INVERSE AND FREE BOUNDARY PROBLEM
FOR THE STRONGLY DEGENERATE HEAT EQUATION

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In the domain $Q_T \equiv \{(x, t) : 0 < x < h(t), t \in [0, T]\}$ with unknown part of boundary $x = h(t)$ we consider the inverse problem for the heat equation

$$u_t = a(t)u_{xx} + f(x, t)$$

with unknown coefficient $a(t) > 0, t \in (0, T]$ which vanishes at the point $t = 0$. Besides the initial condition

$$u(x, 0) = \varphi(x), \quad 0 \leq x \leq h(0)$$

and boundary conditions

$$u(0, t) = \mu_1(t), \quad u(h(t), t) = \mu_2(t), \quad t \in [0, T],$$

we imply two overdetermination conditions

$$a(t)u_x(0, t) = \mu_3(t), \quad t \in [0, T], \quad \int_0^{h(t)} u(x, t)dx = \mu_4(t), \quad t \in [0, T].$$

Under some assumptions we establish existence and uniqueness of solution $(a(t), h(t), u(x, t))$ of the problem (1)-(5) from the space $C[0, T] \times C^1[0, T] \times C^{\alpha,1}(Q_T) \cap C(\bar{Q_T})$ such that $a(t)$ tends to zero as a power $t^\beta, \beta \geq 1$ when $t \to 0 +$. 

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