INVERSE AND FREE BOUNDARY PROBLEM FOR THE STRONGLY DEGENERATE HEAT EQUATION

Nadiya Hryntsiv, Mykola Ivanchov

Department of Mechanics and Mathematics, Ivan Franko National University of Lviv,
1, Universytetska st., 79602 Lviv, Ukraine
E-mail: ivanchov @ franko.lviv.ua

In the domain $Q_T \equiv \{(x,t) : 0 < x < h(t), t \in [0,T]\}$ with unknown part of boundary $x = h(t)$ we consider the inverse problem for the heat equation

$$u_t = a(t)u_{xx} + f(x,t) \tag{1}$$

with unknown coefficient $a(t) > 0, t \in (0,T]$ which vanishes at the point $t = 0$. Besides the initial condition

$$u(x,0) = \varphi(x), \quad 0 \leq x \leq h(0) \tag{2}$$

and boundary conditions

$$u(0,t) = \mu_1(t), \quad u(h(t),t) = \mu_2(t), \quad t \in [0,T], \tag{3}$$

we imply two overdetermination conditions

$$a(t)u_x(0,t) = \mu_3(t), \quad t \in [0,T], \tag{4}$$

$$\int\limits_0^{h(t)} u(x,t)dx = \mu_4(t), \quad t \in [0,T]. \tag{5}$$

Under some assumptions we establish existence and uniqueness of solution $(a(t), h(t), u(x,t))$ of the problem (1)-(5) from the space $C[0,T] \times C^1[0,T] \times C^{2,1}(Q_T) \cap C(\overline{Q}_T)$ such that $a(t)$ tends to zero as a power $t^\beta, \beta \geq 1$ when $t \to 0+$.