(t)) = g(t), & t \in [0, +\infty), \end{cases} \tag{1}$$

with u and f unknown. We assume that, $\forall t \in [0, +\infty)$, $A(t)$ is a sectorial operator in the Banach space X, z and h are functions with values in X, and f is an unknown scalar-valued function, Φ is a proper linear functional in X, g is a givn scalar valued function. The knowledge of $\Phi(u(t))$ should provide the further information, which is necessary to determine f together with u.

We show that, under suitable assumption on the data $A(t)$, z, h, u_0, g, (1) has a unique global solution in $[0, +\infty)$. Moreover, under further conditions, if $A(t) = A_0 + t^{-1}A_1 + ... + t^{-k}A_k + o(t^{-k})$ $(t \to +\infty)$, and z, h and g admit analogous expansions, even u and f can be expanded in the same way.